An Axiomatic Approach to the Measurement of Poverty Reduction Failure

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Highlights: We measure poverty reduction failure.

Abstract

A poverty reduction failure index is a measure of the extent of inability of a society to reduce its poverty level. This paper develops an ordering for ranking alternative income distributions in terms of poverty reduction failures. The ordering can be easily implemented using the generalized Lorenz or the Three I's of poverty (TIP) curve dominance criterion. We also characterize an existing index of poverty reduction failure using an axiomatic structure.

Key words: Poverty, Poverty reduction failure, Axioms, Indices, Ordering.
JEL Classification Codes: D31, D63, I32.

1. Introduction

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Poverty, in any dimension of human well-being, is a denial of human rights. A country has to prepare poverty reduction strategy to show how money freed up from different sources will be used for poverty alleviation. Despite targeted poverty alleviation programs undertaken by government and non-governmental organizations, poverty persists at the aggregate level. Often corrective measures adopted by government to overcome its failure for reducing poverty may generate further problems, such as beneficiaries of a program of transferring resources captured from the non-poor may not be the targeted group, which in turn may make the poor still poor. Such a problem may arise from lack of proper coordination and monitoring. A government may use a non-governmental organization to fulfill the government function for poverty reduction. But this may involve high-level regulatory problem. For whatever reasons it may be, despite various attempts undertaken for poverty reduction, poverty is still growing in many parts of our world. As demonstrated by the World Development Indicators released in 2009 by the World Bank, this is certainly the case for Sub Saharan Africa and South Asian countries. Policy makers and social scientists believed for many years that economic growth would have been the solution to the problem. But the South Asian situation does not support this solution. "South Asia presents a depressing paradox. It is among the fastest growing regions in the world. But it is also home to the largest concentration of people living in poverty. While South Asia is at a far more advanced stage of development than Sub-Saharan Africa, it has many more poor people than Sub-Saharan Africa."(See http://blogs.worldbank.org/developmenttalk/archive/201102, the blog hosted by the World Bank Chief Economist, Justin Yifu Lin.)

From this discussion it is evident that a poverty reduction program may entail some difficulty and since poverty is a violation of human dignity, measurement of the extent of poverty reduction failure is an important issue of investigation. To evaluate the success of public policies, several contributions have appeared on the topic of measuring pro-poor growth, that is, on evaluating whether poor people benefit more from growth than the rest of the population (see, among others, Duclos, 2009, Zheng, 2011 and the references cited in these papers).

An alternative approach has been proposed by Kanbur and Mukherjee (2007, KM hereafter). They assumed that income is the only source of well-being and characterized a class of indices that measures precisely poverty reduction failure without linking it to growth. In view of Justin Yifu Lin's argument, this seems to be a neat methodology; poverty reduction can as well be evaluated without relating it with the promotion of growth.

KM proposed a class of indices which, instead of looking at the total amount of resources available, focused exclusively on the resources available to the rich. "If we view the resources for poverty eradication as coming from those above the poverty line, then an increase in these resources should increase the capacity to reduce shortfalls below the poverty line. If the shortfalls nevertheless remain unchanged, this tells us something about the society in question. We would argue that the same absolute poverty is "worse" if the resources available to address poverty are greater." (KM, p.53). KM's major discussion and empirical application of poverty reduction failure using data prepared by Chen and Ravallion (2004) are based on some members of the family they have characterized. However, none of these indices have been characterized separately.

In this article we develop a poverty reduction failure ordering, which enables us to rank two income distributions in terms of their levels of failures. More precisely, given the poverty line, the ordering tells us whether one distribution corresponds to at least as high poverty reduction failure as another for all poverty reduction failure indices satisfying certain desirable postulates. Thus, once two distributions can be ranked using this ordering it does not become necessary to calculate the values of failure indices to judge whether one distribution is affected by higher or lower levels of failures. The ordering can be easily implemented by seeking generalized Lorenz dominance or second order stochastic dominance of the incomes of the poor. Given that the poverty line is fixed, we can also use the Three I's of Poverty (TIP) curve dominance for ranking purposes (Jenkins and Lambert, 1997). We clearly indicate that there are interesting policy applications of this ordering. We also characterize one of the KM indices of poverty reduction failure. This particular form, which is quite easy to understand, becomes helpful in designing empirical applications of the ordering. The characterization enables us to understand the index from an alternative perspective.

The next section of the paper presents the ordering and the characterization theorem is reported in Section 3. Finally, Section 4 makes some concluding remarks.

2. The Poverty Reduction Failure Ordering

For a population of size n, let $x = (x_1, x_2, ..x_n)$ be the distribution of income, where x_i, the income of person i, is taken from a non-degenerate interval $[a,b]$ in the non-negative part $\Re^1{}_+$ of the real line $\Re$. Let D^n be that subset of $[a,b]^n$, the n-fold Cartesian product of $[a,b]$, in which all income distributions are non-decreasingly ranked, that is, for all $x \in D^n$, $x_1 \le x_2 \le .... \le x_n$. Assume that D^n is the set of all income distributions in this n-person economy .The set of all possible income distributions is $D = \bigcup_{n \in N} D^n$, where N is the set of natural numbers. Often it will be necessary to assume that $[a,b]$ is a subset of $\Re^1_{++}$, the strictly positive part of the real line.

It is assumed that the exogenously given poverty line z is positive and takes values in the subset $[z_-, z_+]$ of the real line, where $z_- > 0 \ge a$ and $z_+ < b$ are the minimum and maximum poverty lines. For any income distribution x, person i is said to be poor if $x_i < z$. Person i is called non-poor or rich if he is not poor. Assume that there are q poor persons in the society. For any $x \in D^n$, let x^p (x^r) be the income distribution of the poor (rich). Since x is non-decreasingly ordered, $x^p = (x_1, x_2, ..., x_q)$ and $x^r = (x_{q+1}, x_{q+2}, ..., x_n)$. For a given population size n, a poverty index P is a real valued function defined on $D \times [z_-, z_+]$. Thus, given any income distribution $x \in D^n$, $n \in N$ and a poverty line $z \in [z_-, z_+]$, $P(x,z)$ determines the extent of poverty associated with the distribution x.

