RESEARCH CONFERENCES

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Algebraic Aspects in Geometry

Mathematical Research and Conference Center,
Będlewo ▪ Poland
17-23 October 2007

Chair: **Norbert Poncin**, University of Luxembourg, LU
Vice-Chair: **Janusz Grabowski**, Polish Academy of Sciences, Warszawa, PL

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Speakers' abstracts

Thursday 18 October

General Session

Chair: Norbert Poncin, University of Luxembourg, LU

09.15-10.15

Giovanni Landi
Università di Trieste, IT
Dirac operators on noncommutative manifolds

Professor Landi motivated the use of noncommutative geometry for singular spaces and gave an introduction to the use of Dirac operators (and of spectral triples) for metric structures on noncommutative manifolds. The general theory has been clarified by basic examples coming from toric and quantum group (co)-actions.

10.45-11.45

Serge Skryabin

Chebotarev Research Institute, Kazan, RU

Algebraic characterization of differential geometric structures

Given a transitive Lie algebra L of vector fields on a smooth manifold X, the quest for a geometric structure on X having all vector fields in L as infinitesimal automorphisms, is a highly relevant objective. The simplest conditions that might be imposed on L can be formulated in terms of representations of the isotropy subalgebras of L in the tangent spaces of X. Professor Skryabin presented a purely algebraic framework in which this aforementioned problem can be extended and solved. In particular, he gave an algebraic interpretation of Riemannian pseudo-metrics and of volume, Hamiltonian and contact forms.

Session 1: Weyl algebra and related conjectures

Chair: Serge Skryabin, Chebotarev Research Institute, Kazan, RU

14.15-15.15

Dmitry Roytenberg

IHES, Bures-sur-Yvette, FR

Weak Lie 2-algebras

A Lie 2-algebra is a linear category equipped with a functorial bilinear operation satisfying skew-symmetry and Jacobi identity up to natural transformations, which themselves obey appropriate coherence laws. Functors and natural transformations between Lie 2-algebras can also be defined, yielding a 2-category. Passing to the normalized chain complex gives an equivalence of 2-categories between Lie 2-algebras and certain "up to homotopy" structures on the complex; for strictly skew-symmetric Lie 2-algebras these are L-infinity algebras, by a result of Baez and Crans. Lie 2-algebras appear naturally as infinitesimal symmetries of solutions of the Maurer-Cartan equation in some differential graded Lie algebras and L-infinity algebras. In particular, (quasi-) Poisson manifolds, (quasi-) Lie bialgebroids and Courant algebroids provide large classes of examples.

15.45-16.45

Vladimir Bavula

University of Sheffield, UK

The Jacobian map, the Jacobian group, and the group of automorphisms of the Grassmann algebra

There are nontrivial dualities and parallels between polynomial algebras and the Grassmann algebras. Professor Bavula reported on the Grassmann algebras at the angle of the Jacobian conjecture for the polynomial algebras and introduced the Jacobian set for the Grassmann algebra, which turns out to be a group - the Jacobian group - a sophisticated part of the group of automorphisms of the Grassmann algebra. Further, he proved that the Jacobian group is a connected unipotent algebraic group. A (minimal) set of generators for the Jacobian group, its dimension and coordinates were found explicitly. The same has been done for the Jacobian ascents - some natural overgroups of the Jacobian group. Eventually, it could be shown that the Jacobian map is surjective for odd n, and is not for even n.

16.45-17.45

Alexei Kanel-Belov

Bar-Ilan University, Ramat-Gan, IL

Authomorphisms of Weyl Algebras

Let $W_n=k[x_1,\dots,x_n,\partial_1,\dots,\partial_n]$ be a Weil algebra. Dixmier's conjecture (DC_n) asserts that $Aut(W_n)=End(W_n)$. It is well-known that DC_n implies JC_n, where JC_n is Jacobian conjecture for n an dimensional space.

Recently Tsushimotto and lately Kanel-Belov and Kontsevich gave independent proof of the implication $JC_{(2n)}$ entails DC_n. The proof is based on reduction modulo infinitely large prime and use of Poisson brackets on the center of Weil algebra modulo p.

Their conjecture maintains that $Aut(W_n)=sympl(k^{(2n)})$. It is known for n=1. The construction of a homomorphism is based on the reduction modulo infinitely large prime, and if this homomorphism is an isomorphism, then (and this is a nontrivial result) this homomorphism is independent on the choice of infinitely large prime. Uniqueness is based on investigation of the group $Aut(Aut(W_n))$ and $Aut(Aut(k[x_1,\dots,x_n]))$. The action of maximal torus has been considered and Bialickii-Birula's theorem has been used.

