Parallel simulations of soft-tissue using an adaptive quadtree/octree implicit boundary finite element method

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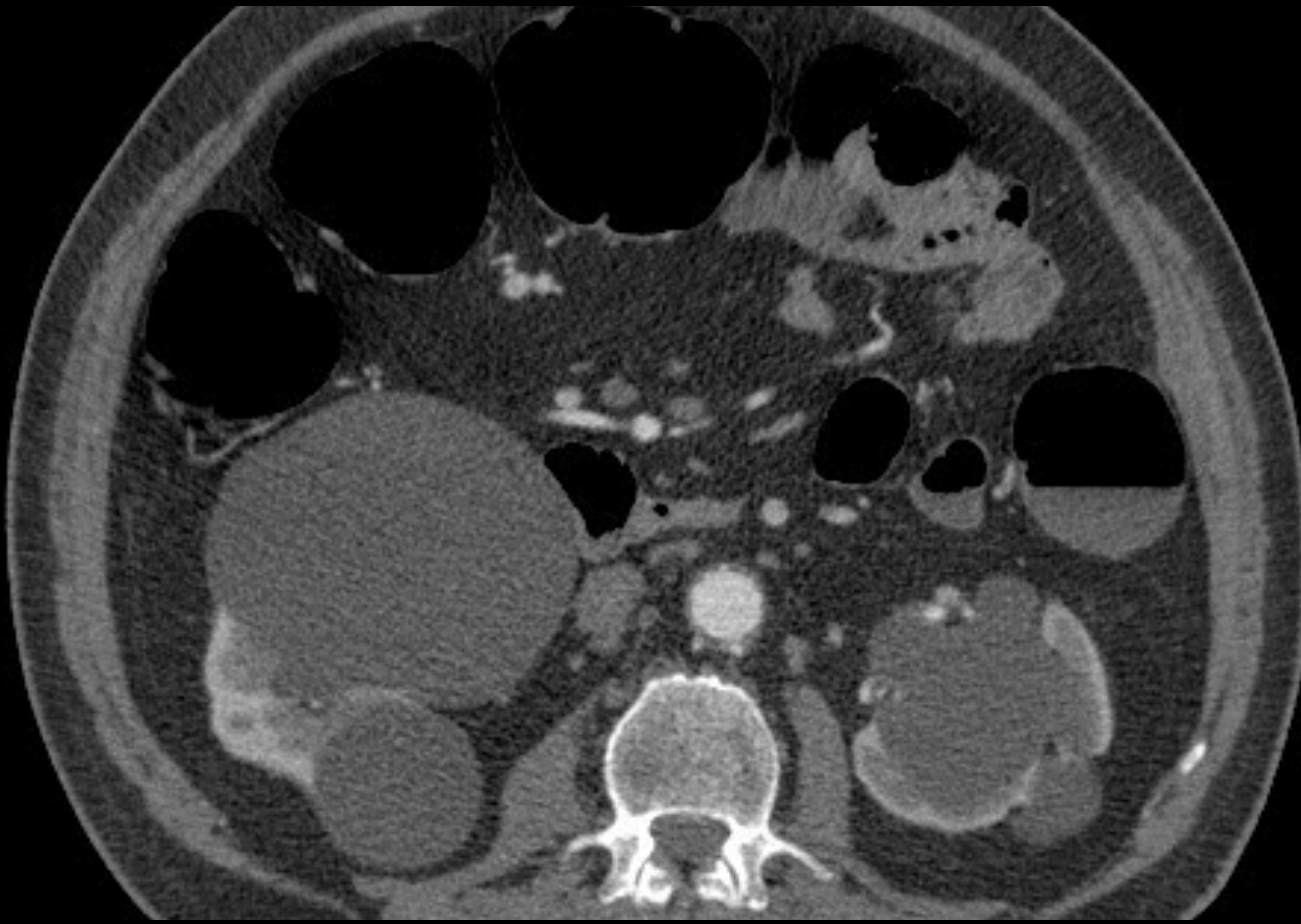
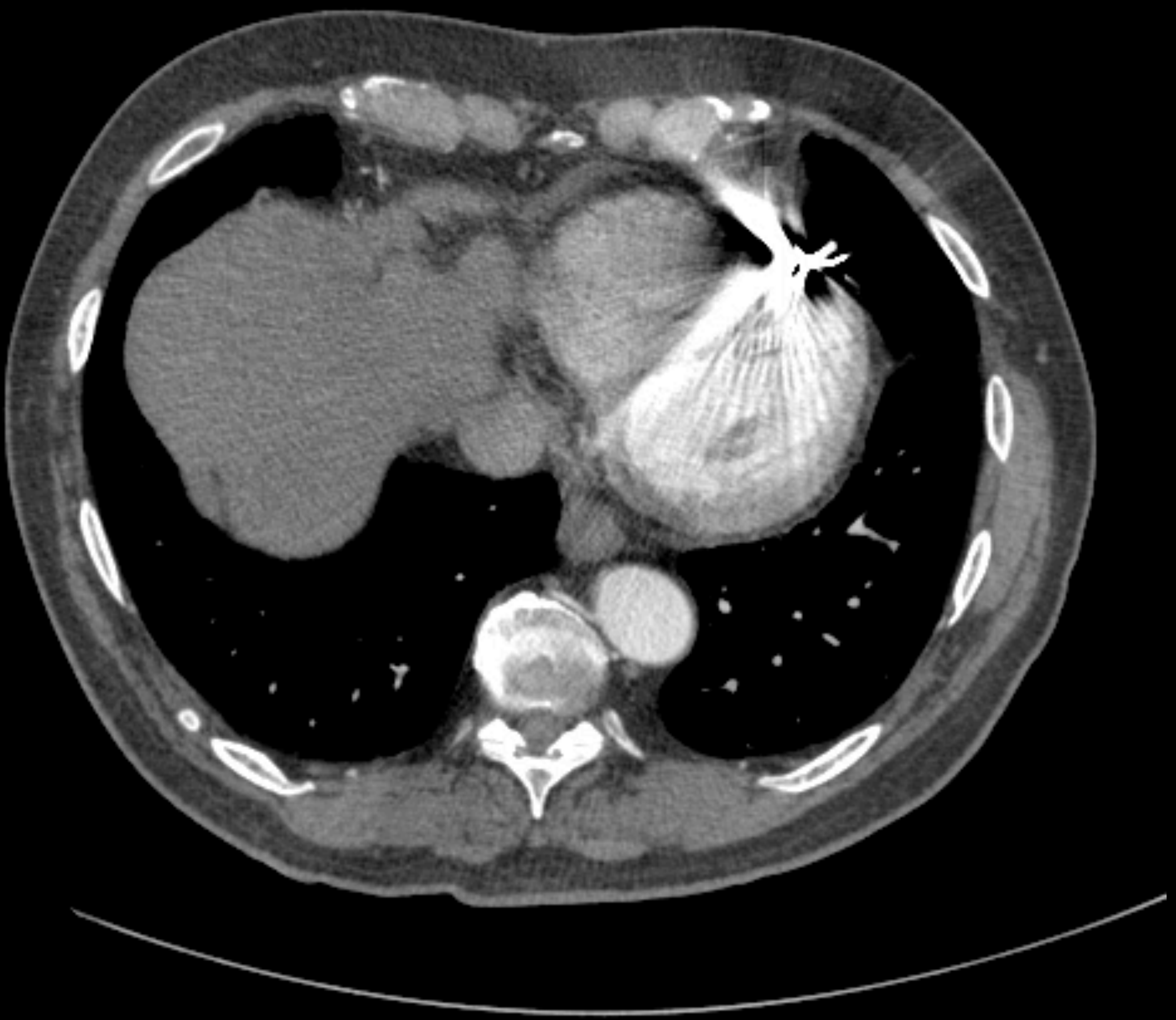
How can we move from an image…



