DISCUSSION PAPER SERIES

No. 4266

EXTENSIVE AND INTENSIVE GROWTH IN A NEOCLASSICAL FRAMEWORK

Andreas Irmen

INTERNATIONAL MACROECONOMICS and TRANSITION ECONOMICS

Centre for Economic Policy Research

www.cepr.org

Available online at: www.cepr.org/pubs/dps/DP4266.asp

ISSN 0265-8003

EXTENSIVE AND INTENSIVE GROWTH IN A NEOCLASSICAL FRAMEWORK

Andreas Irmen, Universität Mannheim and CEPR

Discussion Paper No. 4266
February 2004

Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

CEPR Discussion Paper No. 4266

February 2004

ABSTRACT

Extensive and Intensive Growth in a Neoclassical Framework*

Extensive growth based on the expansion of inputs is likely to be subject to diminishing returns. Therefore it is often viewed as having no effect on per capita magnitudes in the long run. This Paper argues that periods of extensive growth through capital accumulation may be a precursor to periods of intensive growth during which output per unit of input grows through endogenous technical change. Such a sequence of stages of development occurs as capital accumulation affects the incentives to engage in labour-saving technical change. A steady rise in the capital-labour ratio affects the relative scarcity of factors of production, their (expected) relative price, and induces innovation investments.

JEL Classification: D24, J30, O33 and O41
Keywords: endogenous technical change, induced innovation, neoclassical growth model and productivity growth

Andreas Irmen
Universität Mannheim
Fakultät fuer Volkswirtschaftslehre
L7
D-68131 Mannheim
GERMANY
Tel: (49 621) 181 1834
Fax: (49 621) 181 1837
Email: airmen@pool.uni-mannheim.de

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=131023

*Financial assistance from Deutsche Forschungsgemeinschaft under the programme 'Industreiökonomik und Inputmärkte' and research grant IR 44 1/1 is gratefully acknowledged. I wish to thank Christoph Buchheim, Martin Hellwig and Berthold Wigger for helpful comments.

Submitted 10 November 2003

1. Introduction

Extensive growth refers to a growth strategy based on the expansion of inputs. As such, capital accumulation and growth of the labor force raise the growth rate of aggregate output. Yet, due to diminishing returns these growth effects are without a lasting effect on per-capita income. Soviet growth in the second half of the 20th century seems to be a case in point. In Krugman's (1994) words: "It was simply not possible for the Soviet economies to sustain the rates of growth of labor force participation, average education levels, and above all the physical capital stock that had prevailed in previous years." What was missing in the Soviet economic sphere was arguably *intensive growth*, i.e. growth in output per unit of input.

This paper studies the interplay between extensive and intensive growth and argues that both strategies are complementary to each other. This complementarity manifests itself at two levels. First, the absence of intensive growth over a certain period of time does not validate the prediction of stationarity in the long run. Periods of extensive growth may be interpreted as a precursor of periods exhibiting intensive growth because the incentive to engage in productivity enhancing investments depends to some extend on the amount of extensive growth that the economy has already experienced. Second, elements of extensive and intensive growth strategies occur simultaneously and generate positive mutual repercussions that overcome diminishing returns associated with extensive growth and allow for sustained growth in the long run.

The focus of the analysis is on growth through capital accumulation (extensive growth) and growth through endogenous technical change (intensive growth). The intuitive argument that is central for their interplay emphasizes the relative scarcity of factors of production, their relative price, and the incentive to undertake innovation investments. Capital accumulation understood as a rise in the capital-labor ratio steadily raises the scarcity of labor with respect to capital and leads to a rise in the cost of labor relative to the price of capital. This movement in relative factor prices acts as an incentive for profit-maximizing firms to invest in labor-saving technologies. Typically, an innovation investment must be financed on the capital market because the proceeds of the investment materialize only later. A reduction in the price of capital implies a lower interest rate and thus reduces investment costs. A rise in the real wage increases the gains from an investment that saves on labor. Capital accumulation may thus reduce the cost and increase the gain associated with such innovation investment.

The growth performance of countries may then be characterized by an endogenous run through stages of development. In early stages, the driving force of economic growth is capital accumulation because the return to physical capital is high and labor is cheap. In later stages, however, labor is sufficiently expensive so that firms invest in new technologies that economize on labor. Thus, an economy may evolve from a regime of pure extensive growth (stationary regime) into one with both extensive and intensive growth (innovation regime).