For any $x \in D^n$, let $d_i = \dfrac{z - x_i}{z}$ be the normalized deprivation of poor person i with respect to z, where $1 \le i \le q$. Similarly, $e_i = \left(\dfrac{x_i - z}{z}\right)$, $q+1 \le i \le n$, represents the normalized excess income of rich person i over z. Clearly, $d_i \in \left(0, \dfrac{z-a}{z}\right]$, $1 \le i \le q$ and

$e_i \in \left[0, \frac{b-z}{z}\right]$, where $q+1 \le i \le n$. For any $n \in N$, $x \in D^n$, d and e stand respectively for the corresponding deprivation and normalized excess income vectors $\left(d_1, d_2, ..d_q\right)$ and $\left(e_{q+1}, e_{q+2}, ..e_n\right)$, which, by assumption, are ordered non-increasingly and non-decreasingly respectively. For any $q, n \in N$, where $q \le n$, let Γ^q and Ω^{n-q} be respectively those subsets of $\left(0, \frac{z-a}{z}\right]^q$ and $\left[0, \frac{b-z}{z}\right]^{n-q}$ whose elements are ordered non-increasingly and non-decreasingly respectively. Clearly, for all $q, n \in N$, where $q \le n$, $(d, e) \in \Gamma^q \times \Omega^{n-q}$.

A poverty reduction failure (PRF) index A is a real valued function defined on the set of all possible deprivation and normalized excess income distributions. More precisely, $A: \bigcup_{q \in N, q \le n,\, n \in N} \bigcup \Gamma^q \times \Omega^{n-q} \to \Re^1_+$ That is, for all $n \in N$, $(d, e) \in \Gamma^q \times \Omega^{n-q}$, $A(d, e)$ gives the level of poverty reduction failure associated with the deprivation-normalized excess income distribution (d, e).

KM suggested the following axioms for an arbitrary poverty reduction failure (PRF) index A.

Continuity (CON): For all $q, n \in N$, where $q \le n$, A is continuous in its arguments.

Symmetry (SYM): For all $q, n \in N$, where $q \le n$, $(d, e) \in \Gamma^q \times \Omega^{n-q}$, $A(d, e) = A(dB, eC)$, where B and C are two permutation matrices of orders q and $(n-q)$ respectively.[1]

Monotonicity (MON**):** (i) For all $q, n \in N$, where $q \le n$, $(d, e), (d^1, e^1) \in \Gamma^q \times \Omega^{n-q}$ if $d^1_j = d_j$ for all $j \le q, j \ne i$, $d^1_i > d_i, i \le q$; $e^1_j = e_j$ for all $j \in \{n-q, ..., n\}$ then $A(d, e) < A(d^1, e^1)$. (ii) For all $q, n \in N$, where $q \le n$, $(d, e), (d^1, e^1) \in \Gamma^q \times \Omega^{n-q}$ if

[1] A $J \times J$ matrix with entries 0 and 1 is called a permutation matrix of order J if each of its rows and columns sums to 1.

$d_j^1 = d_j$ for all $j \in \{1,....,q\}$; $e_j^1 = e_j$ for all $j \in \{n-q,....,n\}$, $j \neq i$, $e_i^1 > e_i, n-q \leq i \leq n$, then $A(d,e) < A(d^1,e^1)$.

Minimality (MIN**)**: (i) For all $q,n \in N$, where $q \leq n$, if $(d,e) \in \Gamma^q \times \Omega^{n-q}$ is such that $d_j = 0$ for all $j \in \{1,....,q\}$, then $A(d,e) = 0$. (ii) For all $q,n \in N$, where $q \leq n$, if $(d,e) \in \Gamma^q \times \Omega^{n-q}$ is such that $e_j = 0$ for all $j \in \{n-q,....,n\}$, then $A(d,e) = 0$.

Transfers Principle (TRP**)**: For all $q,n \in N$, where $q \leq n$, $y \in D^n$ suppose that $x \in D^n$ is obtained from y by the following transformation: for some i, j $(i < j \leq q)$ and $\theta > 0$, $x_l = y_l$ for all $l \neq i, j$; $x_i - y_i = y_j - x_j = \theta$, where $\theta \leq (y_j - y_i)/2$ if $j = i+1$; $\theta \leq \min\{(y_{i+1} - y_i),(y_j - y_{j-1})\}$ if $j > i+1$. Then $A(d(x),e(x)) < A(d(y),e(y))$, where $d(x)$ is the deprivation vector based on the distribution x and so on.

Subgroup Consistency for Deprivations (SCD): For all $q,n \in N$, where $q \leq n$, $(d^1,d^2,e) \in \Gamma^q \times \Omega^{n-q}$ and $(\hat{d}^1,d^2,e) \in \Gamma^q \times \Omega^{n-q}$, where $d^1,\hat{d}^1 \in \Gamma^{q_1}$, $d^2 \in \Gamma^{q-q_1}$, if $A(\hat{d}^1,e) > A(d^1,e)$, then $A(\hat{d}^1,d^2,e) > A(d^1,d^2,e)$.

Subgroup Consistency for Normalized Excess Incomes (SNI): For all $q,n \in N$, where $q \leq n$, $(d,e^1,e^2) \in \Gamma^q \times \Omega^{n-q}$ and $(d,\hat{e}^1,e^2) \in \Gamma^q \times \Omega^{n-q}$, where $e^1,\hat{e}^1 \in \Omega^{n_1-q}$, $e^2 \in \Omega^{n-n_1}$, if $A(d,\hat{e}^1) > A(d,e^1)$, then $A(d,\hat{e}^1,e^2) > A(d^1,e^1,e^2)$.