Source: *COLONIX, OSIRIX*

…or a series of images…



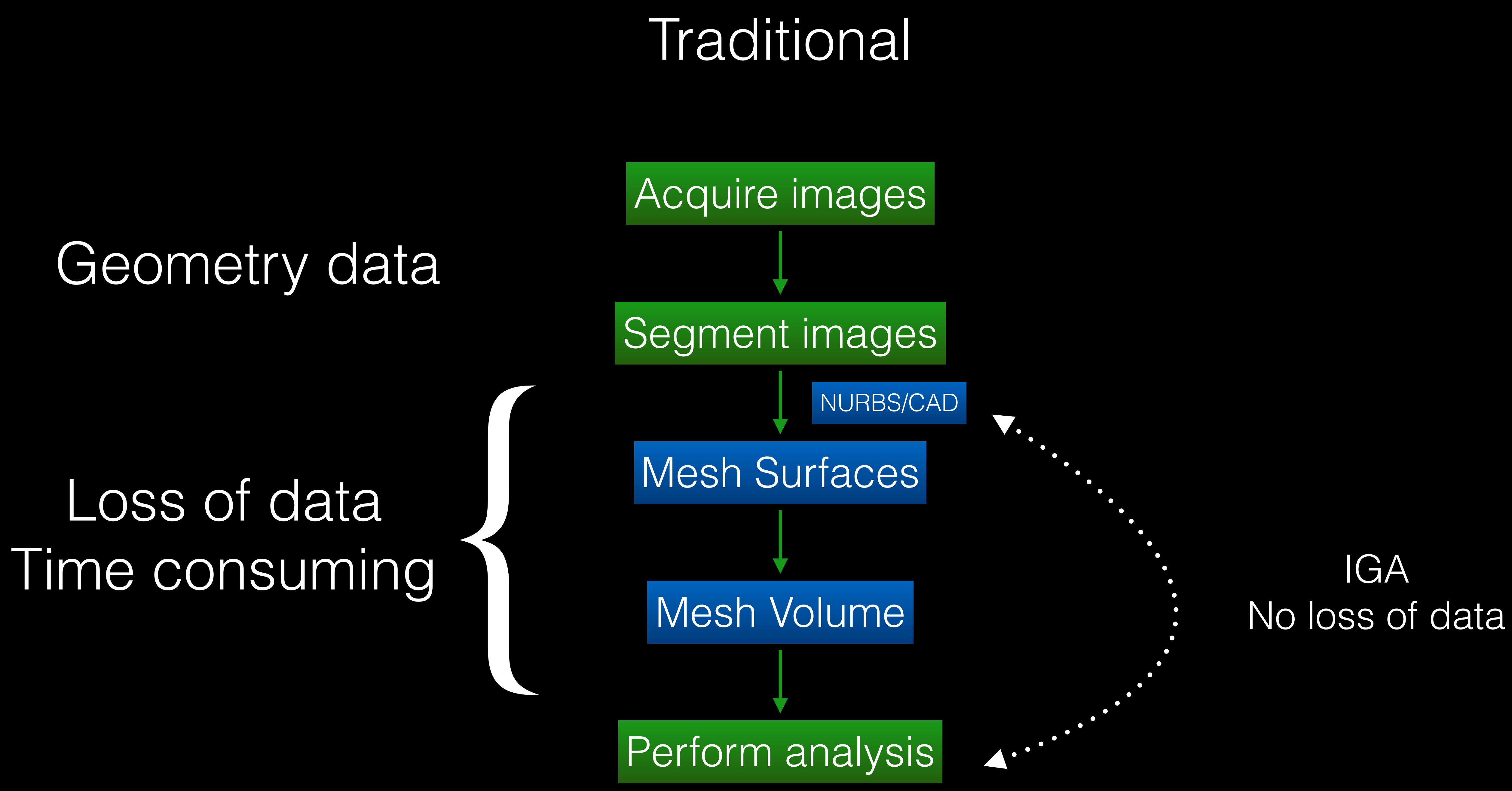
Source: *COLONIX, OSIRIX*

to a full mechanical analysis?

…with the following constraints

- The developed methods should be usable in a day to day *clinical* environment.
- What does that mean?
  - Ease of use - hospital technician, surgeon.
  - Sit comfortably within an *existing* image segmentation and analysis pipeline.
  - Timeframe for results in seconds to one hour, not weeks.
    - CPU time - increasingly cheap.
    - User time - increasingly expensive.
  - Guaranteed results.
    - Adaptivity, error estimation (ongoing work with Pierre Kerfriden).

Pipeline to analysis



Current methods in Medical Image Segmentation, Pham et al. 2000, *Annu Rev. Biomed. Eng.*

Each voxel j is a 32-bit floating point measurement

z

Planes (64)

x

Rows (64)

y

Cols (64)

Image: CGAL Project

Current methods in Medical Image Segmentation, Pham et al. 2000, *Annu Rev. Biomed. Eng.*

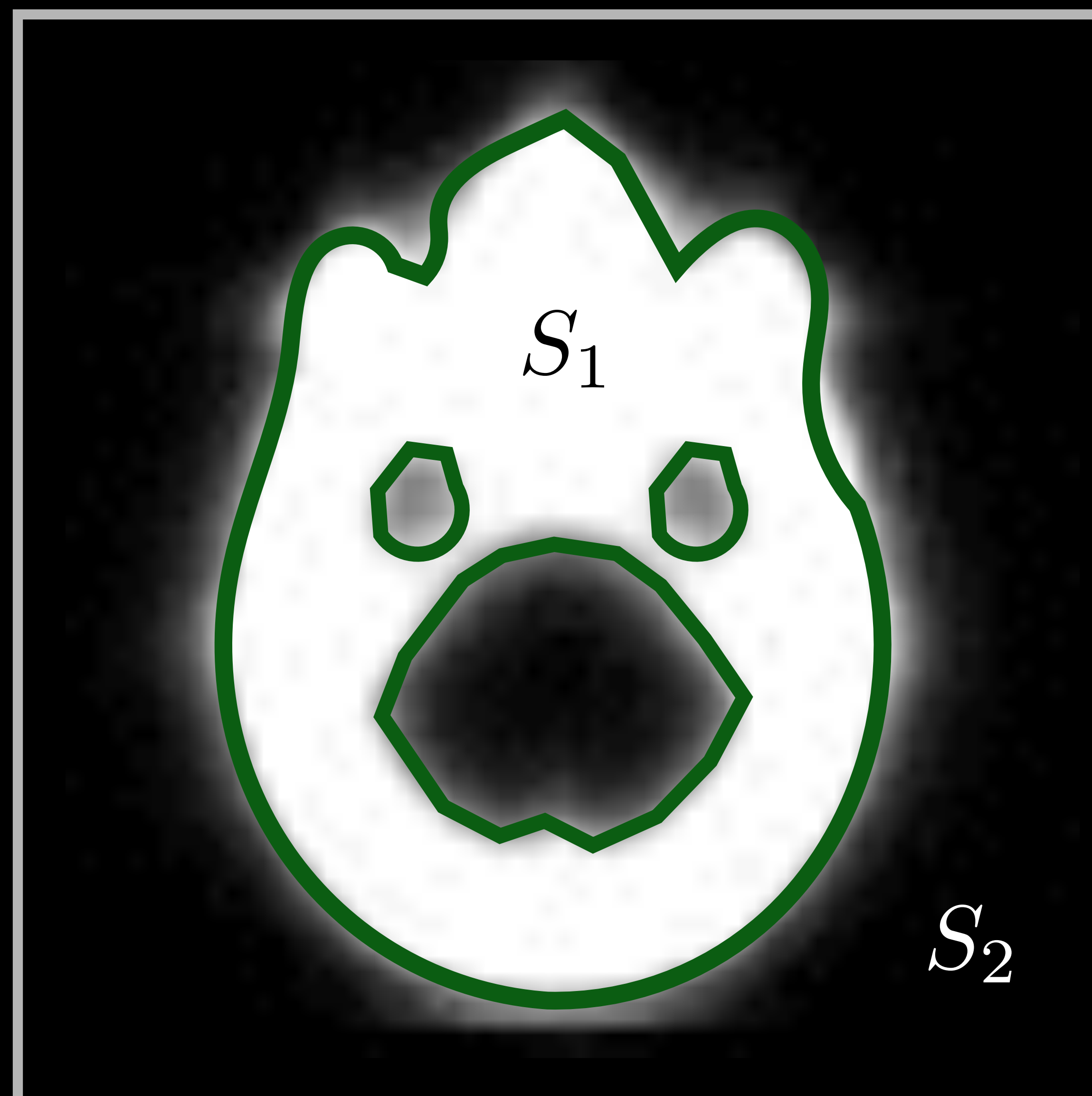
Soft or 'fuzzy' segmentation

$$0 < m_k(j) < 1 \quad \forall j,\, k$$

$$\sum_{k=1}^{K} m_k(j) = 1 \quad \forall j$$

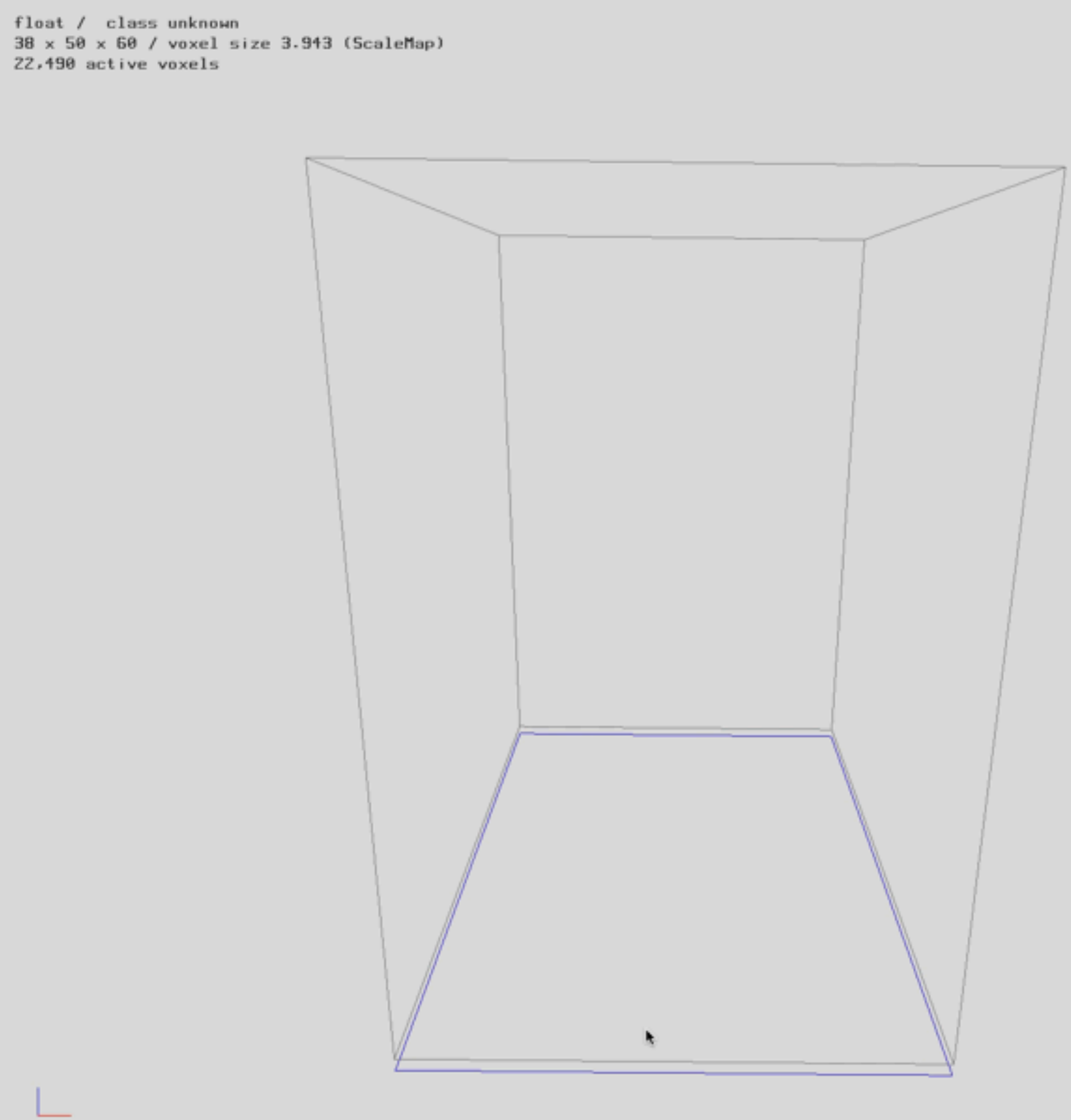
Current methods in Medical Image Segmentation, Pham et al. 2000, *Annu Rev. Biomed. Eng.*