A run through stages is by no means evident but depends on exogenous characteristics of an economy such as the propensity to save, the efficiency of the innovation technology, government intervention, and the population growth rate. For instance, a small savings rate makes it more likely that an economy does not reach the innovation regime. In this case, capital grows but the level of technological knowledge remains constant: extensive growth alone implies a stationary steady state. If the savings rate is high the economy switches into the innovation regime after having accumulated a sufficient amount of capital per worker. Following the switch it converges towards a balanced growth path with a constant endogenous growth rate of per-capita magnitudes. Capital accumulation and technological knowledge can be thought of as complementary state variables in this evolution. On the one hand, technological knowledge is a countervailing force to diminishing returns, on the other hand capital accumulation maintains the relative scarcity of labor such that the expected relative factor price induces innovation investments.

To study this evolution I combine the neoclassical growth paradigm (Solow (1956), Swan (1956)) and the concept of induced innovation. By means of the neoclassical production function the former features a process of capital accumulation reflecting a steady rise of the real wage and a steady fall in the rental rate of capital in response to changing relative scarcities. The latter emphasizes the role of *expected* relative factor prices to induce labor-saving technical change.[1]

The extension of the neoclassical growth model that I suggest allows for a final-good sector and an intermediate-good sector. Both sectors are competitive. Final-good production requires capital and an intermediate good as inputs. The final good is used for consumption, capital, or as an input in innovation. Innovation occurs at the level of competitive intermediate-good firms. The specification of the intermediate-good sector is based on Bester and Petrakis (2003) and Hellwig and Irmen (2001) who develop a setting of endogenous growth for competitive economies.

The analysis leads to a dynamical system that preserves the main dynamical properties of the neoclassical growth model. There is a unique stable steady state. Yet, depending on the above mentioned parameters characterizing an economy this steady state is stationary or exhibits endogenous technical change. In the former case, the evolution of the economy is as in the neoclassical growth model. In the latter case we observe "growth through stages." An economy starting in the stationary regime will at some point in time switch into the innovation regime and converge towards a steady state with a constant growth rate of technological knowledge.

The related literature includes studies that emphasize other potential links between extensive and intensive growth. For instance, Galor and Moav (2001)

[1] The idea that labor-saving inventions occur in circumstances where real wages are expected to rise relative to the real interest rate already appears in an informal note by Fellner (1961).

share my view that the evolution of relative factor prices may ignite a regime switch that fosters economic growth. They emphasize a falling profit rate as an incentive to augment labor via human capital accumulation and interpret the establishment of public education in the second half of the 19th century as a cooperative endeavor made by workers and capitalists which let to the eventual demise of class structure. According to Galor and Moav, capitalists were ready to financially support public education as education augments labor which in turn raised their profit rates due to the complementarity between physical and human capital in aggregate production.

A similar effect on the profit rate is present here. While technical change is labor-saving at the level of the individual firm it is labor-augmenting at the aggregate level. Accordingly, technical change implies more output of the intermediate good which raises the marginal productivity of capital and the profit rate. Then, the owners of capital may be the prime beneficiaries of labor-saving technical change. Yet, augmenting labor is here the result of individual profit-maximizing behavior.

Howitt and Aghion (1998) consider a Schumpeterian model of endogenous growth with capital accumulation. Capital accumulation matters for the economy's long-run growth rate through a scale effect that augments the profit of innovating firms. Similar to my setup, research and the production of capital use the same inputs. However, in my model the scale effect is absent and capital accumulation matters for innovation incentives on firms' input side through changing expected relative factor prices.

Scale effects are also at the heart of Matsuyama (1999) who studies a variant of Rivera-Batiz and Romer's (1990) lab equipment model. He finds that capital accumulation may give rise to cyclical growth with an economy switching between a 'Solow' (stationary) and a 'Romer' (innovation) regime. Contrary to my study factor price movements do not affect the incentives of innovating firms and are therefore inessential for his findings. However, the present paper adds a new argument to Matsuyama's critique of Krugman's (1994) pessimistic prediction concerning the economic performance of the East-Asian Tigers. Indeed, if capital accumulation along neoclassical lines is a satisfactory explanation of the past growth performance of the Tigers (Lau and Kim (1994), Young (1995)) the present study suggests that the Tigers may or may have switched into a regime of endogenous technical change as past capital accumulation has rendered labor sufficiently expensive. This seems to be consistent with Pack's recent estimation that Asian NIEs have absorbed international knowledge in the form of new equipment, intermediates, or disembodied knowledge, and improved on it (Pack (2001), p. 134).