Principle of Population (POP): For all $q,n \in N$, where $q \leq n$, if $(d,e) \in \Gamma^q \times \Omega^{n-q}$, $A(d,e) = A((d,e)^l)$, where $(d,e)^l$ is the $l-$fold replication of (d,e), $l \geq 2$ being any integer.

CON ensures that the index will not be over sensitive to minor changes in deprivations and normalized excess incomes. SYM means that all characteristics other than deprivations and excesses are irrelevant to the measurement of PRF. Part (i) of MON is Sen's (1976) Monotonicity Axiom which demands that a reduction (an increase) in the income (deprivation) of a poor person increases the PRF index. Since all income distributions are ordered, only rank preserving reductions and increments in incomes are allowed. Given the poverty line, part (ii) requires increasingness of the PRF if incomes of the non-poor increase. Because of increase in the income of a person above the poverty line, inequality is likely to increase. Particularly, the ratios and absolute differences between incomes of all persons who have lower incomes than him and his income increase. However, the scope of redistribution of income from rich to poor also increases. In other words, as KM argued, if we regard a society's ability to eliminate poverty in terms of availability of resources of persons above the poverty line, then the ability, accompanied by inequality augmentation, increases as resources increase. Thus, there is a trade-off between higher inequality and higher possibility of income redistribution. Part (ii) of MON demands that the trade-off balances out in favor of higher inequality. As we have argued earlier, the source of increase in inequality can be poor coordination and monitoring problems or some other difficulty of income redistribution from rich to poor.

Recently, in the literature on richness measurement this issue has been addressed from alternative perspectives. For instance, more resource at the hands of the richer rich resulting from non-egalitarian transfers from poorer rich to richer rich increases the command of the rich in terms of richness. Therefore, under progressive taxation more funds can now be raised for financing production of public goods and other beneficial programs like targeted poverty alleviation (see Atkinson, 2007; Peichl et al. 2010 and Bose et al., 2012).

MIN says that failure is minimum (zero) if there is no poor person in the society or there is no excess income to redistribute.[2] TRP, which demands that a rank preserving transfer from a poor to a poorer decreases deprivation, is Donaldson-Weyamark's (1986) 'Minimal Transfer Axiom'. The two subgroup consistency axioms parallel the Foster-Shorrocks (1991) poverty subgroup consistency axiom and require that for any partitioning of the population into subgroups if shortfalls (respectively normalized excess incomes) in a subgroup fall then PRF reduces. POP demands invariance of PRF under replications of the population.

[2] Since we use the term 'normalization' in a different context, here we use the term 'minimality' instead of 'normalization', as used by Kanbur and Mukherjee (2007).

The axioms CON, SYM, MON, TRP, SCD, SNI and POP, MIN are consistent in the sense that there exists a PRF index satisfying these axioms. KM showed that these axioms along with a proportionality condition, which demands that a proportional increase in deprivation argument can be balanced by a proportional decrease in excess income argument, are necessary and sufficient for the following family of PRF indices:

$$A_{K,M}(d,e) = g\left(\left\{ \frac{1}{n} \sum_{i=1}^{q} \varphi(d_i) \right\} . \left\{ \frac{1}{n} \sum_{i=q+1}^{n} \psi(e_i) \right\}^{1/\delta} \right), \tag{1}$$

where δ is a non-zero constant and h is continuous and increasing in its argument. For the purpose of discussion and empirical application, they assumed that g and ψ are identity mapping so that $A_{K,M}$ in (1) becomes

$$A_\delta(d,e) = \left(\frac{1}{n} \sum_{i=1}^{q} \varphi(d_i) \right) \left(\frac{1}{n} \sum_{i=q+1}^{n} e_i \right)^{1/\delta}. \tag{2}$$

Note that part (ii) of MON requires positivity of δ. A rank preserving transfer of income from a richer rich to a poorer rich will decrease or increase A_δ according as $0 < \delta < 1$ or $\delta > 1$.

Next, we note that if the poverty line coincides with the median income, then MON becomes the increased spread axiom of bipolarization measurement, which is concerned with the dispersion of incomes from the median. Since a clustering of incomes on the either side of the median should increase polarization (see Wolfson, 1994 and Chakravarty, 2009), TRP shows that polarization and poverty reduction failure are two different concepts.

In order to develop a PRF ordering of income distribution, we assume that a general PRF index can be written as $H\left(F\left(d_1, d_2, ..., d_q\right), L\left(e_{q+1}, e_{q+2}, ..., e_n\right)\right)$, where F and L are non-decreasing and population replication invariant, and F is also S-convex. Since it is not clear how a PRF index should change under a redistribution of income between two rich persons, without loss of generality, we aggregate the normalized excess incomes of the rich in the function H through the non-decreasing function L. Non-decreasingness of F ensures that it does not decrease under a reduction in the income of a poor. S-convexity means that the individuals are treated symmetrically and a rank-preserving transfer of income from a poor to a poorer poor does not increase F and hence H. Thus, non-decrasingness and S-convexity of F along with non-decreasingness of H ensure that the PRF index satisfies SYM, the weak versions of TRP and part (i) of MON. Likewise, non-decreasingness of L guarantees that part (ii) of MON is satisfied by H. Evidently, H fulfills POP. A PRF satisfying these conditions will be called regular.

