

4th Conference on  
**Poisson Geometry**  
and related fields

**University of Luxembourg**  
June 7-11, 2004

**Speakers:**

Anton Alekseev (Genève)  
Martin Bordemann (Mulhouse)  
Henrique Bursztyn (Toronto)  
Rui L. Fernandes (Lisbonne)  
Philip Foth (Tucson, Arizona)  
Janusz Grabowski (Varsovie)  
Marco Gualtieri (Montréal)  
Kirill C. H. Mackenzie (Sheffield)  
Yoshiaki Maeda (Yokohama)  
Philippe Monnier (Bordeaux)

Serge Parmentier (Lyon)  
Olga Radko (Los Angeles)  
Tudor S. Ratiu (Lausanne)  
Pavol Severa (Bratislava)  
Izu Vaisman (Haifa)  
Theodore Voronov (Manchester)  
Aissa Wade (Philadelphia)  
Alan Weinstein (Berkeley)  
Ping Xu (Philadelphia)  
Nguyen Tien Zung (Toulouse)

**Schedule and speakers' abstracts:**

**Monday, June 7**

**10h50 – 11h50**

**Alan Weinstein, Prequantization of Poisson manifolds associated with contact manifolds**

Given a manifold  $C$  with a cooriented contact structure, a construction of LeBrun (1991) produces a Poisson manifold  $P$  with boundary. The interior of  $P$  is the symplectization of  $C$ , while the boundary of  $P$  is a copy of  $C$  with the zero Poisson structure (nevertheless, Poisson automorphisms of  $P$  preserve the contact structure on the boundary).

In this talk, Alan Weinstein proposed that an appropriate prequantization of  $P$  is given by a Jacobi manifold  $Q$  and a Jacobi map  $p: Q \rightarrow P$ . The fibers of  $p$  are, as usual, the orbits of a circle action, but the circle action is not free on the part of  $Q$  lying above  $C$ . The proposal is based on considerations coming from Berezin-Toeplitz quantization of bounded domains like the unit ball in complex  $n$ -space.

**13h50 – 14h50**

### **Izu Vaisman, Poisson structures on foliated manifolds**

Izu Vaisman presented a survey of results on relationships between a Poisson structure  $P$  and a foliation  $F$  on a differentiable manifold  $M$ . First, he discussed transversally-Poisson structures of  $F$ . These are bivector fields  $P$  that define a Poisson algebra structure on the subalgebra of  $F$ -leaf-wise constant smooth functions of  $M$ . Such structures are relevant for physical systems with gauge parameters.  $P$  is equivalent with a foliation of  $M$  by pre-symplectic leaves, which contains  $F$  as a sub-foliation. Then, he discussed leaf-wise Poisson structures, i.e., Poisson bivector fields  $P$  of  $M$  such that the symplectic leaves of  $P$  are submanifolds of the leaves of  $F$ . Here, the main result is a spectral sequence for the Poisson cohomology of  $P$ . Finally, he presented the coupling Poisson structures of Vorobiev. They extend Sternberg's symplectic form, which describes the coupling between a particle and a field. He presented the characterization of an  $F$ -coupling Poisson bivector field  $P$  via a normal bundle of  $F$ , and gave a simpler construction of Vorobiev's structure on the dual bundle of the kernel of a transitive Lie algebroid over a symplectic manifold.

**15h10 – 16h10**

### **Philip Foth, Toric degenerations of polygon spaces and applications**

There are remarkable integrable systems on the moduli spaces of spatial polygons, where the angle variables define the "bending flows". Philip Foth showed that there exist flat degenerations of these spaces to toric varieties where the polytopes are defined by the action variables. He also constructed toric degenerations of moduli spaces of stable marked projective lines, which also illuminate interesting integrable systems. He generalized this approach to higher dimensions, like Flaschka-Millson integrable systems and Chow quotients of grassmannians by torus action. These constructions can be applied to finding algebraic invariants of these spaces and to representation theory.