The conjecture can be extended in following way: is there a basic correspondence between holonomical D-moduli and Lagrangian varieties. The correspondence is based on reduction modulo p, where p is infinitely large prime, and consideration of annihilator in the center. For n=1, this correspondence has been established. Let D be in W_1, and $P(x^p,\partial_p)=\det(D_p)$, where D_p is reduction of D modulo infinitely large prime. Then, $P(x,y)=0$ is an equation of corresponding curve. It has the same degree as D.

A general construction exists; the problem is to prove its correctness.

Friday 19 October

Session 2: Infinite dimensional Lie algebras and Field Theory

Chair: Martin Schlichenmaier, University of Luxembourg, LU

09.00-10.00

Ctirad Klimcik

Institute of Mathematics of Luminy, Marseille, FR

$q\to\infty$ limit of the quasitriangular WZW model

The objective of this talk was to study the $q\to\infty$ limit of the q-deformation of the WZW model. It could be shown that the commutation relations of the $q\to\infty$ current algebra are underlain by certain affine Poisson structure on the group of holomorphic maps from the disc into the complexification of the target group. The Lie algebroid corresponding to this affine Poisson structure has been integrated to a global symplectic groupoid which turned out to be nothing but the phase space of the $q \to \infty$ limit of the q-WZW model.

Further, it has been proved that this symplectic groupoid admits a chiral decomposition compatible with its (anomalous) Poisson-Lie symmetries. Finally, the chiral theory has been dualized in a remarkable way and the exchange relations have been evaluated for the $q \to \infty$ chiral WZW fields in both, the original and the dual pictures.

10.00-11.00

Glenn Barnich

Université Libre de Bruxelles & International Solvay Institutes, BE

Algebraic structure of gauge systems: theory and applications

The general construction of the BRST-antifield formalism for gauge systems has been reviewed with special emphasis on the role played by locality. Applications, ranging from anomalies and counterterms in quantum field theory to symmetries and consistent deformations in the classical framework, have been discussed.

11.30-12.30

Oleg Sheinman

Steklov Mathematical Institute, Moscow, RU

Lax operator algebras and integrable systems

A Lax operator algebra is an infinite-dimensional Lie algebra given by a classical simple or reductive Lie algebra, a Riemannian surface, and Tyurin data on it. Such algebras appeared in course of development of the general concept of Lax operators on algebraic curves, due to I. Krichever. It turned out that general Lax operators admit multiplicative and Lie structures, and those have orthogonal and symplectic generalizations, as well as generalizations related to other classical Lie algebras. These results in a kind of current or gauge algebras generalizing (non-twisted) Kac-Moody and Krichever-Novikov algebras.

In his talk, Professor Sheinman introduced Lax operator algebras, explained their relations to Krichever-Novikov and Kac-Moody algebras. He gave an outline of the Hamiltonian theory of Lax equations due to I. Krichever and its generalization related to classical complex Lie algebras. He stressed the tight relation between the almost-graded structure on the Lax operator algebras and the problem of consistency of Lax equations. Moreover, he showed how the more general class of equations, namely zero curvature equations on Riemann surfaces, appears in connection with Kadomtsev-Petviashvili equation. Eventually, he explained the classification result on the local central extensions of the Lax operator algebras.

15.00-16.00

Karl-Hermann Neeb

Darmstadt University of Technology, DE

Infinite-dimensional Lie algebras beyond Kac-Moody and Virasoro algebras

In his lecture, Professor Neeb discussed some aspects of infinite-dimensional Lie algebras and groups attached to principal bundles over a compact manifold M. The case where M is the circle S^1 corresponds to loop groups, affine Kac-Moody groups, and the Virasoro group. Passing from one to higher dimensional base manifolds, the variety of the corresponding Lie algebras and groups increases tremendously and it becomes a challenging problem to organize this variety and to understand the structural features of the corresponding groups.

16.30-17.30

Alice Fialowski

Eotvos Lorand University, Budapest, HU

Infinite dimensional Lie algebras, deformations, and geometry

The concepts symmetry and deformations are considered to be two fundamental guiding principles for developing the physical theory further. From the mathematical point of view, considering deformed objects can describe the "neighboring" objects. This can be made precise with the notion of moduli space, classifying inequivalent objects of the same type. The moduli space should be equipped with a "geometric" structure such that "nearby points" should be also "nearby" in the sense of deforming the structure of the initial object.