Hard segmentation



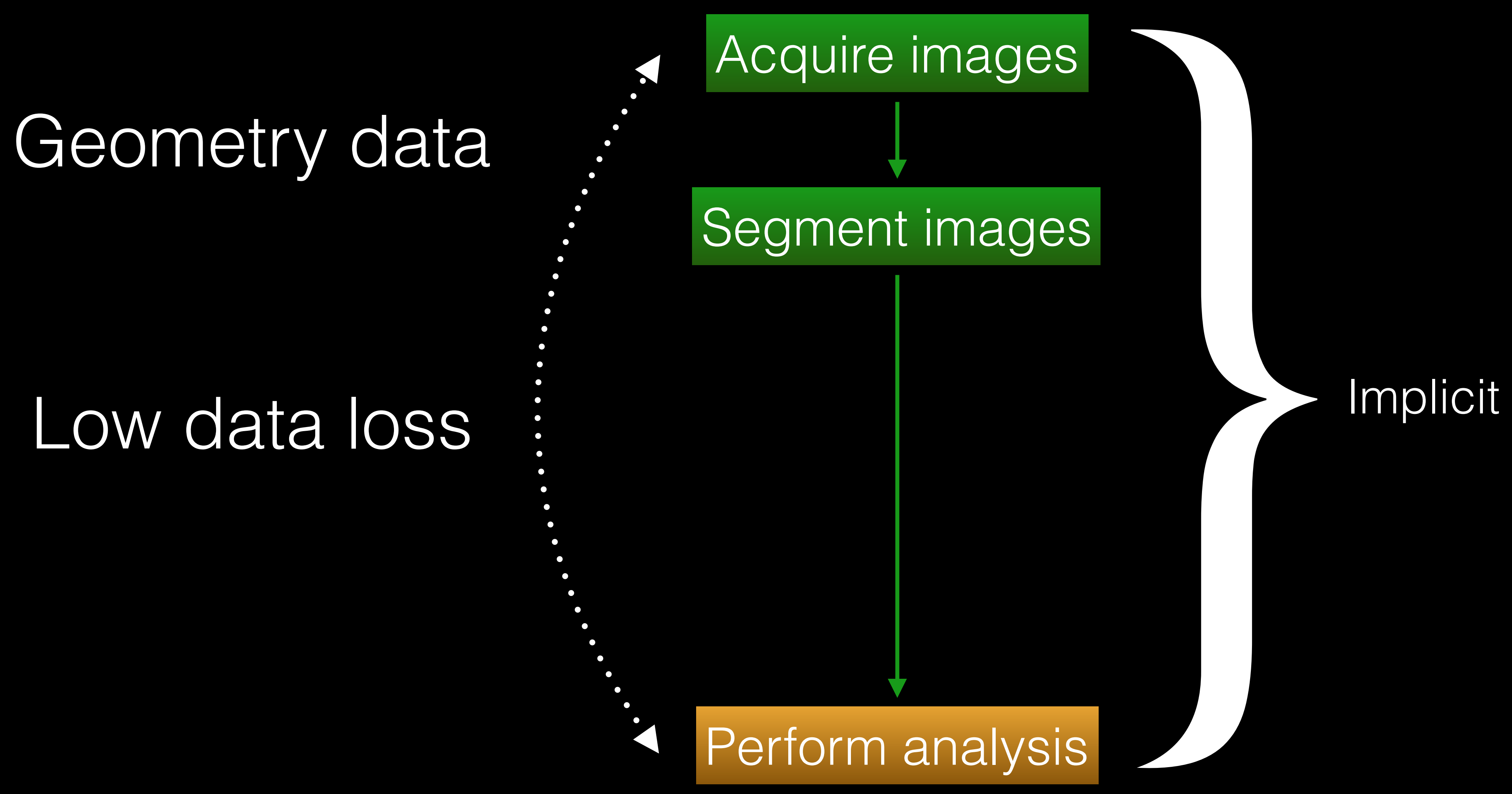
$$\Omega = \bigcup_{k=1}^{K} S_k \qquad S_k \cap S_j = \varnothing \quad \forall k \neq j$$

Hard Segmentation at 0.2f, OpenVDB and ITK



Pipelines to analysis

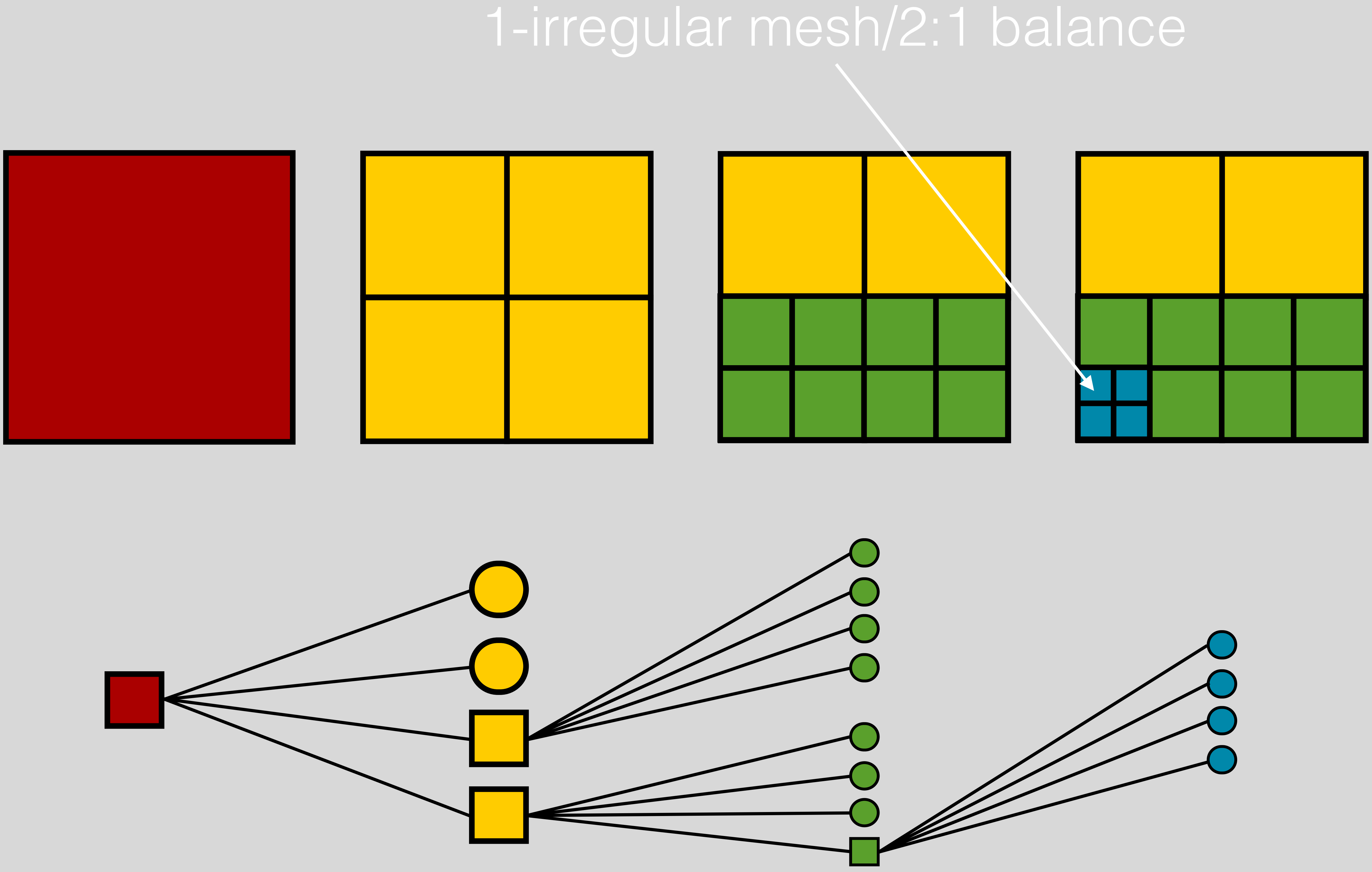
Adaptive Implicit Boundary Pipeline



Influences

- **Uncoupling discretisation and approximation.**
  - Bordas et al. - *Geometry-Independent Field Theory.*
  - Legrain et al. - *Double Nested Quadtree.*
- **PDEs on octrees.**
  - Museth et al. **-** *OpenVDB Library.*
  - Bangerth et al. **-** *deal.ii Library.*
- **Numerical methods for softly segmented medical images.**
  - J. J. Ródenas et al. **-** *Adaptive Quadtree.*
  - Miller et al. - *Adaptive Meshfree.*
- **Implicit boundary/immersed boundary/XFEM etc.**
  - Belytschko et al. **-** *XFEM.*
  - Hansbo and Burman **-** *Ghost penalty methods.*
  - Peskin et al. - *Immersed boundary methods.*
    - *And many, many others…*