The paper is organized as follows. I present the details of the model in section 2. Section 3 studies the intertemporal general equilibrium, characterizes the dynamical system, and analyzes possible equilibrium paths. In Section 4, I discuss extensions. Section 5 concludes.

2. The Model

The economy has a household sector, a final-good sector, and an intermediate-good sector in an infinite sequence of periods $t = 1, 2, ...$ There are four objects of exchange, a manufactured final good, a manufactured intermediate good, labor, and bonds. I call 'final good' a commodity which serves for consumption as well as for investment. If invested, this commodity is either used as future capital in the final-good sector or as an immediate input into innovation undertaken by firms of the intermediate-good sector.

In each period t, there are markets for all four objects of exchange. Treating the final good as the numéraire, I let p_t denote the real price of the intermediate good, w_t the real wage, and p_t^b the real bond price at t. A bond at t is a claim on one unit of the final good at $t+1$. Working with real interest rates rather than bond prices, I write $p_t^b = 1/(1 + r_{t+1})$ where r_{t+1} is the real interest rate from t to $t+1$.

2.1. The Household Sector

The household sector has an initial endowment of B_1 bonds coming due at $t = 1$ and owns the shares of all firms in the economy. Moreover, it supplies L units of labor in each period. The allocation of per-period income to consumption and savings is subject to the budget constraint

$$C_t + \frac{B_{t+1}}{1 + r_{t+1}} = w_t L + B_t + \Pi_t, \tag{2.1}$$

where C_t is consumption of the final good, B_{t+1} is bond demand in t, $w_t L$ is wage income, B_t capital income from the repayment of bonds due in t, and Π_t denotes the aggregate dividend distribution.

As to the consumption-savings decision of the household sector I take a behavioristic point of view and assume that real aggregate savings in t is a fixed fraction of aggregate income in t, i.e.,[2]

$$\frac{B_{t+1}}{1 + r_{t+1}} = s\left(w_t L + B_t + \Pi_t\right), \tag{2.2}$$

with $s \in (0, 1)$ denoting the marginal and average propensity to save.

2.2. The Final-Good Sector

The final-good sector produces according to the production function

$$Y_t = F\left(K_t, X_t\right); \tag{2.3}$$

[2] The main results of the analysis can also be obtained using e.g. a standard two-period lived overlapping generations framework.

here Y_t is aggregate output of the final good, K_t is capital input in t, and X_t denotes the amount of the intermediate good used in period-t production. The function $F : \Re_+^2 \rightarrow \Re_+$ exhibits constant returns to scale, satisfies standard concavity, differentiability as well as Inada conditions. The latter are sufficient but often not necessary for my results.

Capital in t must be installed one period before its use in production and fully depreciates after being used.[3] A capital investment of K_t units undertaken in period $t-1$ is financed by an issue of $(1+r_t) K_t$ bonds.

In terms of the final good of period t as numéraire the profit in t of the final-good sector is

$$Y_t - (1+r_t) K_t - p_t X_t, \tag{2.4}$$

where $(1+r_t) K_t$ is capital service payments and $p_t X_t$ is the cost of the intermediate-good input. The price for K_t in units of the final good in t is $(1+r_t) > 1$. This reflects the fact that K_t must be carried over from period $t-1$ before its use in production in period t.