In her talk, Professor Fialowski dealt with deformations of Lie algebras. The algebras, which are of relevance in conformal field theory, integrable systems related to partial differential equations, etc., are typically infinite dimensional. She concentrated on these Lie algebras. In the infinite dimensional case, several new phenomena appear. First of all the relation of cohomology and deformations is not so tight anymore as in finite dimension. In particular, the Witt and Virasoro algebras, the current algebras have nontrivial geometric deformations, despite the fact that the cohomology spaces for those algebras are trivial and hence the algebras are formally rigid. She reported on theses constructions and raised further questions.

Session 3: L-infinity algebras

Chair: Zoran Skoda, **Institute Rudjer Boskovic, HR**

09.00-10.00

Martin Markl

The Czech Academy of Sciences, Prague, CZ

L-infinity algebras: an overview

Professor Markl started by recalling various definitions of L-infinity algebras and formulating basic properties of these objects and their maps.

In the second part of the talk he explained that L-infinity algebras are homotopy invariant versions of Lie algebras and indicated a general approach of handling structures of this type.

The talk was self-consistent.

10.00-11.00

Pavol Severa

Comenius University, Bratislava, SK

L-infinity algebras as first approximations

It has been shown how L-infinity algebras (or rather dg-manifolds) arise from Kan simplicial manifolds (i.e. from "Lie n-groups" and "Lie n-groupoids"), and more generally, from presheaves on the category of surjective submersions. This is a straightforward generalization of the procedure Lie groups → Lie algebras. It is done by restricting the presheaves to submersions with the odd line as the fibre. It has also been discussed what happens if one allows the odd plane etc., as the fibre, when differential forms get replaced by their higher-order analogues.

Session 4: Short talk – Lie algebras

Chair: Cornelia Vizman, **West University of Timisoara, Timisoara, RO**

11.30-12.00

Oksana Hentosh

Ukrainian National Academy of Sciences, Lviv, UA

The Lie-algebraic structure of Lax-integrable (2/2+1)-dimensional nonlinear dynamical systems

Lax-type representations for integrable (1+1)-dimensional nonlinear dynamical system hierarchies on functional manifolds and their super-analogs can be interpreted as Hamiltonian flows on dual spaces to the operator Lie algebras of integral-differential operators. Their Hamiltonian structure is given by the R-deformed canonical Lie-Poisson bracket and the corresponding Casimir functionals as Hamiltonians. Every Hamiltonian flow of such a type can be written as a compatibility condition of the spectral relationship for the corresponding integral-differential operator and a suitable eigenfunction evolution. If the spectral relationship admits a finite set of eigenvalues an important problem of finding the Hamiltonian representation for the hierarchy of Lax-type coupled with the evolutions of eigenfunctions and appropriate adjoint eigenfunctions naturally arises.

The problem is solved partially for the Lie algebra of super-integral-differential operators of two anticommuting variables by means of the variational Casimir functionals' property under some Lie-Backlund transformation.

For the coupled Lax-type nonlinear dynamical system hierarchy the corresponding hierarchies of additional or so called "ghost" symmetries for the coupled Lax-type flows are established to be Hamiltonian also. It is proved that the additional hierarchy of Hamiltonian flows is generated by the Poisson structure which is obtained from the tensor product of the R-deformed canonical Lie-Poisson bracket with the standard Poisson bracket on the related eigenfunction and adjoint eigenfunction superspace and the corresponding natural powers of a suitable eigenvalue are their Hamiltonians. The relation of these hierarchies with both Lax integrable (2|2+1)-dimensional nonlinear dynamical systems and their triple Lax-type linearizations has been investigated.

Sunday 21 October

Session 5: Lie and Poisson structures

Chair: Alice Fialowski, Eötvös Loránd University, Budapest, HU

09.00-10.00

Mathieu Stienon

ETH Zürich, CH

Holomorphic Poisson Structures and Groupoids

Mathieu Stienon described holomorphic Poisson manifolds, Lie algebroids and Lie groupoids from the viewpoint of real Poisson geometry. He gave a characterization of holomorphic Poisson structures in terms of the Poisson Nijenhuis structures of Magri-Morosi and described a double complex, which computes the holomorphic Poisson cohomology. To that purpose, he showed that a holomorphic Lie algebroid structure on a vector bundle A→X is equivalent to a matched pair of complex Lie algebroids (T^(0,1)X,A^(1,0)), in the sense of Lu. The holomorphic Lie algebroid cohomology of A is isomorphic to the cohomology of the elliptic Lie algebroid T^(0,1)X \bowtie A^(1,0). In the case when (X,π) is a holomorphic Poisson manifold and A=(T^*X)_π, such an elliptic Lie algebroid coincides with the Dirac structure corresponding to the associated generalized complex structure of the holomorphic Poisson manifold. Furthermore, Dr. Stienon explained why a holomorphic Lie algebroid is integrable if and only if its underlying real Lie algebroid is integrable. This proves that the integrability criteria of Crainic and

Fernandes do also apply in the holomorphic context without any modification.