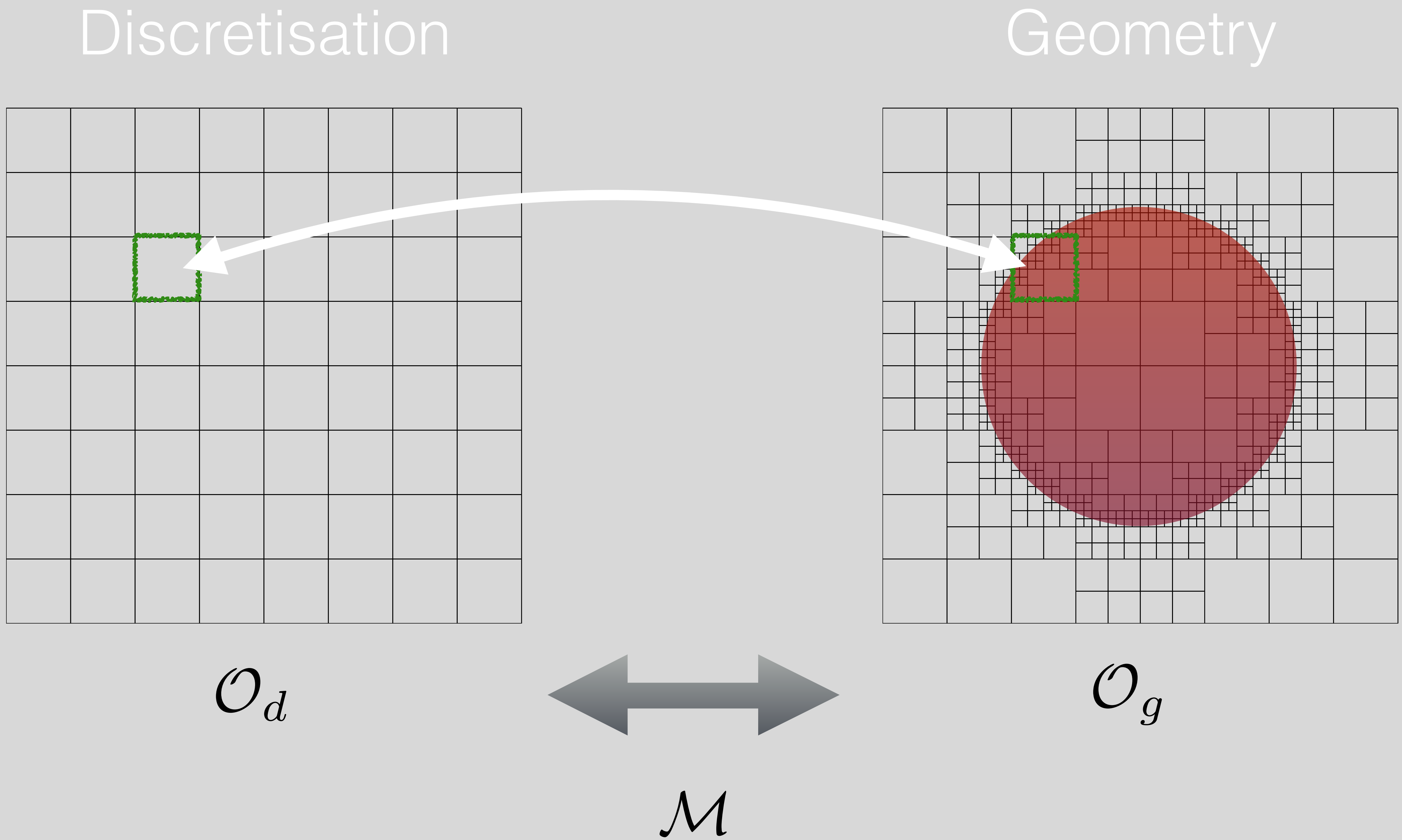
The method



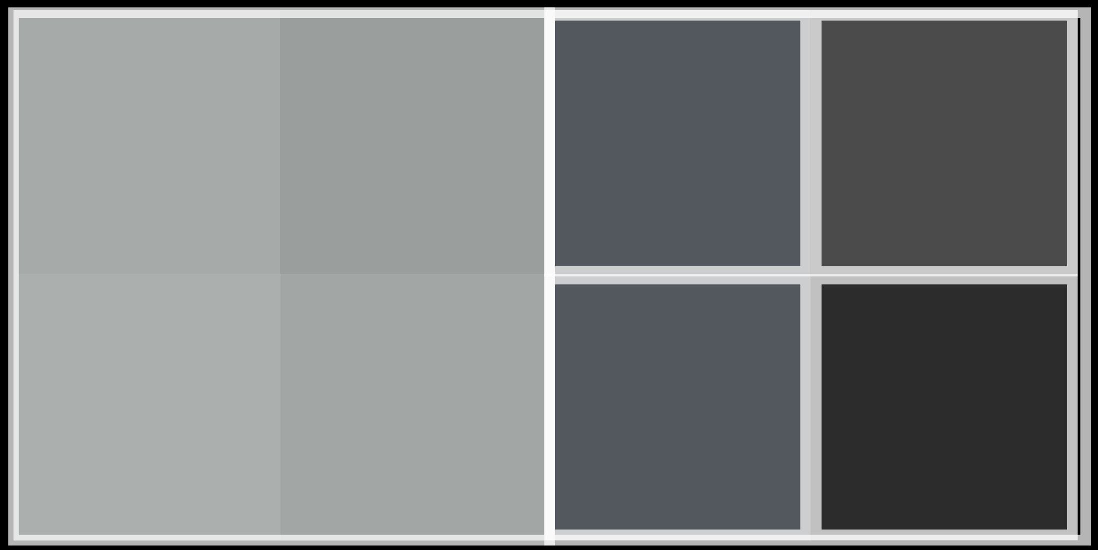
Octree or Quadtree data structure

Uncoupling discretisation and geometry

Nested Octree

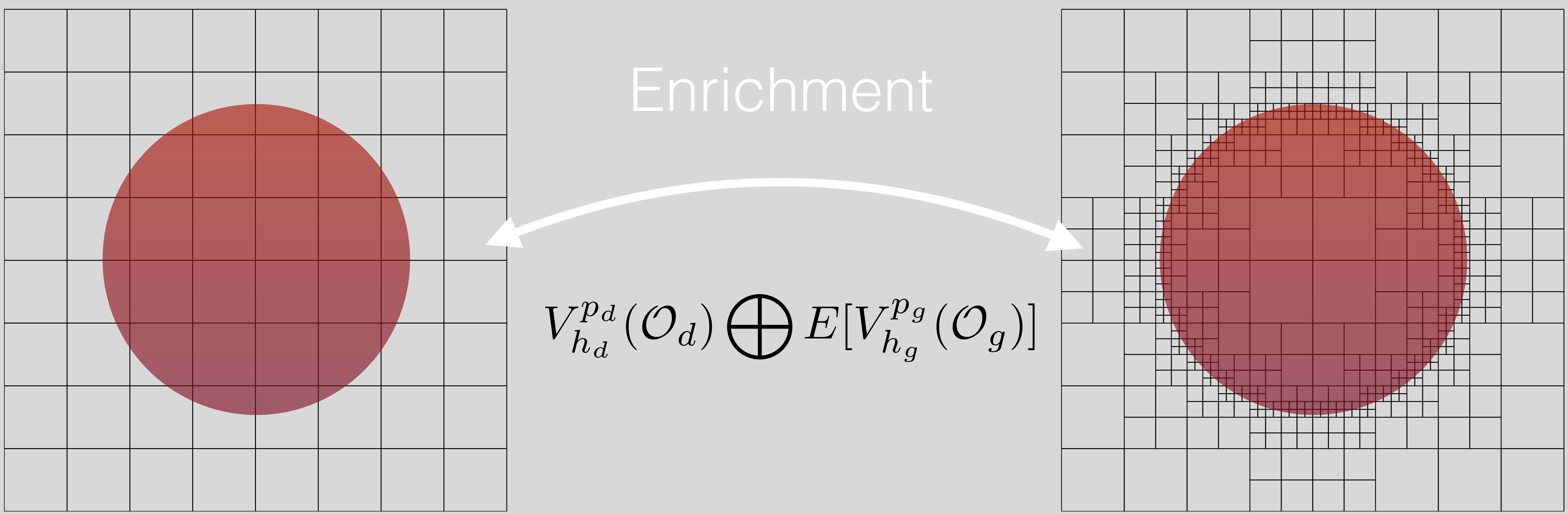


Strategy 1: Soft or 'fuzzy' segmentation



Fast, automatic. But no so good for contact mechanics and setting boundary conditions…