The final-good sector takes the sequence $\{p_t, r_t\}$ of prices and interest rates as given and maximizes the sum of the discounted present values of profits in all periods. Since it simply buys capital and intermediate goods for each period, its maximization problem is equivalent to a series of one-period maximization problems. The respective first-order conditions for capital and intermediate goods state that the value of the respective marginal product must be equal to the input price. Define the period-t capital intensity in the final good-sector as

$$k_t \equiv \frac{K_t}{X_t}. \tag{2.5}$$

Then, for $t = 1, 2, ...$ with $f(k_t) \equiv F(k_t, 1)$ the first-order conditions are

$$K_t \quad : \quad f'(k_t) = 1 + r_t \tag{2.6}$$

$$X_t \quad : \quad f(k_t) - k_t f'(k_t) = p_t \tag{2.7}$$

Initially, the final-good sector has $K_1 = \bar{K}_1$ units of capital at its disposal.[4] It stems from investment decisions prior to period $t = 1$ and causes outstanding debt obligations equal to $(1+r_1)\bar{K}_1$. This is the counterpart of the initial bond holdings of the household sector:

$$B_1 = (1+r_1)\bar{K}_1. \tag{2.8}$$

[3] This assumption is only made to simplify the exposition. All results extend readily to the case of depreciation rates strictly smaller than one.

[4] Condition (2.6) for $t = 1$ implies that $\bar{K}_1$ is consistent with profit-maximizing behavior.

2.3. The Intermediate-Good Sector

The set of all intermediate-good firms is represented by the set $\Re_+$ of nonnegative real numbers.

Technology With respect to the production of output in periods $t = 1, 2, ...$, all firms have the same technology. Each firm has a capacity limit of one unit of output per period.[5] Its output in period t is given by

$$x_t = \min\{1, a_t l_t\}, \tag{2.9}$$

where x_t is output, a_t is the firm's labor productivity in period t, and l_t its labor input. The firm's labor productivity is equal to

$$a_t = A_{t-1}(1 + q_t); \tag{2.10}$$

here A_{t-1} is an indicator of the economy-wide labor productivity in period $t-1$, and q_t is an indicator of productivity growth at this firm.

To achieve a productivity growth rate $q_t > 0$ from period $t-1$ to period t, the firm must invest $i(q_t)$ units of the final good in period $t-1$. The function $i(.)$ is twice continuously differentiable and satisfies

$$i(0) = 0;\ i'(q_t) > 0 \text{ and } i''(q_t) \geq 0 \text{ for } q_t \geq 0. \tag{2.11}$$

Observe that (2.11) implies $i'(0) > 0$, i.e., the marginal cost of innovative activity is assumed to be strictly positive. In other words, the marginal productivity of the first small unit of input in innovation is finite and the technology of intermediate-good producers does not comply with the Inada conditions. It will become clear later on that this assumption is crucial for the presence of stages of growth. I shall interpret the level of $i'(0)$ as an indicator of a country's innovation technology.

If the firm decides not to innovate in period $t-1$ it can avail itself for production in t of the production technique of period $t-1$ in which case $a_t = A_{t-1}$.

Following Hellwig and Irmen (2001) I assume that an innovation in period t is proprietary knowledge of the firm only in t, i.e., in the period when it is made. Subsequently, the innovation becomes embodied in the economy-wide productivity indicators A_t, $A_{t+1}, ...$, with no further scope for proprietary exploitation.

[5] The analysis is easily extended to a more general specification involving variable capacity based on prior capacity investments, with investment outlays a strictly convex function of capacity. In such a setting profit-maximizing behavior implies that a large innovation investment is accompanied by a large capacity investment. Thus, the simpler specification treated here abstracts from effects on firm size in an environment with changing levels of innovation investments.

Profit Maximization The innovation investment $i(q_t)$ in period $t-1$ is financed by an issue of $(1+r_t)i(q_t)$ bonds. In terms of the final good of period t as numéraire, a production plan (q_t, l_t, x_t) for period t thus yields the profit

$$\pi_t = p_t x_t - w_t l_t - (1+r_t)i(q_t), \tag{2.12}$$

where $p_t x_t = p_t \min\{1,\ A_{t-1}(1+q_t)l_t\}$ is the firm's revenue from output sales, $w_t l_t$ its wage bill at the real wage rate w_t, and $(1+r_t)i(q_t)$ its debt service.

I assume that the firm takes the sequence $\{p_t, w_t, r_t\}$ of real prices as well as the sequence $\{A_t\}$ of aggregate productivity indicators as given and chooses its production plan so as to maximize the sum of the discounted present values of its profits in all periods. Because production choices for different periods are independent of each other, for each period t, it will in fact choose the plan (q_t, l_t, x_t) to maximize the profit π_t from this plan in period t.