For any $m,n \in N$, let $x \in D^m$ and $y \in D^n$ be arbitrary. Denote the corresponding deprivation-normalized excess income vectors by $\left(d\left(x^p\right),e\left(x^r\right)\right)$ and $\left(d\left(y^p\right),e\left(y^r\right)\right)$, where $\left(d\left(x^p\right),e\left(x^r\right)\right),\in \Gamma^{q(x)} \times \Omega^{m-q(x)}$ and $\left(d\left(y^p\right),e\left(y^r\right)\right) \in \Gamma^{q(y)} \times \Omega^{n-q(y)}$, $q(x)\left(q(y)\right)$ being the number of poor persons in $x(y)$. Then we say that x poverty reduction failure dominates y, what we write $x \geq_{PRF} y$, if and only for any non-decreasing $H: \Re^1{}_+ \times \Re^1{}_+ \to \Re^1{}_+$, $H\left(F\left(d\left(x^r\right)\right),L\left(e\left(x^r\right)\right)\right) \geq H\left(F\left(d\left(y^p\right)\right),L\left(e\left(y^r\right)\right)\right)$ for all $(F,L): \bigcup_{q\in N, q\le n} \bigcup_{n\in N} \Gamma^q \times \Omega^{n-q} \to \Re^1{}_+ \times \Re^1{}_+$ that satisfy POP and weak forms of MON, with F being also S-convex.

For any $x \in D^n$ the generalized Lorenz curve $GLC(x, j/n)$ of x is a plot of the $\sum_{i=1}^{j} x_i \Big/ n$ against j/n, where $j = 0,1,...,n$ and $GLC(x,0)=0$. For $x \in D^m$, $y \in D^n$, x generalized Lorenz dominates y if the generalized Lorenz curve of x is nowhere below that of y (Shorrocks, 1983).

The following theorem can now be stated:

Theorem 1: For any $m,n \in N$, let $x \in D^m$ and $y \in D^n$ be arbitrary. Then the following conditions are equivalent:

(i) $x \geq_{PRF} y$.

(ii) y^p generalize Lorenz dominates x^p and $L\left(e\left(x^r\right)\right) \geq L\left(e\left(y^r\right)\right)$.

Proof: $(i) \Rightarrow (ii)$: Define $H\left(F\left(d\left(x^p\right)\right),L\left(e\left(x^r\right)\right)\right) = \left(L\left(e\left(x^r\right)\right)\right)^{\theta} F\left(d\left(x^p\right)\right)$, where $\theta \geq 0$ and F and L are any arbitrary non-negative, non-decreasing, population replication invariant functions and F is also S-convex . Then H is non-decreasing and $H\left(F\left(d\left(x^p\right)\right), L\left(x\left(e^r\right)\right)\right) \geq H\left(F\left(d\left(y^p\right)\right), L\left(y\left(e^r\right)\right)\right)$ implies that $\left(L\left(e\left(x^r\right)\right)\right)^{\theta} F\left(d\left(x^p\right)\right) \geq \left(L\left(e\left(y^r\right)\right)\right)^{\theta} F\left(d\left(y^p\right)\right)$ **.** By choosing $\theta = 0$ **,** we get $F\left(d\left(x^p\right)\right) \geq F\left(d\left(y^p\right)\right)$**.** Now**,** let $\left(x^p\right)^{q(y)}\left(\left(y^p\right)^{q(x)}\right)$be the $q(y)\left(q(x)\right)-$fold replication of $x^p\left(y^p\right)$so that the total number of poor persons in the two replicated distributions is $q(x)q(y)$. Since F is population replication invariant$F\left(d\left(\left(x^p\right)^{q(y)}\right)\right) = F\left(d\left(x^p\right)\right)$ and $F\left(d\left(\left(y^p\right)^{q(x)}\right)\right) = F\left(d\left(y^p\right)\right)$, where $\left(x^p\right)^{q(y)}\left(\left(y^p\right)^{q(x)}\right)$ is the $q(y)\left(q(x)\right)-$ fold replication of $x^p\left(y^p\right)$. But the arbitrary function F is non-decreasing and S-convex as well. This is equivalent to the condition that $\left(y^p\right)^{q(x)}$ generalized Lorenz dominates $\left(x^p\right)^{q(y)}$ (see Foster, 1984 and Chakravarty, 2009, p. 55-56.). In view of population replication invariance, the generalized Lorenz curve of $\left(y^p\right)^{q(x)}$ ($\left(x^p\right)^{q(y)}$) coincides with that $x^p\left(y^p\right)$. Hence y^p generalize Lorenz dominates x^p .

Note that by population replication invariance, the value of the function $L\left(e\left(x^r\right)\right)$ for the distribution x^r is same as that for the distribution $\left(x^r\right)^{n-q(y)}$, the $\left(n-q(y)\right)$ -fold replication of x^r . By choosing $F\left(d\left(x^p\right)\right) = 1$, in the inequality $\left(L\left(e\left(x^r\right)\right)\right)^{\theta} F\left(d\left(x^p\right)\right) \geq \left(L\left(e\left(y^r\right)\right)\right)^{\theta} F\left(d\left(y^p\right)\right)$ and using population replication invariance of L, we get $L\left(e\left(x^r\right)\right) = L\left(e\left(\left(x^r\right)^{n-q(y)}\right)\right) \geq L\left(e\left(\left(y^r\right)^{m-q(x)}\right)\right) = L\left(e\left(y^r\right)\right)$**.**

$(i) \Rightarrow (ii)$: Using population replication invariance of the generalized Lorenz curve, we can show that the generalized Lorenz superiority of y^p over x^p is equivalent to the condition that $F\left(d\left(x^p\right)\right) \geq F\left(d\left(y^p\right)\right)$ for all non-decreasing, population replication invariant and S-convex F . We also have the following population replication invariant inequality $L\left(e\left(x^r\right)\right) \geq L\left(e\left(y^r\right)\right)$. Since H is non-decreasing in its arguments, we must have $H\left(F\left(d\left(x^r\right)\right), L\left(e\left(x^r\right)\right)\right) \geq H\left(F\left(d\left(y^p\right)\right), L\left(e\left(y^r\right)\right)\right)$ for all $H : \Re^1{}_+ \times \Re^1{}_+ \to \Re^1{}_+$. Hence $x \geq_{PRF} y$. Δ