Strategy 2: Hard Segmentation



$V_{h_d}^{p_d}(\mathcal{O}_d)$

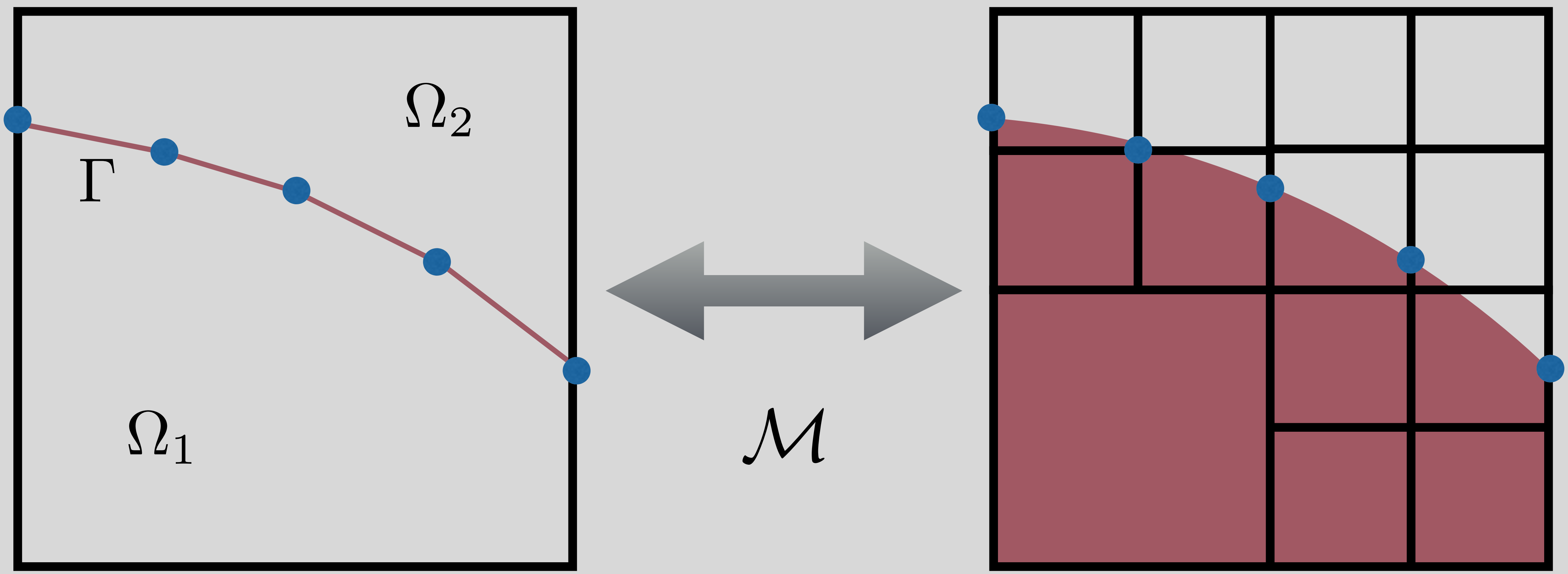
$p_d > p_g$

$V_{h_g}^{p_g}(\mathcal{O}_g)$

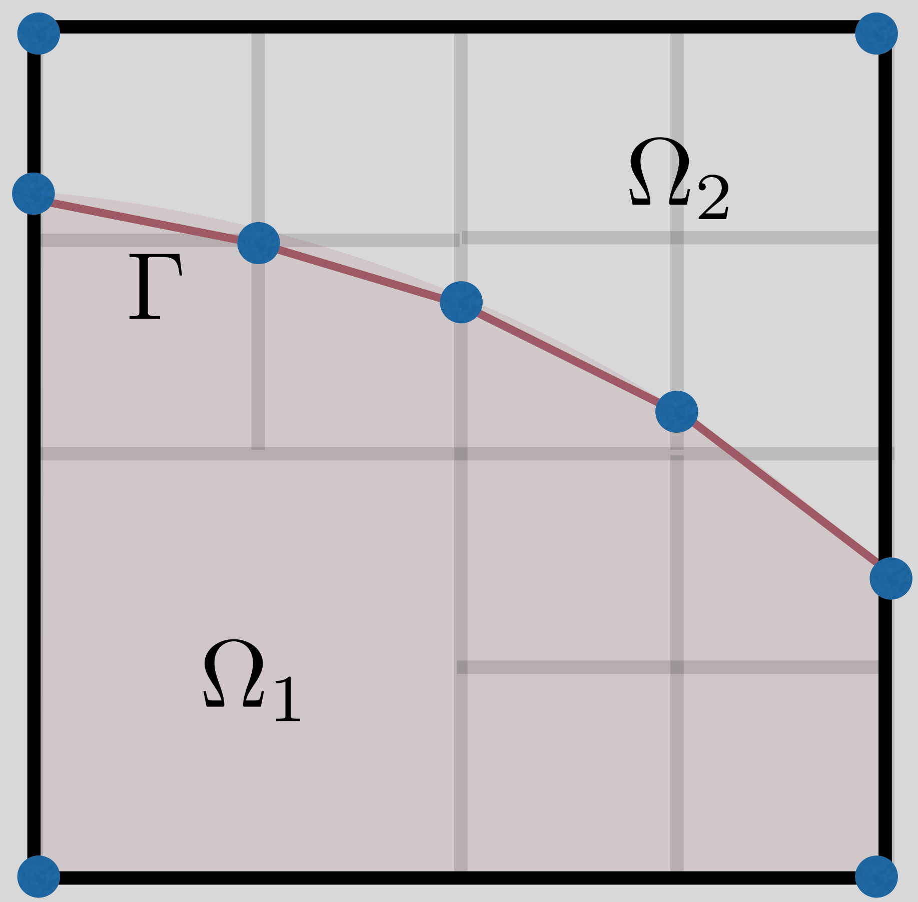
$h_d > h_g$

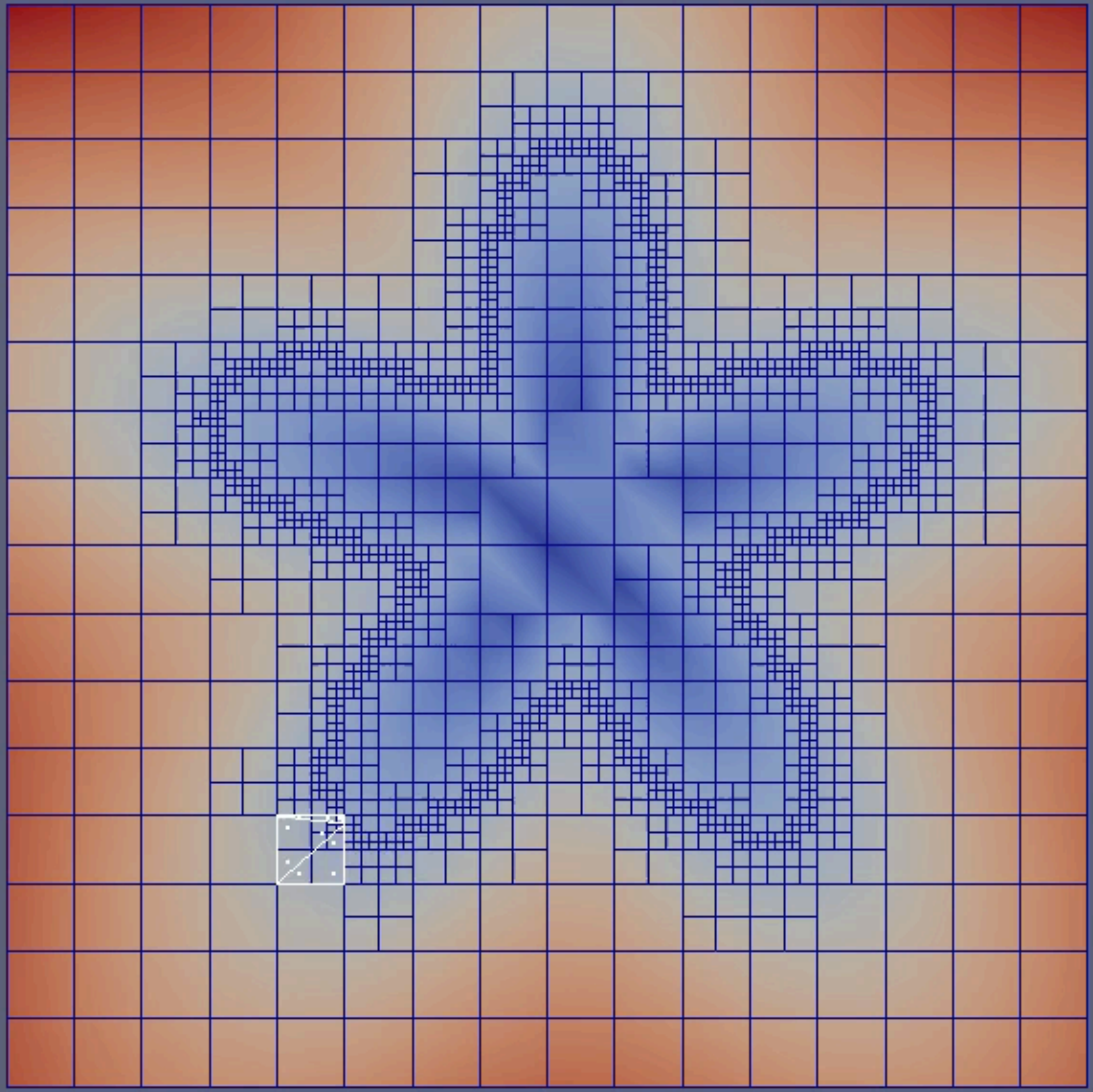
$p_g = 1$

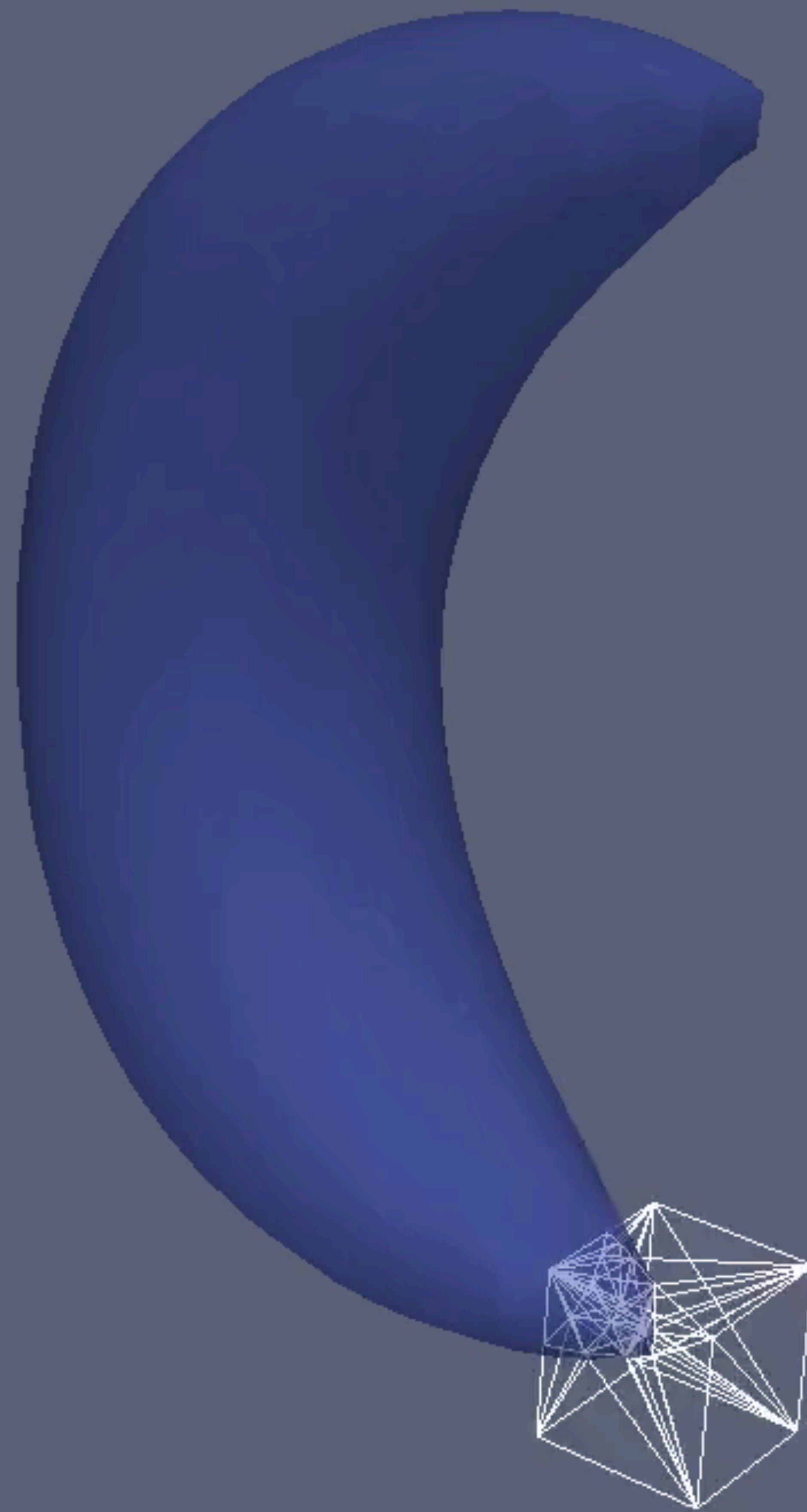
For each enriched cell in the discretisation…