10.00-10.45

Sergei Silvestrov

Lund University, SE

Quasi Lie algebras

Quasi-Lie algebras, their subclasses Hom-Lie algebras and Quasi-Hom-Lie algebras and examples have been presented.

Examples of Quasi-Lie algebras include algebras of twisted or quantized differential operators and vector fields and related non-commutative differential calculi; known and new one-parameter and multi-parameter deformations of infinite-dimensional Lie algebras of Witt and Virasoro type appearing in conformal field theory, string theory and deformed vertex models; Krichever-Novikov type algebras and their generalizations and deformations arising in connection with algebraic geometric methods for non-linear integrable continuous and discrete systems; almost graded algebras and their generalizations; multi-parameter families of quadratic and almost quadratic algebras for special choices of parameters including algebras from non-commutative algebraic geometry; various classes of braided Lie algebras; universal enveloping algebras of Lie algebras, Lie superalgebras and general group graded color Lie algebras.

The common uniting feature for all these algebras is the appearance of twisted generalizations of Jacoby identities providing new structures of interest for investigation from the side of associative algebras, the non-associative algebras generalizing associative and Lie algebras, generalizations and deformations of Leibniz algebras, twisted generalizations of Hopf algebras, the non-commutative differential calculi beyond usual differential calculus and q-differential calculi, generalized central and other (co-)homological extensions of algebras and interplay with algebraic geometry and deformation theory.

The aim of this talk was to give an introduction and a review some main examples, constructions and open directions in the area of Quasi-Lie algebras.

Session 6: Short talks – quantization

Chair: Sergei Silvestrov, Lund University, SE

11.30-12.00

Armen Sergeev

Steklov Mathematical Institute, Moscow, RU

Geometric quantization of the universal Teichmueller space

The universal Teichmueller space consists of normalized quasisymmetric homeomorphisms of the circle. It contains all classical Teichmueller spaces of compact Riemann surfaces of finite genera and, as a smooth part, the space of normalized diffeomorphisms of the circle. The speaker explained how to quantize geometrically the smooth part and reported on the difficulties arising when trying to quantize the whole universal Teichmueller space.

12.00-12.30

Fabian Radoux
University of Luxembourg, LU
Natural and projectively equivariant quantization

Equivariant quantization, in the sense of C. Duval, P. Lecomte, and V. Ovsienko, has developed as from 1996 in a rather small community. This quantization procedure requires equivariance of the quantization map with respect to the action of a symmetry group G, is well-defined globally on manifolds endowed with a flat G-structure, and leads to invariant star-products. It was first studied on vector spaces for the projective and the conformal groups, and then extended in 2001 to arbitrary manifolds. In this setting, equivariance with respect to all diffeomorphisms has been restored, which has finally led to the concept of natural and projectively invariant quantization. Existence of such quantization maps has been investigated in several recent works, which have briefly been addressed in this talk.

Session 7: Forward look

Chair: Janusz Grabowski, Polish Academy of Sciences, Warsaw, PL

15.00-16.00 Forward look plenary discussion

Poster Session

Chair: Pierre Mathonet, University of Liège, BE

16.30-18.30 Poster presentations

17 participants presented a poster:

1. Ataguema, 2. Domitrz, 3. George, 4. Hinterleitner, 5. Kieserman, 6. Lombardo, 7. Makhlouf, 7. Matsumura, 8. Michel, 9. Oleinick, 10. Panasyuk, 11. Seredynska, 12. Shchukin, 13. Skoda, 14. Szablikowski, 15. Vizman, 16. Voznischeva

The session began at 4 pm and ended at 7 pm.

Session 8: Algebroids and supergeometry

Chair: Martin Markl, **The Czech Academy of Sciences, Prague, CZ**

09.00-10.00

Theodore Voronov

University of Manchester, UK

Q-manifolds and Mackenzie theory

Q-manifolds are supermanifolds endowed with a homological vector field. They should be regarded as a nonlinear extension of the notion of Lie algebras, together with Poisson and Schouten manifolds. Q-manifolds provide an effective geometric language for describing structures such as, e.g., strong homotopy Lie algebras and Lie algebroids.