What theorem 1 says is the following. Of two income distributions x and y , if the incomes of the poor in y generalized Lorenz dominates (equivalently, second order stochastically dominates) that in x and a non-decreasing function of the excess incomes of the non-poor in x is at least as large as that in y , then the level of poverty reduction failure associated with x is at least as large as that for y for all regular poverty reduction failure indices. The converse is also true. Note that for any $\delta > 0$ the KM family given by (2) is a member of this class of indices. Note also that we do not need subgroup decomposability of F for this general result to hold. Thus, for a fixed poverty line, the Sen (1976) index, which is non-subgroup decomposable, can be used to represent F. An attractive feature of our result is that population size is not assumed to be fixed for the distributions under consideration. This result has an important policy application. Given the exogenously determined poverty line, if one region of a country dominates another in the sense of condition (ii) of the theorem, then since the first region is affected by higher poverty reduction failure; it may need prior policy attention for poverty reduction failure. This observation applies equally well to inter-country comparisons under ceteris paribus assumptions.

For any $x \in D^n$, Jenkins and Lambert (1997) defined the TIP curve of x , $TIP\left(x, {}^{i}\!/\!_{n}\right)$ as a plot of $\frac{1}{nz}\sum_{j=1}^{i}(z - x_j^*)$ against $\frac{i}{n}$, where $1 \leq i \leq n$, $x^* = (x_1^* ,..., x_n^*)$ is the censored income distribution corresponding to $x = (x_1,..., x_n)$ and $TIP(x,0) = 0$. In a censored income distribution all incomes above the poverty line are replaced by the poverty line itself.

We can now restate our Theorem 2 as follows:

Theorem 2: For any $m, n \in N$, let $x \in D^m$ and $y \in D^n$ be arbitrary. Then the following conditions are equivalent:

(iii) $x \geq_{PRF} y$.

(iv) x TIP curve dominates y , that is, the TIP curve of x is nowhere below that of y and $L\left(e\left(x^r\right)\right) \geq L\left(e\left(y^r\right)\right)$.

Clearly, while theorem 1 is stated in terms of welfare superiority of the poor, theorem 2 is stated in terms of their deprivation dominance. Condition (ii) of Theorem 1 and condition (iv) of Theorem 2 explicitly state these conditions, which are very easy to check.

By definition, for a fixed poverty line, the function *F* is a focused poverty index, that is, it is independent of the incomes rich. Since the poverty line has been fixed arbitrarily, it may change over the interval $\left[z_-, z_+\right]$. Consequently, the set of poor person changes. Therefore, income distribution of every group of poor persons should be strictly separable from that of the non-poor persons. This implies that the poverty index is completely strictly recursive (see Blackorby et al., 1978). This rules out all poverty indices $P(x, z)$ that depend on ranks of the poor persons in the entire income distributions (see, for example, Thon, 1979). However, all subgroup decomposable poverty indices are completely strictly recursive. Assuming that the income distributions are non-increasingly ordered, the Donaldson-Weymark (1980) illfare-ranked single series Ginis also satisfy this condition (see Bossert, 1990). This leaves a quite large class of functions from which to choose.

Theorem 1 is can be used to test empirically whether any pair of distributions can be ranked under the ordering $\geq_{PRF}$. For this assume for simplicity that $L\left(e\left(y^r\right)\right) = \frac{1}{n}\sum_{i=q+1}^{n} e_i(y)$**.** Note that this specification is insensitive to the redistribution of incomes among the rich**.** Then one method of doing this is to calculate $\frac{1}{n}\sum_{i=q+1}^{n} e_i(y)$ for each of the distributions and make pair-wise comparisons of these values and check for generalized Lorenz domination of income distributions of the poor. In case we do not have rankings in the desired directions, we cannot unambiguously say anything about the ordering of the distributions in terms of PRF. Thus, the ordering $\geq_{PRF}$ is transitive but not complete.

2. The Characterization Theorem

In this section we develop a characterization of a particular form of the KM family. Later in this section we explain the merit of this particular form. For our characterization in addition to some of the KM axioms, we will need the following axioms also.

Normalization (NOM): (i) For all $q,n \in N$, where $q \leq n$, if $n \in N$ if $(d,e) \in \Gamma^q \times \Omega^{n-q}$ is such that for all $j \in \{n-q,....,n\}$, $e_j = c$, where $c > 1$ is a constant, then $A(d,e) = A\left(d, \frac{n-q}{n}c\right)$. (ii) For all $q,n \in N$, where $q \leq n$, if $n \in N$ if $(d,e) \in \Gamma^q \times D^{n-q}$ is such that $\frac{1}{n}\sum_{i=q+1}^{n} e_i = 1$, then $A(d,e) = h(d)$, where $h : [0,1)^q \to \Re^1{}_+$ is increasing and continuous.

Scaling Principle (SCP)**:** (i) For all $q,n \in N$, where $q \leq n$, $(d,e) \in \Gamma^q \times \Omega^{n-q}$, $A(d,ke) = kA(d,e)$, where $k > 1$ is a scalar. **(**ii) For all $q,n \in N$, where $q \leq n$, $(d,e) \in \Gamma^q \times \Omega^{n-q}$, $A(kd,e) = kA(d,e)$, where $0 < k < 1$ is a scalar.