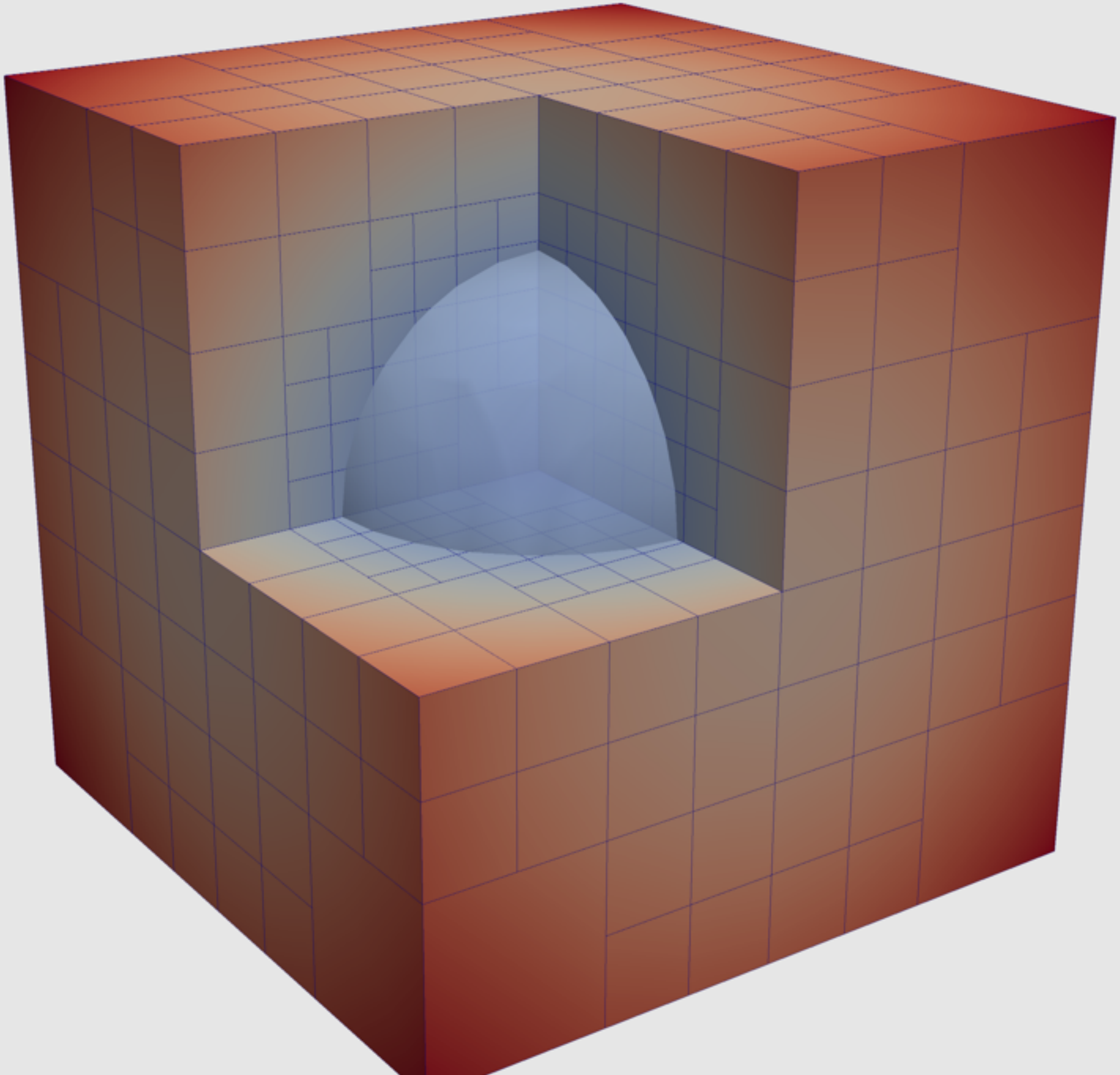


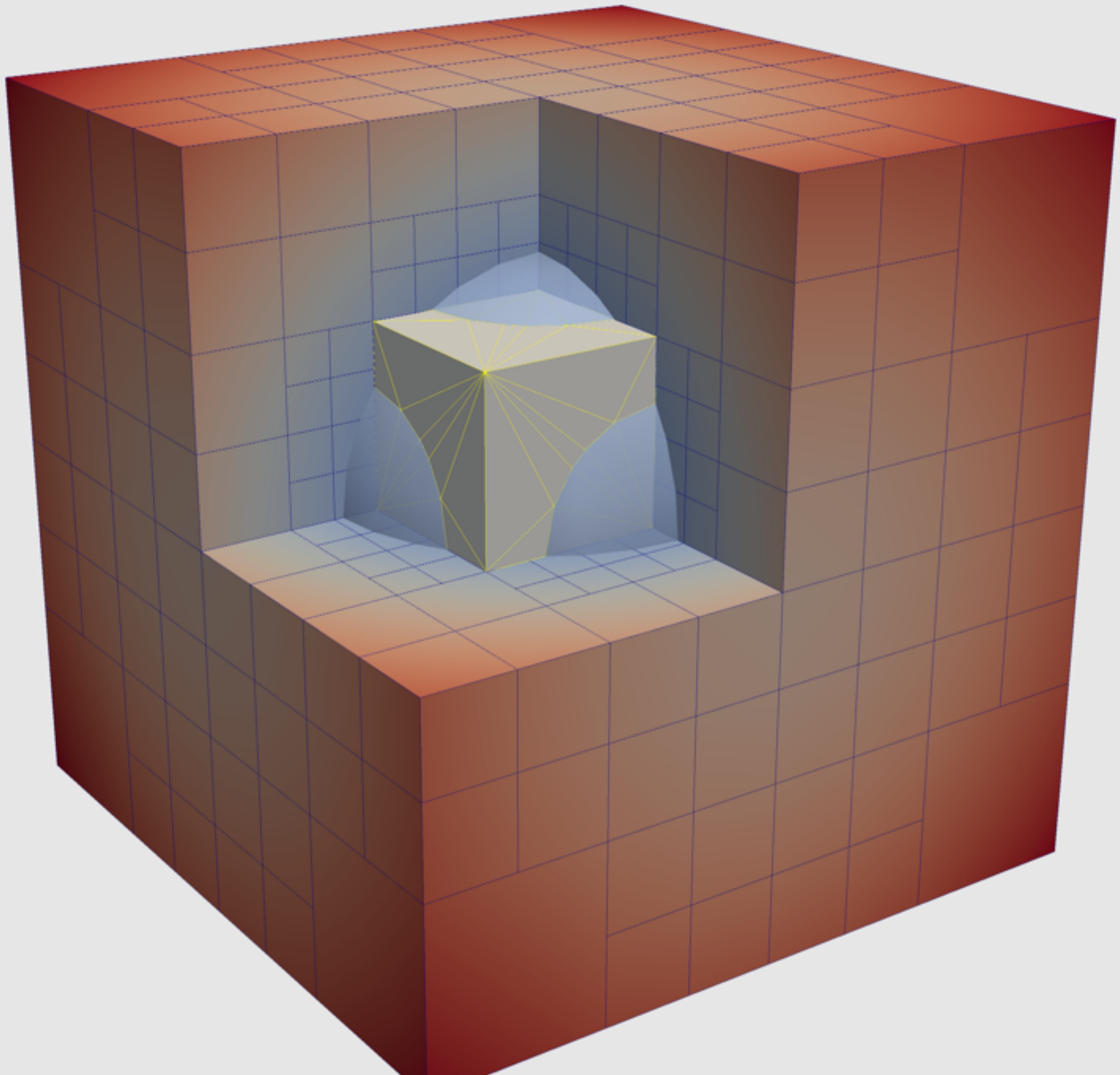
generate local Delaunay triangulation…

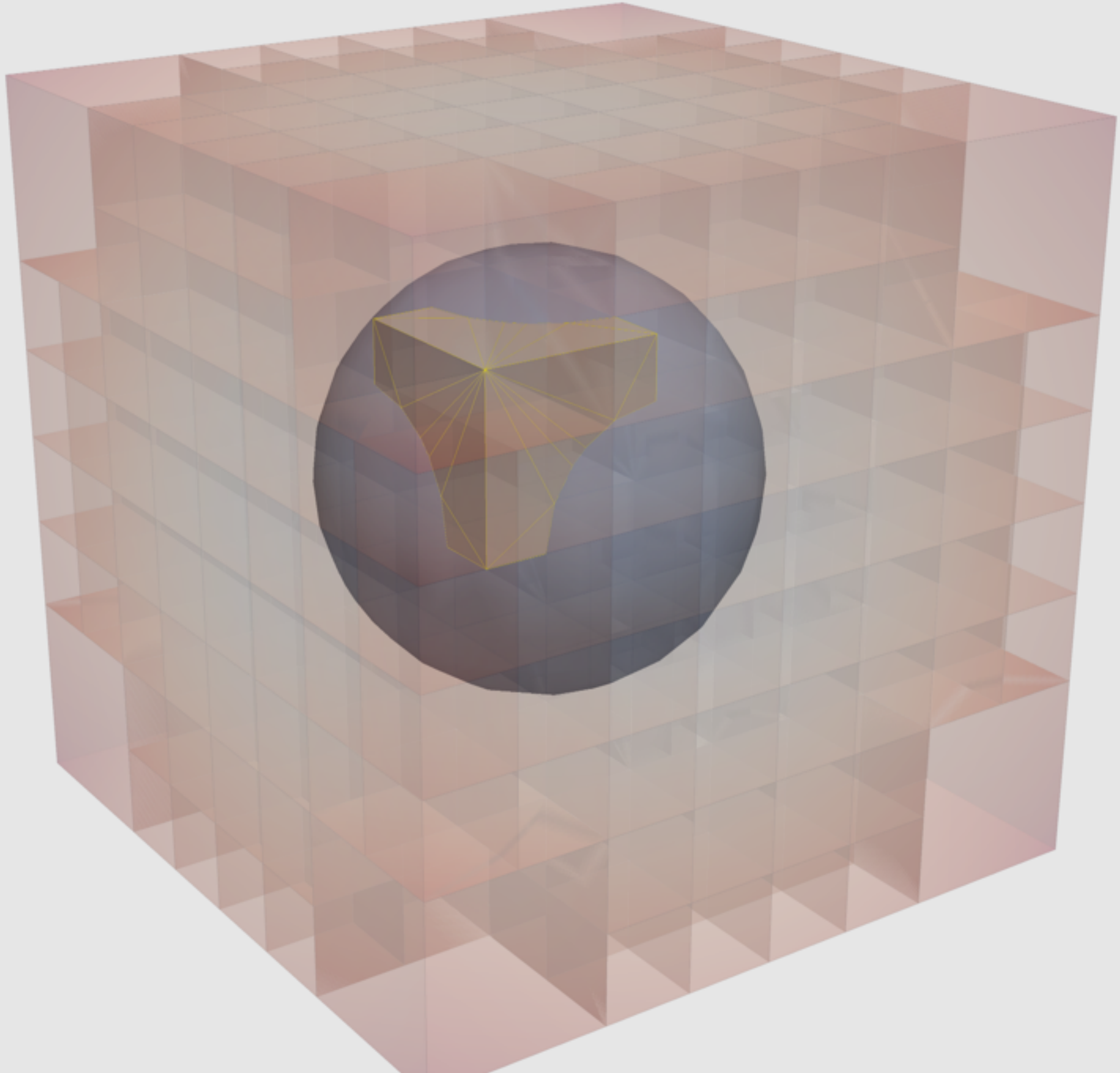




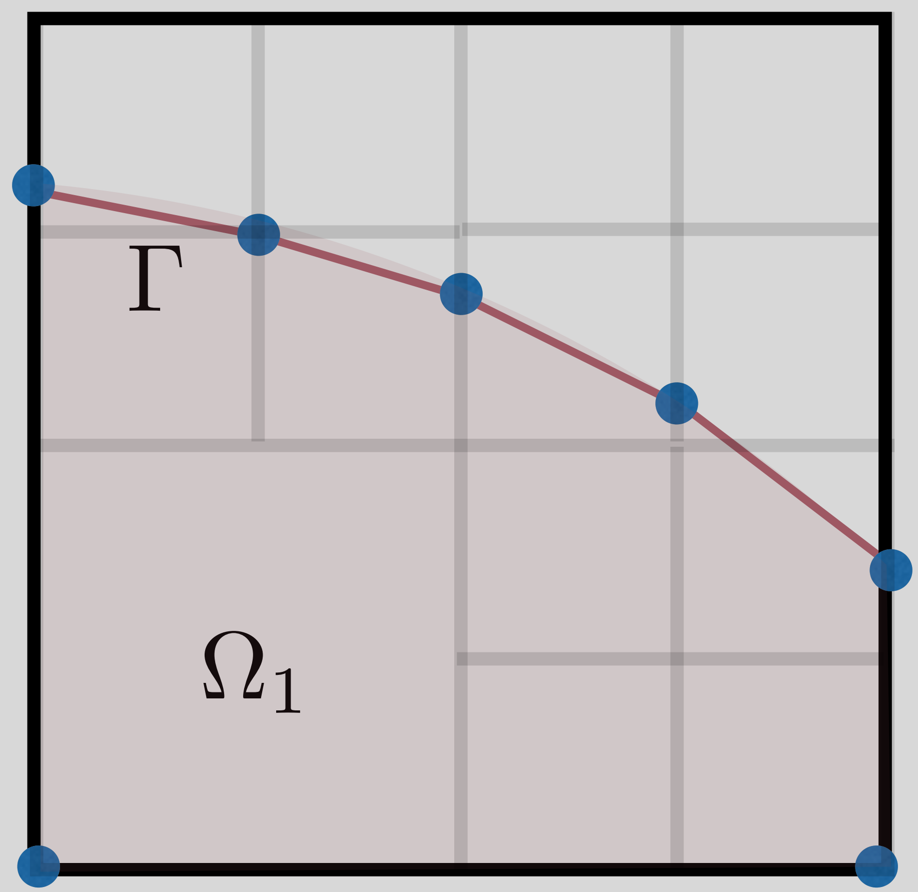




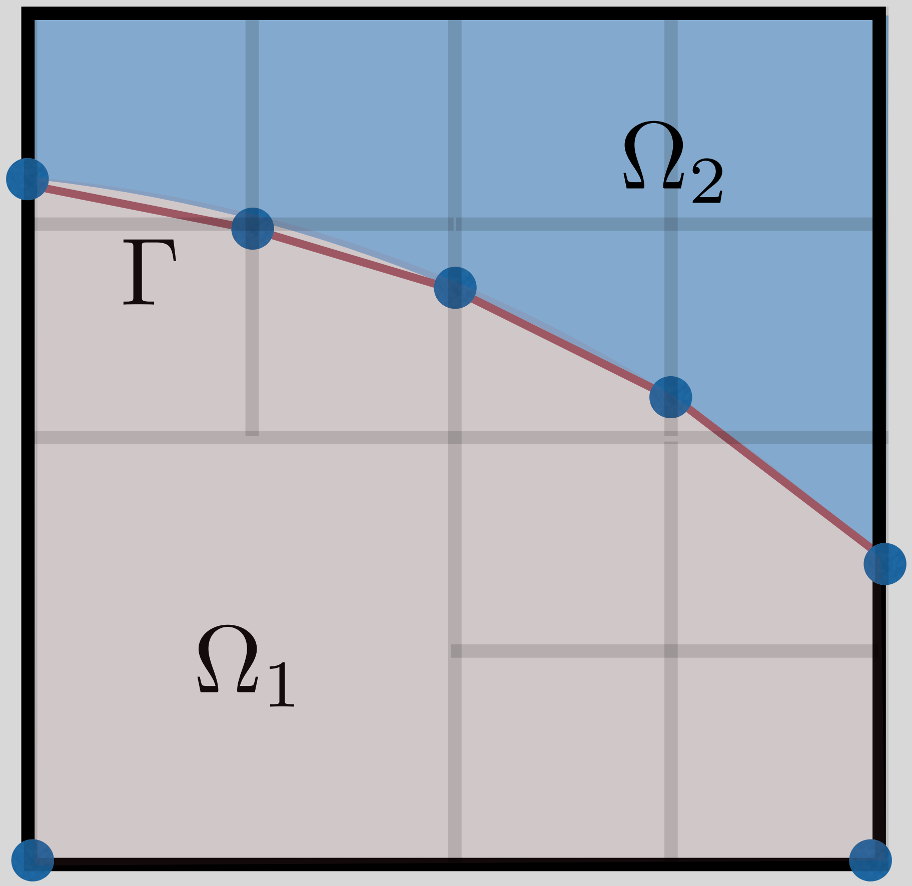




Case 1: boundary



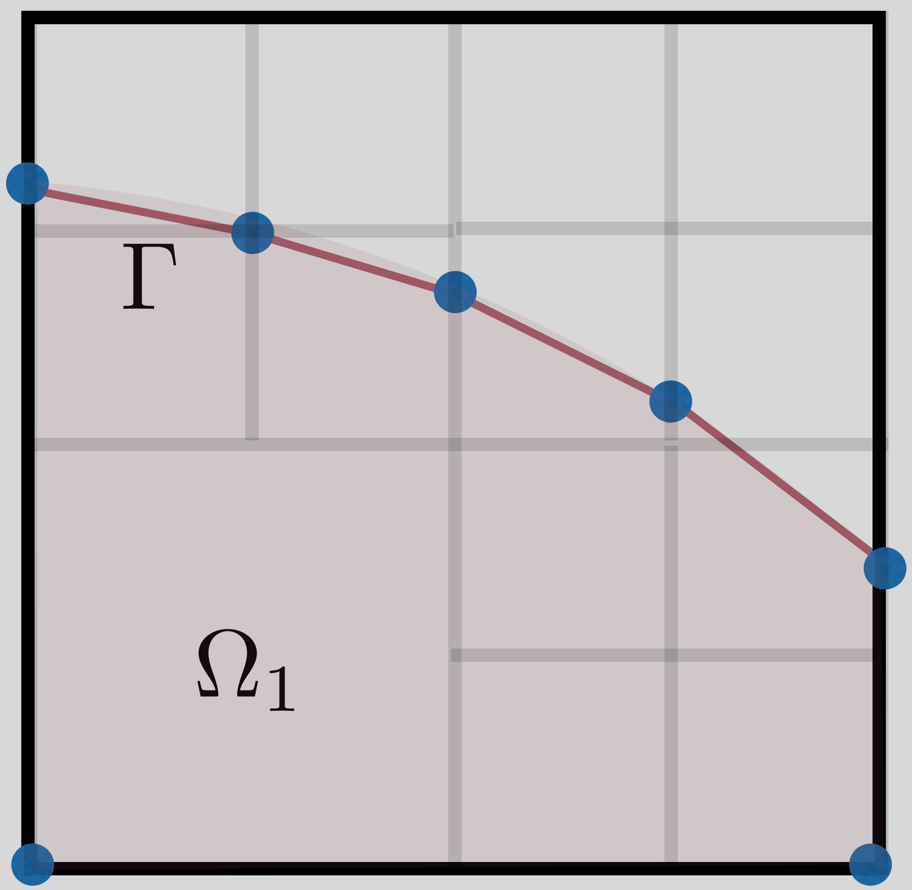
Case 2: Material Interface



$$\mathbf{u}_h(\mathbf{x}) = \sum_{i=1}^{N} \mathbf{N}_i u_i + \sum_{i=1}^{N} \mathbf{N}_i \sum_{j=1}^{M} \psi_j(\mathbf{x}) a_i^j$$

Case 3: Dirichlet Boundary

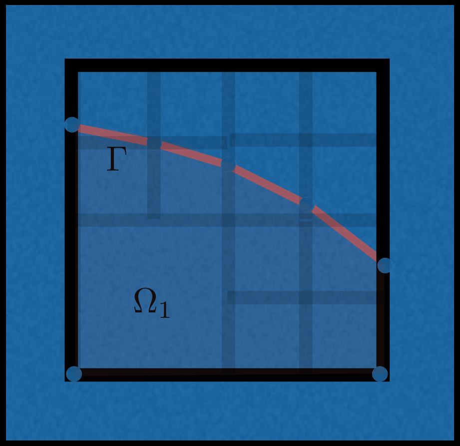
Nitsche's method, Lagrange multipliers…



Distributed memory parallelisation

Two work units.

