

## STABLE EXTENDED FINITE ELEMENT METHOD: CONVERGENCE, ACCURACY, PROPERTIES AND DIFFPACK IMPLEMENTATION

**D.A. Paladim**

Institute of Mechanics and Advanced Materials, Cardiff School of Engineering, Cardiff University, Wales, UK.  
AlvesPaladimD@cardiff.ac.uk

**S. Natarajan**

School of Civil and Environmental Engineering, The University of New South Wales, Sydney, Australia.  
sundararajan.natarajan@gmail.com

**S.P.A Bordas<sup>a</sup>, P. Kerfriden<sup>b</sup>**

Institute of Mechanics and Advanced Materials, Cardiff School of Engineering, Cardiff University, Wales, UK.  
<sup>a</sup>stephane.bordas@gmail.com, <sup>b</sup>pierre.kerfriden@gmail.com

**F. Vogel**

inuTech GmbH, Germany. frank.vogel@inutech.de

**Key Words:** *extended/generalized finite element method, stable G/XFEM, blending, convergence, Diffpack implementation.*

### ABSTRACT

Problems involving singularities and moving boundaries, especially when they involve discontinuities, create difficulties for the finite element method. On another, albeit related, front, two diametrically opposed approaches are attempting to simplify the CAD to Analysis pipeline: isogeometric methods on the one hand [1] aim at coupling the geometry and field approximations, whilst implicit boundary definition-based methods attempt to decouple them [3,4,5].

We examine in this paper one instance of the latter approach, and rely on partition of unity enrichment of the field variable to capture discontinuities along material interface or domain boundaries. We study in particular the stable generalized finite element method of Babuška and Banerjee [6] for higher order approximations in two and three dimensions and propose a generic implementation within the C++ library Diffpack from inuTech GmbH [7]. In a companion paper, the implementation of enrichment within Diffpack is presented in more detail. We will present results obtained with our 3D implementation of partition of unity enrichment within Diffpack. This implementation represents the interfaces through level-sets and palliates blending problems using various approaches. We study here the stabilisation approach proposed in [6] in more detail and pay particular attention to the global convergence rate of the approach and to the stability and the local flux convergence close to the interfaces.

### REFERENCES

- [1] T.J.R Hughes, J.A. Cottrell and Y. Bazilevs. Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement. *Computer Methods in Applied Mechanics and Engineering.*, Vol. **194**, 4135–4195, 2005.
- [2] G. Legrain, N. Chevaugeon, N. and K. Dréau. High order X-FEM and level sets for complex microstructures: Uncoupling geometry and approximation. *Computer Methods in Applied Mechanics and Engineering.*, Vol. **194**, 4135–4195, 2005.
- [3] G. Legrain, R. Allais, N. and P. Cartraud. On the use of the extended finite element method with quadtree/octree meshes. *International Journal for Numerical Methods in Engineering.*, Vol. **86**, 717–743, 2011.
- [4] M. Moumnassi, S Belouettar, E Béchet, S.P.A Bordas, D Quoirin and M. Potier-Ferry. Finite element analysis on implicitly defined domains: An accurate representation based on arbitrary parametric surfaces. *Computer Methods in Applied Mechanics and Engineering.*, Vol. **200**, 774–796, 2011.
- [5] N. Moës, M. Cloirec, P. Cartraud and J.F. Remacle. A computational approach to handle complex microstructure geometries. *Computer Methods in Applied Mechanics and Engineering.*, Vol. **192**, 3163-3177, 2003.
- [6] I. Babuška and U Banerjee. Stable generalized finite element method (SGFEM). *Computer Methods in Applied Mechanics and Engineering.*, Vol. **201–204**, 91–111, 2012.
- [7] P.H. Langtangen. *Computational Partial Differential Equations - Numerical Methods and Diffpack Programming*, Series: Texts in Computational Science and Engineering, 2<sup>nd</sup> ed., Vol. **1**, Springer-Verlag, 2003.