Part (i) of NOM is non-poor counterpart to Sen's (1976) poverty normalization axiom. Sen argued that when all the poor persons have the same income, product of the poor-headcount ratio q/n and the income gap ratio $\left(\sum_{i=1}^{q}(z - x_i)\right)\Big/qz$ can gave us an adequate picture of poverty. Likewise, when all the rich persons have the same income, the product of the non-poor-head count ratio $(n-q)/n$ and the excess income ratio $\left(\sum_{i=q+1}^{n}(x_i - z)\right)\Big/(n-q)z$ can be a suitable indictor of non-poverty. Part (ii) of the axiom says that if the sum of the normalized excesses of the rich, as a fraction of the total population size, is one, then the level of poverty reduction failure is determined by the deprivations of the poor. This is a cardinality principle. Note that apart from recognizing the dependence of the PRF index on the deprivations of the poor, it incorporates the excess incomes of the rich and their population size in a particular way. It may be worthwhile to mention that this condition is satisfied by the general functional form considered by KM for discussion and empirical application. According to part (i) of SCP, given deprivations of the poor, if the normalized excess incomes of all the rich are scaled up by a common factor, then the PRF index also gets scaled up by the same factor. Likewise, given normalized excess incomes of the rich, a proportionate reduction in the deprivations of the poor decreases the PRF index accordingly. Note that the scaling factor k is chosen such that the number of poor persons in the society does not change. Thus, SCP is simply a variant of the linear homogeneity condition, a postulate used in index number theory extensively (see Chakravarty, 2009). According to linear homogeneity of a function, an equiproportionate change in all arguments of the function leads to an equiproportionate change in the functional value.

We can now state the following theorem.

Theorem 3: The only poverty reduction failure index $A: \bigcup_{q \in N, q \leq n} \bigcup_{n \in N} \Gamma^q \times \Omega^{n-q} \to \Re^1_+$ satisfying axioms CON, SYM, MON, TRP, SCD, SNI, POP, NOM and SCP is a positive multiple of

$$A_\varphi(d,e) = \left(\frac{1}{n} \sum_{i=1}^{q} \varphi(d_i) \right) \left(\frac{1}{n} \sum_{i=q+1}^{n} e_i \right), \tag{3}$$

where $\varphi : \left[0, \frac{z-a}{z}\right) \to \Re^1_+$ is continuous, increasing and strictly convex.

Proof: Theorem 1 of Kanbur and Mukherjee (2007) shows that the only PRF index satisfying CON, SYM, MON, TRP, SCD, SNI and POP is given by

$$A(d,e) = f\left(\left\{ \frac{1}{n} \sum_{i=1}^{q} \varphi(d_i) \right\}, \left\{ \frac{1}{n} \sum_{i=q+1}^{n} \psi(e_i) \right\} \right), \tag{4}$$

where φ and ψ are continuous and increasing, φ is strictly convex and f is continuous and increasing in its arguments.

Now, define the function $u_\varphi(e) = f\left(\left\{ \frac{1}{n} \sum_{i=1}^{q} \varphi(d_i) \right\}, \left\{ \frac{1}{n} \sum_{i=q+1}^{n} \psi(e_i) \right\} \right)$. Increasingness of f in its arguments implies that $u_\varphi(e)$ is also increasing. Suppose that for all $j \in \{n-q, ..., n\}$, $e_j = c > 1$. Then by part (i) of NOM, we have

$u_\varphi\left(\frac{1}{n}\sum_{i=q+1}^{n}\psi(c)\right)=u_\varphi\left(\frac{(n-q)}{n}c\right)$. Since $u_\varphi(e)$ is increasing, it must be the case that $\frac{1}{n}\sum_{i=q+1}^{n}\psi(c)=\frac{(n-q)}{n}c$, which implies that $\psi(c)=c$. Continuity of ψ ensures that $\psi(e_i)=e_i$ for all $e_i\in\Re^1_+$. Substituting this form of ψ in (4) we get $f\left(\left\{\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)\right\},\left\{\frac{1}{n}\sum_{i=q+1}^{n}\psi(e_i)\right\}\right)= f\left(\left\{\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)\right\},\left\{\frac{1}{n}\sum_{i=q+1}^{n}e_i\right\}\right)$.

In view of part (i) SCP, for any $k>1$, it follows that

$$f\left(\left\{\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)\right\},\left\{\frac{k}{n}\sum_{i=q+1}^{n}e_i\right\}\right)=kf\left(\left\{\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)\right\},\left\{\frac{1}{n}\sum_{i=q+1}^{n}e_i\right\}\right) \tag{5}$$

This relationship is true for all values of $\frac{1}{n}\sum_{i=q+1}^{n}e_i$. Therefore, it is true for $\frac{1}{n}\sum_{i=q+1}^{n}e_i=1$ as well.

Now, if $\frac{1}{n}\sum_{i=q+1}^{n}e_i=1$, then from (5), by part (ii) of NOM, we get

$$f\left(\left\{\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)\right\},k.1\right)=kf\left(\left\{\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)\right\},1\right)=kh(d). \tag{6}$$

If $h(d)=0$, then there is nothing to prove. We, therefore, assume that $h(d)>0$. It then follows from (6) that

$$k=\frac{f\left(\left\{\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)\right\},k\right)}{h(d)}. \tag{7}$$

Plugging the value of k from (7) into (5) we get

$$f\left(\left\{\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)\right\},\left\{\frac{k}{n}\sum_{i=q+1}^{n}e_i\right\}\right)=\frac{f\left(\left\{\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)\right\},k\right)}{h(d)}\ f\left(\left\{\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)\right\},\left\{\frac{1}{n}\sum_{i=q+1}^{n}e_i\right\}\right). \tag{8}$$

Define

$$G\left(\left\{\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)\right\},\left\{\frac{1}{n}\sum_{i=q+1}^{n}e_i\right\}\right)=\frac{f\left(\left\{\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)\right\},\left\{\frac{1}{n}\sum_{i=q+1}^{n}e_i\right\}\right)}{h(d)}. \tag{9}$$