**16h40 – 17h40**

### **Martin Bordemann, (Bi)modules, morphisms and reduction of star-products: the symplectic case, foliations and obstructions**

(Bi)modules, morphisms and reduction of star-products are studied in a framework of multidifferential operators along maps: morphisms deform Poisson maps and representations on functions spaces deform coisotropic maps. If a star-product is representable on a coisotropic submanifold, it is equivalent to a star-product for which the vanishing ideal is a left ideal. If the reduced phase space exists, a star-product with suitable Deligne class is representable and the reduced algebra is the commutant of this module (hence a bimodule). Obstructions to representability to third order are related to the Atiyah-Molino class of the foliation of the coisotropic submanifold, and the same kind of obstructions occurs for the quantization of a Poisson map between symplectic manifolds. For vanishing Atiyah-Molino class, the representation and morphism problem is solvable.

**Tuesday, June 8**

**9h30 – 10h30**

**Kirill C. H. Mackenzie, Duality for double and higher structures**

The duality between linear Poisson structures on a vector bundle and Lie algebroid structures on its dual is best expressed in terms of the tangent and cotangent double vector bundles associated to the vector bundle. The duality of double vector bundles is also fundamental to the concept of double Lie algebroid, which includes the cotangent double of a Lie bialgebroid. In this talk Kirill

Mackenzie showed that the somewhat surprising duality properties of double vector bundles are easily understood in terms of triple vector bundle structures and indicate the general phenomena in the case of higher structures.

**10h50 – 11h50**

**Serge Parmentier, On dynamical Poisson groupoids**

In this talk, Serge Parmentier reported on a joint work with L.-C. Li and with R. Pujol. The lecture surveyed several properties of Poisson groupoids of Etingof-Varchenko type. In particular, the description of Poisson groupoid duality together with reduced duality diagrams, and the link with Lie quasi-bialgebras have been given. Applications thereof to the description of the moduli space of dynamical  $r$ -matrices and double constructions have also been addressed.

**13h50 – 14h50**

**Janusz Grabowski, Frame independent mechanics: Lie and Poisson brackets on sections of affine bundles**

A geometric framework for a frame-independent formulation of different problems in analytical mechanics has been developed. In this approach affine bundles replace vector bundles of the standard description. In particular, momenta take affine values and energy functions are replaced by sections of certain affine line bundles called AV-bundles. Categorical constructions for affine bundles and AV-bundles as well as natural analogs of Lie algebroid structures on affine bundles – Lie affgebroids – have been presented. Certain Lie algebroids and Lie affgebroids canonically associated with an AV-bundle turn out to be closely related to affine analogs of Poisson and Jacobi brackets. Homology and cohomology of the latter are canonically defined. The developed concepts find an application in solving some problems of frame-independent geometric description of mechanical systems.

**15h10 – 16h10**

**Nguyen Tien Zung, Linearization of Lie groupoids and symplectic groupoids**

In this talk Nguyen Tien Zung presented some recent results concerning the linearization of Lie groupoids and symplectic groupoids, as well as some applications.

**16h40 – 17h40**

**Ping Xu, On the universal lifting theorem**

For a Lie groupoid  $G$ , the space of multiplicative multivector fields on  $G$  is naturally a graded Lie algebra. Ping Xu introduced, for a Lie algebroid  $A$ , the notion of the so called "multi-differentials", which is also a graded Lie algebra. He proved that for an  $s$ -connected and  $s$ -simply-connected Lie groupoid, these two graded Lie algebras are isomorphic.

As an application, he gave the integration theorem of quasi-Lie bialgebroids. Application to quasi-Poisson manifolds was discussed.

**18h00 – 19h30**

**Poster session**

Fifteen posters were presented during this session.

**Wednesday, June 9**

**9h30 – 10h30**

**Marco Gualtieri, Generalized Geometry**

In his talk Marco Gualtieri described generalized complex geometry, which is a way of unifying complex and symplectic geometry using the framework of complex Dirac structures. He described the main results in this field and some implications of this unification for mirror symmetry.

