A generalised finite mixture model

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Outline

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1. Nagin's Finite Mixture Model

2. Generalization of the basic model

3. The Luxemburgish salary trajectories

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This is still an inter-individual model, but unlike other classical models such as standard growth curve models, it allows the existence of subpolulations with completely different behaviors.

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Aim of the analysis: Find r groups of trajectories of a given kind (for instance polynomials of degree 4, $P(t) = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3 + \beta_4 t^4$.

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- mixture : population composed of a mixture of unobserved groups

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Hence,

$$p^j(y_{i_t}) = \frac{1}{\sigma}\phi\left(\frac{y_{i_t} - \beta^j t_{it}}{\sigma}.\right) \quad (5)$$

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Software:

SAS-based Proc Traj procedure
by Bobby L. Jones (Carnegie Mellon University).

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Software:

Mplus package by L.K. Muthén and B.O Muthén.

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1. Difficult to extend to other types of data.
2. Group cross-over effects.
3. Can create the illusion of non-existing groups.

Model Selection

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Rule:

The bigger the BIC, the better the model!

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To be classified into a small group, an individual really needs to be strongly consistent with it.

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$$L = \frac{1}{\sigma}\prod_{i=1}^{N}\sum_{j=1}^{r}\frac{e^{x_i\theta_j}}{\sum_{k=1}^{r} e^{x_i\theta_k}}\prod_{t=1}^{T}\phi\left(\frac{y_{i_t} - \beta^j t_{i_t}}{\sigma}\right). \tag{12}$$

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Confidence intervals for the probabilities of group membership can be computed by a parametric bootstrap technique.

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$$y_{i_t} = \beta_0^j + \beta_1^j t + \beta_2^j t^2 + \beta_3^j t^3 + \beta_4^j t^4 + \alpha_1^j z_{1t} + ... + \alpha_L^j z_{Lt} + \varepsilon_{i_t}, \quad (13)$$

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Unfortunately the estimation of parameters α_l^j is not implemented in proc traj procedure; it is just possible to plot the impact of the covariates.

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- year of birth of children

Result for 9 groups (dataset 1)

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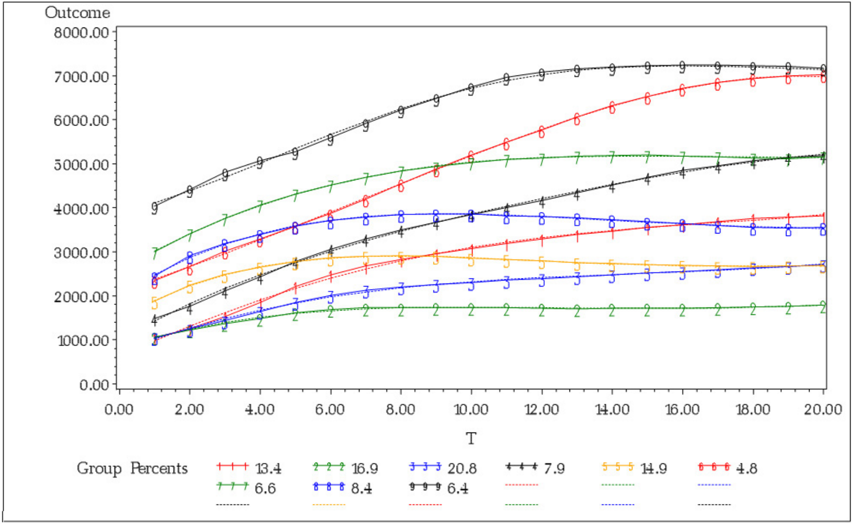

Adding covariates to the trajectories (dataset 2)

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