Then from (8) and (9) it follows that

$$G\left(\left\{\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)\right\},\left\{\frac{k}{n}\sum_{i=q+1}^{n}e_i\right\}\right)=\frac{f\left(\left\{\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)\right\},\left\{\frac{k}{n}\sum_{i=q+1}^{n}e_i\right\}\right)}{h(d)}.$$

$$=\frac{1}{h(d)}\left[\frac{f\left(\left\{\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)\right\},k\right)\left(\left\{\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)\right\},\frac{1}{n}\sum_{i=q+1}^{n}e_i\right)}{h(d)}\right]$$

$$=\left[\frac{f\left(\left\{\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)\right\},k\right)}{h(d)}\right]\left[\frac{\left(\left\{\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)\right\},\frac{1}{n}\sum_{i=q+1}^{n}e_i\right)}{h(d)}\right]$$

$$=G\left(\left\{\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)\right\},k\right)G\left(\left\{\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)\right\},\left\{\frac{1}{n}\sum_{i=q+1}^{n}e_i\right\}\right). \tag{10}$$

Now, using steps similar to equations (33)-(39) of Chakravarty (2009a) we can show that $G\left(\left\{\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)\right\},\left\{\frac{1}{n}\sum_{i=q+1}^{n}e_i\right\}\right)=\frac{1}{n}\sum_{i=q+1}^{n}e_i$, which in view of (9), gives

$$f\left(\left\{\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)\right\},\left\{\frac{1}{n}\sum_{i=q+1}^{n}e_i\right\}\right)=h(d)\left(\frac{1}{n}\sum_{i=q+1}^{n}e_i\right)=f\left(\left\{\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)\right\},1\right)\left(\frac{1}{n}\sum_{i=q+1}^{n}e_i\right). \tag{11}$$

Since the separable function $h(d)=f\left(\left\{\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)\right\},1\right)$ depends essentially on $\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)$, we can rewrite it as $h(d)=\xi\left(\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)\right)$, where $\xi:\Re_{+}^{1}\to\Re^{1}$ is continuous and increasing. This in turn gives

$$f\left(\left\{\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)\right\},\left\{\frac{1}{n}\sum_{i=q+1}^{n}e_i\right\}\right)=\xi\left(\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)\right)\left(\frac{1}{n}\sum_{i=q+1}^{n}e_i\right). \tag{12}$$

Equation (12) along with the definition of the PRF shows that ξ is non-negative valued.

Now, applying part (ii) of NOM to the form of f given by (12), we get

$$k\xi\left(\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)\right)\left(\frac{1}{n}\sum_{i=q+1}^{n}e_i\right)=\xi\left(\frac{1}{n}\sum_{i=1}^{q}\varphi(kd_i)\right)\left(\frac{1}{n}\sum_{i=q+1}^{n}e_i\right), \tag{13}$$

where $0<k<1$ is arbitrary. Since ξ is a single coordinated function, (13) holds for all (d,e) if and only if $\xi(0)=0$ and $\xi(t)=\lambda t$, where $\lambda\geq 0$ is a scalar. MON rules out the trivial solution $\xi(0)=0$ and $\lambda=0$. So, $\lambda>0$. Hence we have

$$f\left(\left\{\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)\right\},\left\{\frac{1}{n}\sum_{i=q+1}^{n}e_i\right\}\right)=\lambda\left(\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)\right)\left(\frac{1}{n}\sum_{i=q+1}^{n}e_i\right). \tag{14}$$

This establishes the necessity part of the theorem. The sufficiency is easy to verify. Δ

The index in (14) drops out as a particular member of A_δ in (2) if $\delta=1$. (Since λ is a scale parameter, we assume without loss of generality that λ =1.) For $\delta=1$, A_δ (that is, the index in (14)) remains unaffected under a rank preserving redistribution of income from a richer rich to a poorer rich. Since an inequality index should register a decrease unambiguously under such a change, it is clear that a PRF index is different from an inequality index.

The function φ is called an individual poverty function and $\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)$ is a subgroup decomposable poverty index, where subgroup decomposability demands that for any partitioning of the population into subgroups overall poverty is given by the sum of population share weighted subgroup poverty levels (see Foster and Shorrocks, 1991, Zheng, 1997 and Chakravarty, 2009, for a discussion). Therefore, the index in (14) is the product of a subgroup decomposable poverty index and a symmetric linear function $\frac{1}{n}\sum_{i=q+1}^{n}e_i$ of the excess incomes of the rich. This product is very easy to understand in the sense that its direct relationship with rich persons' excess incomes over the poverty line and the extent of poverty suffered by the poor are specified in an extremely simple manner. Furthermore, we have already noted how the simple specification $\frac{1}{n}\sum_{i=q+1}^{n} e_i$ becomes helpful for empirical applications of the dominance Theorems 1 and 2. These are two major reasons for developing a characterization of the index given by (14).

In order to illustrate the formula in (14), let. $\varphi(t)=t^{\alpha}$, where $\alpha>1$. Then $\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)$ becomes the Foster-Greer-Thorbecke (1984) poverty index and the explicit formula for the corresponding PRF index in (3) becomes

$$A_{\alpha}(d,e)=\left(\frac{1}{n}\sum_{i=1}^{q}(d_i)^{\alpha}\right)\left(\frac{1}{n}\sum_{i=q+1}^{n}e_i\right)=\left(\frac{1}{n}\sum_{i=1}^{q}\left(1-\frac{x_i}{z}\right)^{\alpha}\right)\left(\frac{1}{n}\sum_{i=q+1}^{n}\left(\frac{x_i-z}{z}\right)\right). \tag{15}$$