**10h50 – 11h50**

**Aissa Wade, On the local structure of Dirac manifolds**

Aissa Wade gave a local normal form for Dirac structures. As a consequence, she showed that the dimensions of the pre-symplectic leaves of a Dirac manifold have the same parity. She also explained that, given a point  $m$  of a Dirac manifold  $M$ , there is a well-defined transverse Poisson structure to the pre-symplectic leaf  $P$  through  $m$ . Finally, she described the neighborhood of a pre-symplectic leaf in terms of geometric data. This description agrees with that given by Vorobjev for the Poisson case.

**13h50 – 14h50**

**Yoshiaki Maeda, Complex Maslov blurred bundles**

Alan Weinstein constructed Maslov gerbes, which is the complex version of the Maslov bundles. Yoshiaki Maeda gave another approach to construct these objects by using the deformation quantization of the quadratic functions in the Weyl algebra.

**15h10 – 16h10**

**Pavol Severa, Non-commutative differential forms and quantization of the odd symplectic category**

There is a simple and natural quantization of differential forms on odd Poisson supermanifolds, given by the relation  $[f, dg] = \{f, g\}$ , for any two functions  $f$  and  $g$ . This non-commutative differential algebra has a geometrical realization as a convolution algebra of the symplectic groupoid integrating the Poisson manifold.

This quantization is just a part of a quantization of the odd symplectic category in terms of  $\mathbb{Z}_2$ -graded chain complexes. It is a straightforward consequence of the theory of BV operators acting on semi-densities, due to H. Khudaverdian.

**16h40 – 17h40**

**Henrique Bursztyn, Dirac structures, moment maps and quasi-Poisson manifolds**

Henrique Bursztyn described how the usual notions of hamiltonian action, momentum map and symplectic reduction in Poisson geometry can be extended to the context of (twisted) Dirac geometry. He showed how hamiltonian quasi-Poisson manifolds with group-valued moment maps fit into this general framework, and pointed out some nice features of this alternative approach.

**Thursday, June 10**

**9h00 – 10h00**

**Olga Radko, The Picard groups and topologically stable Poisson structures**

The Picard group of a Poisson manifold  $P$ , introduced by Bursztyn and Weinstein, consists of all Morita self-equivalence  $P$ -bimodules, with the operation of the relative tensor product. Olga Radko described the Picard groups of certain Poisson structures on compact oriented surfaces, and discussed some related results.

**10h15 – 11h15**

**Philippe Monnier, Poisson cohomology**

Philippe Monnier explained an explicit computation of the local Poisson cohomology of generic germs at 0 of Poisson structures in dimension 2. While generalizing this computation to higher dimensions, we are led to introduce a new cohomology attached to a function. He then gave some "basic" properties of this cohomology.

**11h45 – 12h45**

**Rui Fernandes, First steps in Poisson topology**

In his lecture Rui Fernandes discussed properties of Poisson manifolds, which have a topological flavor, such as a rigidity, softness and stability.

**Afternoon: guided tour**

**Friday, June 11**

**9h00 – 10h00**

**Theodore Voronov, Higher derived brackets**

Various brackets in algebra and geometry, such as the Lie bracket in an arbitrary Lie algebra or superalgebra, the Poisson bracket on a Poisson manifold, etc., are examples of the so called "derived brackets", having as their source a canonical bracket (such as the commutator of vector fields or the canonical Poisson bracket on the cotangent bundle) and a "deriving element". In all these cases, the deriving element is in a certain sense "quadratic" and the resulting bracket satisfies the Jacobi identity.

"Higher derived brackets" is a generalization of derived brackets, a construction of an infinite

sequence of operations from simple data on a Lie superalgebra. It should be looked at as a proper framework for "non-quadratic" deriving elements. On one hand the main statement is purely algebraic, on the other hand the construction is motivated by various examples from geometry and mathematical physics. It contains, in particular, the standard description of arbitrary SH Lie algebras via homological vector fields and the example of brackets generated by a differential operator (such as the Batalin--Vilkovisky type operators).

**10h15 – 11h15**

**Anton Alekseev, Deformation quantization of coadjoint orbits**

Anton Alekseev explained how Fedosov's quantization applies to coadjoint orbits. In many situations, this method gives rise to explicit formulas for invariant \*-products related to dynamical twists and dynamical R-matrices.