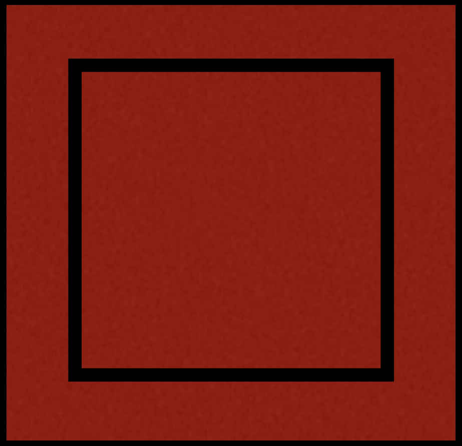
Boundary



$D - 1$

n_b

Interior



D

n_i

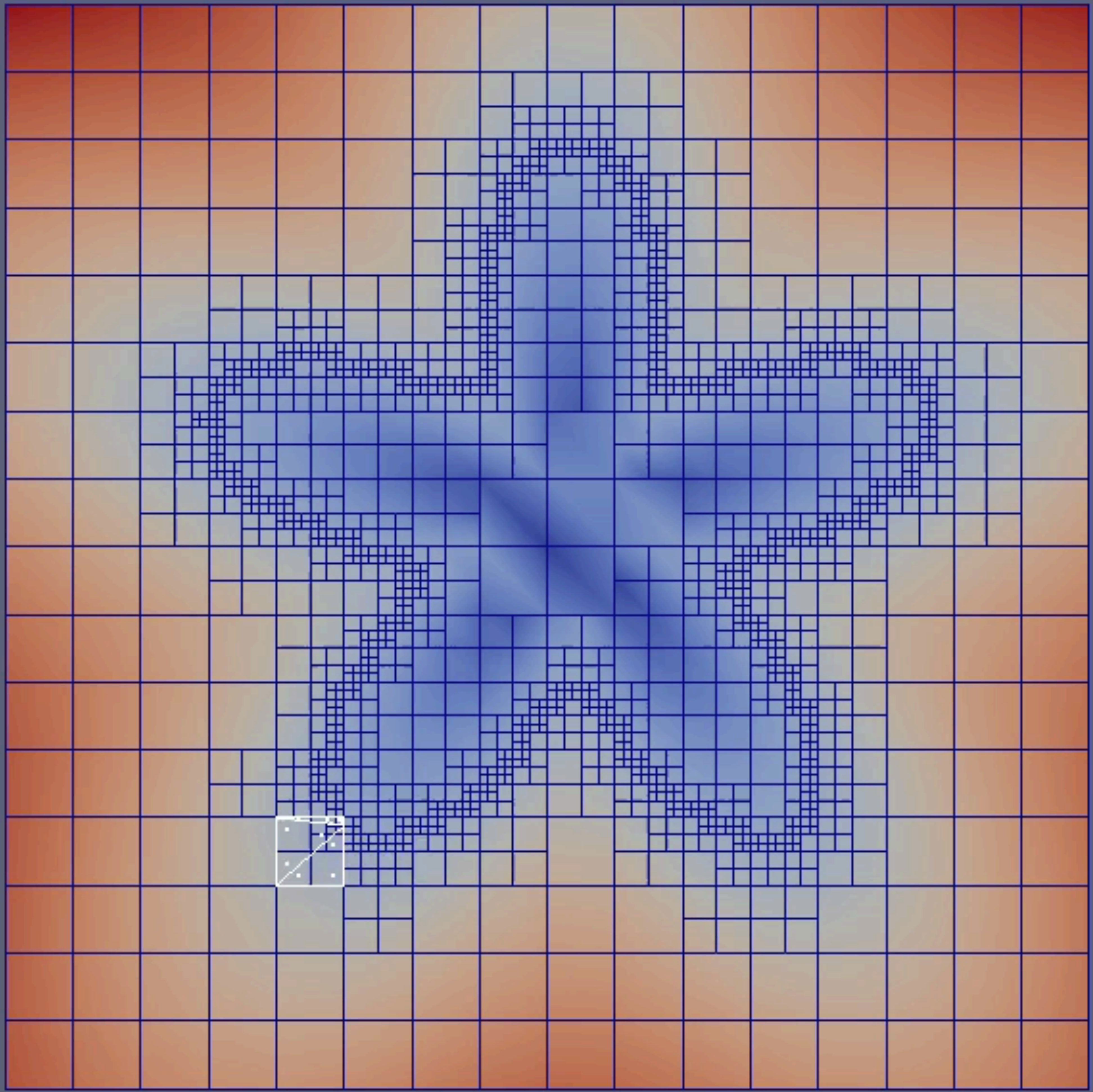
$$n_b >> n_i$$

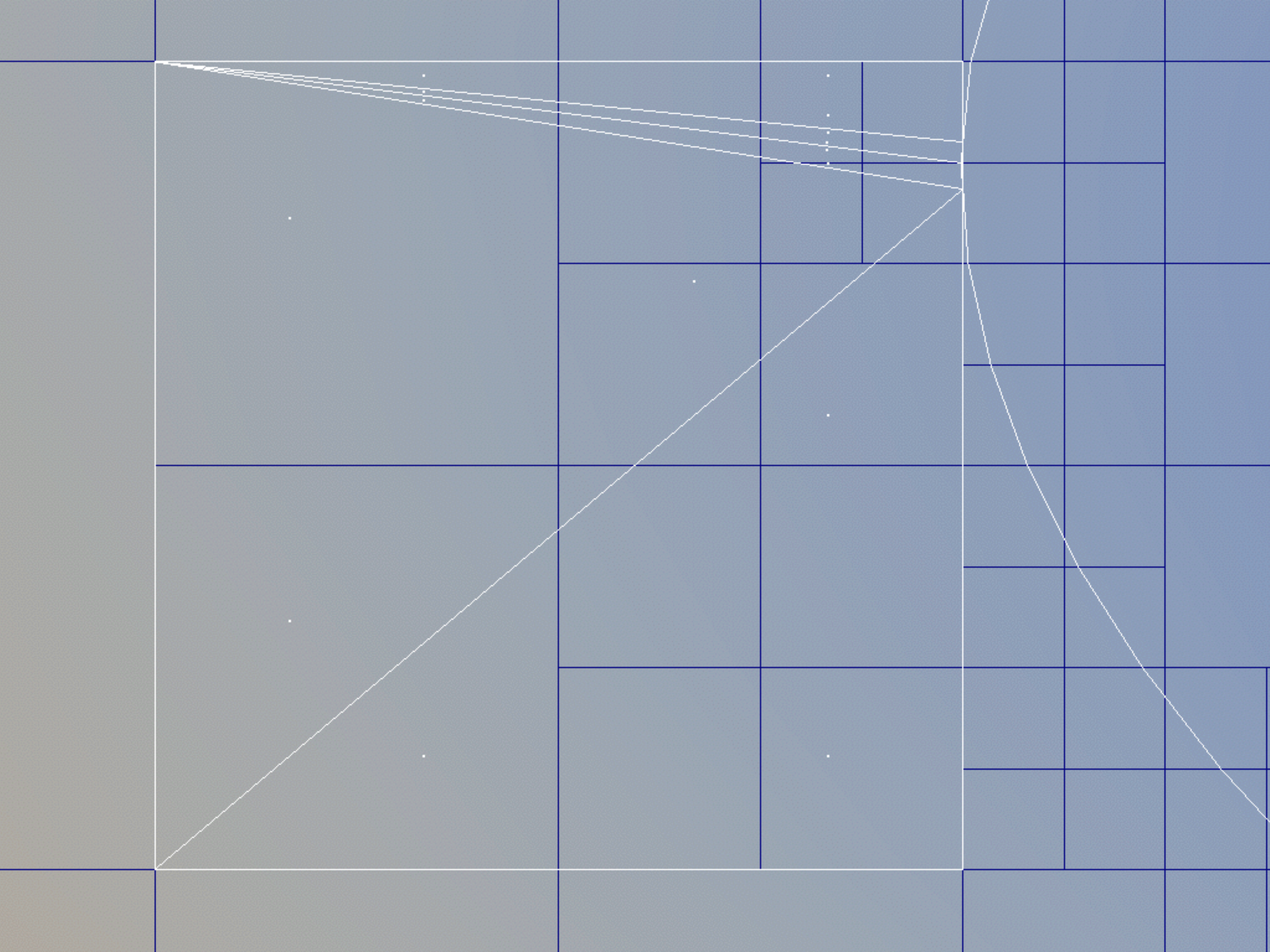
The problem.

- Workload is no longer approximated well by the sparsity pattern of the problem because workload is quadrature bound.
- Some processors are assigned many boundary cells.
- Poor scaling as longest running workers dominate overall runtime.
- **Conclusion:** Superior balancing algorithms are required for optimal scaling.

Conditioning number.

$$\kappa$$





Initial run

Circle	Flower
20 iterations	100 iterations

Conclusion: Methods to keep condition number bounded are a necessity for *practical* computations using iterative solvers.

Summary and Outlook

- We are developing a cartesian grid implicit boundary/enriched finite element method specifically designed for rapid and automatic image based analysis.
- We plan to release the code as an open-source framework. Based upon deal.ii/PETSC/CGAL backend.
- Still outstanding issues with parallelisation and conditioning.
- Complexity is shifted from segmentation and mesh generation (clinician) to the numerical method (the developer); we believe the proposed method has potential to be used as a reference method for automatic image based analysis.

Acknowledgements

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