Accurate evaluation of stress intensity factors using error estimation in quantities of interest based on equilibrated recovery

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ABSTRACT

During the last years the use of error estimators which measure the error in a quantity of interest defined by the analyst, instead of the energy norm, have become increasingly popular as they provide an error indicator for goal oriented adaptivity procedures. In this paper we propose an *a posteriori* recovery-based error estimation procedure which considers the stress intensity factor K typical of singular problems as the quantity of interest in finite element (FE) approximations.

In general, error estimators in quantities of interest have been based on residual techniques and, although recovery techniques have been often preferred when considering the error in energy norm due to their robustness and simplicity, so far, there is no available procedure which considers an equilibrated recovery technique that can be used in standard FE frameworks. In [1] a standard SPR recovery technique is used to obtain an error measure of the J-integral, which is closely related to the value of the SIF. However, it does not consider any equilibrium constraints or the singularity near the crack tip, thus the obtained recovered stress field is not well suited for this kind of problems.

The technique proposed herein relies on the enhanced superconvergent patch recovery technique presented in [2] to evaluate highly accurate recovered stress fields $\boldsymbol{\sigma}^*$ of the primal and dual problems, which are then used to obtain a sharp error estimate. The primal problem is simply the problem under analysis. To formulate the dual problem we consider the linear interaction integral representing K to obtain the applied loads of the dual FE approximation to solve. The high accuracy of the recovered stress fields $\boldsymbol{\sigma}^*$ for both the primal and dual solutions is obtained by decomposing the raw stress field obtained from the finite element approximations into singular and smooth parts, and enforcing the fulfilment of boundary and internal equilibrium equations. The results indicate an accurate estimation of the error in K for benchmark problems with exact solution.

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