**11h45 – 12h45**

**Tudor Ratiu, Orbits in Banach Lie-Poisson spaces**

The notion of Banach Lie-Poisson space has been presented and the main properties of these spaces have been discussed. Special emphasis has been given to the problem of extensions. The various orbits in specific Banach Lie-Poisson spaces associated to  $W^*$ -algebras and operator ideals have been analyzed.

**Closing session:**

**It was decided that the 5<sup>th</sup> Conference on "Poisson Geometry and related fields" will be held in 2006 in Japan.**

**List of participants:**

See below.

## List of Participants

1. Adewole Kayode Ajileye, University of Ado-Ekiti, Nigeria
2. Alekseev Anton, University of Geneva, Switzerland
3. Ammar Faouzi, Faculty of Sciences of Sfax, Tunisia
4. Androulidakis Iakovos, University of Athens, Greece
5. Antunes Paulo, University of Coimbra, Portugal
6. Arnal Didier, University of Bourgogne, Dijon, France
7. Bonneau Philippe, University of Bourgogne, Dijon, France
8. Bordemann Martin, University of Haute Alsace, France
9. Brahic Olivier, University of Montpellier, France
10. Buneci Madalina, University of Targu-Jiu, Romania
11. Bursztyn Henrique, University of Toronto, Canada
12. Calaque Damien, Institut de Recherche Mathématique Avancée, Strasbourg, France
13. Calvo Iván, University of Zaragoza, Spain
14. Cardona Alexander, Keio University, Yokohama, Japan
15. Chloup Veronique, University of Metz, France
16. Ciccoli Nicola, University of Perugia, Italy
17. Cohen Alexandra, University of Paris, France
18. Dufour Jean-Paul, University of Montpellier, France
19. Duval Christian, Center for Theoretical Physics, Luminy, France
20. El Gradechi Amine, University of Artois, France
21. Falceto Fernando, University of Zaragoza, Spain
22. Fernandes Rui Loja, Instituto Superior Tecnico, Lisbon, Portugal
23. Foth Philip, University of Arizona, Tucson, United States
24. Gloden Raoul, Communautés Européennes, Luxembourg, Luxembourg
25. Grabowski Janusz, Polish Academy of Sciences, Warsaw, Poland
26. Gracia-Saz Alfonso, University of California, Berkeley, United States
27. Gramsch Bernhard, University of Mainz, Germany
28. Gratus Jonathan, Bangor University, United Kingdom

29. Greshnov Alexander, Sobolev Institute of Mathematics, Novosibirsk, Russia
30. Gruetzmann Melchior, Friedrich-Schiller University, Jena, Germany
31. Gualtieri Marco, Fields Institute, Montreal, Canada
32. Gutt Simone, Université Libre de Bruxelles, Belgium
33. Hansoul Sarah, University of Liège, Belgium
34. Hirica Iulia-Elena, University of Bucharest, Romania
35. Hofer Laurent, University of Haute Alsace, France
36. Huebschmann Johannes, University of Lille, France
37. Iglesias Ponte David, Penn State University, Philadelphia, United States
38. Indelicato Davide, University of Zurich, Switzerland
39. Kadem Abdelouahab, Ferhat Abbas University, Setif, Algeria
40. Karolinsky Eugene, Kharkiv National University, Ukraine
41. Kass Guy, University of Luxembourg, Luxembourg
42. Khmelynskaya Alena, Kazan State University, Russia
43. Kosmann-Schwarzbach Yvette, Polytechnic School, Paris, France
44. Kotov Alexei, Friedrich-Schiller University, Jena, Germany
45. Krantz Tom, University H. Poincaré, Nancy, France
46. Kubarski Jan, Technical University of Lodz, Poland
47. Kushnirevych Vitaliy, University of Freiburg, Germany
48. Laurent Camille, Penn State University, Philadelphia, United States
49. Le Blanc Ariane, University of Poitiers, France
50. Lecomte Pierre, University of Liège, Belgium
51. Leitner Frederick, University of Arizona, Tucson, United States
52. Lu Jiang-Hua, University of Hong Kong, China
53. Mackenzie Kirill, University of Sheffield, United Kingdom
54. Maeda Yoshiaki, Keio University, Yokohama, Japan
55. Martinez Eduardo, University of Zaragoza, Spain
56. Masmoudi Mohsen, University H. Poincaré, Nancy, France
57. Maspfuhl Oliver, Federal Polytechnic School of Lausanne, Switzerland
58. Mathonet Pierre, University of Liège, Belgium
59. Matveev Vladimir S., University of Freiburg, Germany
60. Mehta Rajan, University of California, Berkeley, United States
61. Mikami Kentaro, Akita University, Japan