A rank preserving transfer of income from a poor to a poorer poor decreases A_{α} by a larger amount the higher is the value of α. For α =2, A_{α} contains the coefficient of variation of the income distribution of the poor as a component. Alternatives of interest arises from the specifications $\varphi(t)=1-(1-t)^{\theta}, 0<\theta<1$, and $\varphi(t)=-\log t$ (assuming positivity of all incomes). The subgroup decomposable poverty indices associated with these specifications are respectively the Chakravarty (1983) and Watts (1967) indices.[3]

Our definition of the PRF index on $d_i's$ and $e_i's$ implicitly assumes that it satisfies a scale invariance axiom- an equiproportional variation in all the incomes and the poverty line does not change the PRF index. An alternative notion of invariance is translation invariance, which demands that an equal absolute change in all the incomes and the poverty line keeps the PRF index unaltered. Indices satisfying these two notions of invariance are referred to as relative and absolute indices respectively. In the absolute case we define the PRF index on the absolute deprivations $\omega_i=(z-x_i)$, $i\leq q$ and absolute excesses $\upsilon_i=(x_i-z), i>q$. Then using our axiomatic structure we can characterize the following absolute PRF index

[3] Zheng (2000) defined a distribution sensitivity measure using the individual poverty function and made a comparison among these three indices of poverty using the sensitivity measure.

$$c\left(\frac{1}{n}\sum_{i=1}^{q}\xi(\omega_i)\right)\left(\frac{1}{n}\sum_{i=q+1}^{n}\upsilon_i\right), \tag{16}$$

where the individual poverty function $\xi:(0,z-a]\to\Re^1_+$ is continuous, increasing and strictly convex and $c>0$ is a constant. In (16) if we set $\xi(t)=(\exp(rt)-1)$, where r is any positive scalar, then the resulting index becomes the Zheng(2000) index.

We now consider the notion of proportional transfers introduced by Fleurbaey and Michal (2001) which are more egalitarian than the transfers considered in TRP and examine their implications on the PRF indices[4].

Proportional Transfers Principle (PTP): For all $q,n\in N$, where $q\le n$, $y\in D^n$ suppose that $x\in D^n$ is obtained from y by the following transformation: for some i,j $(i<j\le q)$ and, $x_l=y_l$ for all $l\ne i,j$; $x_i=y_i(1+\rho)\le x_j=y_j(1-\rho)$, where $\rho>0$ is small such that the ranks of the individuals in the distributions x and y are the same. Then $\sum_{i=1}^{q}U(x_i)>\sum_{i=1}^{q}U(y_i)$, where the individual utility $U:[a,b]\subset\Re^1_{++}\to\Re$ function is continuous and increasing

PTP demands that welfare of the poor should increase under a transfer of income of size $y_j\rho$ from the richer poor j to the poorer poor i given that i actually receives the lower amount $y_i\rho$ instead of the actually transferred amount $y_j\rho$. The difference $(y_j-y_i)\rho$ is the size of the cost or loss associated with the transfer. Fleurbaey and Michal (2001) demonstrated that the symmetric utilitarian welfare function satisfies PTP, that is, $\sum_{i=1}^{q}U(x_i)>\sum_{i=1}^{q}U(y_i)$, if and only if there exists a strictly concave transformation $V:\left[-\frac{1}{a},-\frac{1}{b}\right]\subset\Re^1_-\to\Re^1$ such that $V\left(-\frac{1}{\vartheta}\right)=U(\vartheta)$,where $\Re^1_-$ is the negative part of the real line $\Re$.

Since the transfer considered in PTP affects only two poor persons, the component $\frac{1}{n}\sum_{i=q+1}^{n}e_i$ of the indices in (14) and (16) remains unaltered. Consequently, we

[4] Altrnative notions of redistributions are analyzed further in Chateauneuf and Moyes (2006) and Chakravarty (2009, Chapter 3) .We can similarly examine the behaviors of PRF indices with respect these alternative formulations as well.

have to investigate the impact of the transfer only on the subgroup decomposable poverty indices $\frac{1}{n}\sum_{i=1}^{q}\varphi(d_i)$ and $\frac{1}{n}\sum_{i=1}^{q}\xi(g_i)$. The PRF index in (14) satisfies PTP if and only if exists a strictly convex transformation $\psi:\left(-\infty,-\frac{z}{z-a}\right)\to\Re^1$ such that $\psi\left(-\frac{1}{t}\right)=\varphi(t)$. The ψ functions for the Foster-Greer-Thorbecke and the Chakravarty indices are given respectively $\psi_\alpha(t)=\left(-\frac{1}{t}\right)^\alpha$ and $\psi_\theta(t)=1-\left(1+\frac{1}{t}\right)^\theta$. For the Watts index the ψ function is $\psi(t)=-\log\left(-\frac{1}{t}\right)$. Thus, all these three indices satisfy PTP. It is easy to see that for the Zheng index the associated $\psi-$type unique strictly convex transformation is given by $\left(\exp\left(-\frac{r}{t}\right)-1\right)$ and hence this index also satisfies PTP. Thus, in order to verify the satisfaction of PTP by a particular PRF index, it is necessary to examine the behavior of the underlying individual poverty function with respect to PTP.

To every subgroup decomposable poverty index there exists a corresponding index of PRF. These indices will differ depending upon the specification of the individual poverty function.

4. Conclusion

Poverty reduction failure is a problem in many countries of the world. In this article an ordering for ranking alternative income distributions in terms of poverty reduction failures has been characterized. The ordering, which has interesting policy applications, can be implemented by seeking dominance in terms of the generalized Lorenz curve of the poor (or the TIP curve of the society) and a non-decreasing function of the incomes of the rich. The ordering can be applied empirically under extremely simplified assumptions. We also characterized a very simple index of poverty reduction failure possessing some attractive features. It coincides with a member of a general family of indices suggested by Kanbur and Mukherjee (2007).

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