"Mackenzie theory" stands for the rich circle of notions that have been put forward by Kirill Mackenzie in recent years: double structures such as double Lie groupoids and double Lie algebroids, Lie bialgebroids and their doubles, nontrivial dualities for double and multiple vector bundles, etc.

In this talk the speaker discussed double Lie algebroids and explained how this quite complicated fundamental notion becomes equivalent to a very simple one if the language of Q-manifolds is used. In particular, it shows how the two seemingly different notions of a "Drinfeld double" of a Lie bialgebroid due to Mackenzie and Roytenberg respectively, turn out to be the same object if properly understood. It also allows obtaining generalizations such as multiple Lie algebroids and multiple Lie bialgebroids.

10.00-10.45

Jan Kubarski

Technical University of Łódź, PL

Hirzebruch signature operator, algebraic aspects, applications to Lie algebroids

The aim of this talk was to construct two Hirzebruch signature operators for some transitive invariantly oriented Lie algebroids A. When introducing a Riemannian structure in A, one can define the *-Hodge operator, the codifferential, and the first signature operator. The use of the Hochschild-Serre spectral sequence for A, and the Lusztig and Gromov examples, allow defining a second signature operator. In his lecture, the speaker described a general algebraic approach to the *-Hodge operator, Hodge Theorem, and Hirzebruch operator. He gave applications to four fundamental examples concerning signature: classical example, Lie algebroid example, Lusztig example, and Gromov example.

Session 9: Short talks – topological groups

Chair: Abdenacer Makhlouf, Université de Haute Alsace, Mulhouse, FR

11.30-12.00

Wojciech Wojtynski

Warsaw University, PL

Effective integration of Lie algebras

For integrable Banach Lie algebras the corresponding simply connected Banach Lie groups have been constructed in an explicit way. A concept of discriminant subgroup of a normed Lie algebra has been introduced and it has been shown that a Banach Lie algebra is integrable if and only if its discriminant subgroup is discrete. Relations of the discriminant subgroup with the Ado and Malcev theorems have been investigated.

12.00-12.30

Yuliya Zelenyuk

University of Witwatersrand, Johannesburg, ZA

Transformations and colorings of groups

A subset S of an Abelian group G is symmetric if there is an element g of G, such that S=2g-S. This concept has been introduced by Protasov and it has turned out to be very interesting from the point of view of Ramsey theory. The speaker gave a proof of the following theorem. For every infinite Abelian group G, endomorphism f from G to G, and a finite coloring of G, there exist a subset S of arbitrarily big cardinality and an element g, such that the union S and f(S)-f(g)+g is monochrome.

Session 10: Supersymmetry and sigma models

Chair: Aleksandr Zheltukhin, National Science Center – Kharkov Institute of Physics and Technology, UA

15.00-16.00

Thomas Strobl

Université Claude Bernard, Lyon 1, FR

Yang-Mills type sigma models

Most ordinary sigma models are functionals describing harmonic maps between two given (pseudo-)Riemannian manifolds. From the physics point of view they can be thought of as a generalization of functionals defined on a collection of scalar fields, i.e. functions from spacetime to a linear target, which is then replaced by a curved manifold in the sigma model. Yang-Mills theories play an important role in the understanding of fundamental interactions. They can be thought of as functionals defined on (fiber-linear) maps from the tangent bundle of spacetime to a Lie algebra. The speaker proposed a generalization where the target is replaced by a Lie algebroid. In a further step, he also treated higher degree form fields on spacetime and more complicated algebroids, leading e.g. to functionals for non-abelian gerbes.

16.30-17.30

Alexei Kotov
University of Luxembourg, LU

Sigma models with relation to super structures in Geometry

It turns out that a Q-manifold, invented in Physics, encodes information about basic algebraic notions such as Lie algebras, Lie and Courant algebroids, and their higher analogues. The speaker focused on the notion of Q-bundle, that is, a bundle in the category of Q-manifolds. To each homotopy class of "gauge fields" (sections in the category of graded manifolds) and each cohomology class of a certain subcomplex of forms on the fiber, one associates a cohomology class on the base. In particular, this construction generalizes the Chern-Weil formalism for principal bundles. In the case of a PQ-bundle (a bundle the typical fiber of which is a graded symplectic manifold together with a compatible Q-structure), a canonical characteristic class, which admits a local transgression as an integrand of an AKSZ-type sigma model action, has been described.