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65. Nagy Paul-Andi, Humboldt University, Berlin, Germany
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67. Neumaier Nikolai, University of Frankfurt, Germany
68. Nunes da Costa Joana, University of Coimbra, Portugal
69. Ortega Juan-Pablo, University of Franche-Comté, Besançon, France
70. Ovsienko Valentin, University of Lyon, France
71. Panasyuk Andriy, University of Warsaw, Poland
72. Parfyonova Nataliya, Kharkov University, Ukraine
73. Parmentier Serge, University of Lyon, France
74. Peiffer Monique, University of Luxembourg, Luxembourg
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76. Petalidou Fani, University of Peloponnese, Greece
77. Pichereau Anne, University of Poitiers, France
78. Pier Jean-Paul, University of Luxembourg, Luxembourg
79. Poncin Norbert, University of Luxembourg, Luxembourg
80. Przybylska Maria, Institut national de recherche en informatique et en automatique, Sophia Antipolis, France
81. Racaniere Sebastien, Imperial College London, United Kingdom
82. Radko Olga, University of California, Los Angeles, United States
83. Radoux Fabian, University of Liège, Belgium
84. Rakhnin Andriy, Kharkov University, Ukraine
85. Ratiu Tudor, Federal Polytechnic School of Lausanne, Switzerland
86. Roytenberg Dmitry, Utrecht University, Netherlands
87. Saksida Pavle, University of Ljubljana, Slovenia
88. Sansonetto Nicola, University of Padua, Italy
89. Schaffhauser Florent, Université Pierre et Marie Curie, Paris, France
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91. Schlichenmaier Martin, University of Luxembourg, Luxembourg
92. Seredynska Malgorzata, Polish Academy of Sciences, Warsaw, Poland
93. Ševera Pavol, Comenius University, Bratislava, Slovakia
94. Siby Hassène, University of Montpellier, France

95. Sleewaegen Pierre, Université Libre de Bruxelles, Belgium
96. Stefanini Luca, University of Zurich, Switzerland
97. Sternheimer Daniel, Keio University, Yokohama, Japan
98. Stiénon Mathieu, Université Libre de Bruxelles, Belgium
99. Suzuki Haruo, Hokkaido University, Sapporo, Japan
100. Tang Xiang, University of California, Berkeley, United States
101. Torossian Charles, École Normale Supérieure, Paris, France
102. Tumpach Barbara, Polytechnic School, Paris, France
103. Udodova Olga, Ukrainian State Academy of Railway Transport, Kharkov, Ukraine
104. Ungureanu Viorica, University of Targu-Jiu, Romania
105. Urbanski Pawel, University of Warsaw, Poland
106. Vaisman Izu, University of Haifa, Israel
107. Valery Boyomo Ntokime, University of Liège, Belgium
108. Vankerschaver Joris, University of Ghent, Belgium
109. Voronov Theodore, University of Manchester, United Kingdom
110. Wade Aissa, Penn State University, Philadelphia, United States
111. Weinstein Alan, University of California, Berkeley, United States
112. Wurzbacher Tilmann, University of Metz, France
113. Xu Ping, Penn State University, Philadelphia, United States
114. Yashagin Eugene, Kazan State University, Russia
115. Zambon Marco, University of California, Berkeley, United States
116. Zhu Chenchang, University of California, Berkeley, United States
117. Zung Nguyen Tien, University of Toulouse, France