If the firm innovates, it incurs an investment cost $(1+r_t)i(q_t)$ that is associated with a given innovation rate $q_t > 0$ and is independent of the output x_t. This introduces a positive scale effect, namely if the firm innovates, then it wants to apply the innovation to as large an output as possible and to produce at the capacity limit $x_t = 1$. The choice of (q_t, l_t) must then minimize the costs of producing the capacity output.

Suppose $w_t > 0$, then an input combination (q_t, l_t) that minimizes unit costs must satisfy

$$l_t = \frac{1}{A_{t-1}(1+q_t)}, \tag{2.13}$$

and

$$q_t \in \arg\min_{q \geq 0} \left[\frac{w_t}{A_{t-1}(1+q)} + (1+r_t)i(q) \right]. \tag{2.14}$$

Given the differentiability and convexity of the innovation cost function $i(.)$, (2.14) actually determines q_t *uniquely* as the solution to the first-order condition

$$\begin{gathered} \frac{w_t}{A_{t-1}(1+q_t^*)^2} \leq (1+r_t)i'(q_t^*), \\ \text{with strict inequality } \textit{only} \text{ if } q_t^* = 0. \end{gathered} \tag{2.15}$$

The latter relates the marginal reduction of the firm's wage bill to the marginal increase in its investment costs. As both marginal effects are proportional to the respective factor price condition (2.15) implies

Lemma 1 The unit-cost-minimizing growth rate of labor productivity can be expressed in terms of a map $q : \Re_+ \to \Re_+$,

$$q_t^* = q\left(\frac{w_t}{A_{t-1}(1+r_t)}\right), \tag{2.16}$$

with

$$
\begin{aligned}
q_t^* &= 0 \text{ for } \frac{w_t}{A_{t-1}(1+r_t)} \leq i'(0), \\
q_t^* &> 0 \text{ otherwise.}
\end{aligned}
\tag{2.17}
$$

Moreover,[6]

$$
q'(.) \geq 0 \text{ with strict inequality only if } \frac{w_t}{A_{t-1}(1+r_t)} \geq i'(0).
$$

Hence, for any A_{t-1} the chosen growth rate of labor productivity is an increasing function of relative factor prices. More precisely, given that the innovation decision is made in period $t-1$ the choice of q_t depends on the *expected relative factor price ratio.* The higher this ratio the more pronounced is the incentive to engage in labor saving innovation in $t-1$. It is in this sense that an innovation is *induced.* Clearly, the evolution of factor prices to be determined by general equilibrium conditions will play a crucial role for whether and to what extend intermediate-good firms engage in innovation investment.

Lemma 1 also emphasizes the role of the assumption $i'(0) > 0$. Had I assumed $i'(0) = 0$ any equilibrium with a strictly positive real wage would imply a strictly positive growth rate of labor productivity as the unit-cost minimizing choice.

A choice of q_t^* may not be profit-maximizing if the prices p_t, w_t, r_t, and the productivity index A_{t-1} are such that the profit associated with a production plan (q_t, l_t, x_t) is negative. In this case the firm will prefer not to produce any output at all, i.e., it chooses the production plan $(0, 0, 0)$. Any profit-maximizing production plan with output $x_t > 0$ must therefore satisfy

$$
\pi_t^* := \pi\left(q_t^*; p_t, w_t, r_{t-1}, A_{t-1}\right) \geq 0. \tag{2.18}
$$

I assume that if intermediate-good firms choose to be active, they *always* plan to produce the capacity output $x_t = 1$. This assumption simplifies the exposition because it implies that all active intermediate-good firms choose the same production plan (q_t, l_t, x_t) for period t.[7] No significant loss of generality is involved because in circumstances where intermediate-good firms do plan to produce some output $x_t \neq 1$, their maximized profits as well as their innovation investments are zero, and they would be just as willing to choose the production

[6] The last result follows immediately from total differentiation of (2.15) in conjunction with (2.11).

[7] Indeed, for any constellation of the parameters p_t, w_t, r_t, and A_{t-1}, there may be more than one profit-maximizing production plan. In particular, if we have $\pi_t^* = 0$, maximum profits are zero, and this maximum is attained at both, the plan (q_t, l_t, x_t) satisfying (2.13), (2.16), and $x_t = 1$, *and* the plan $(0, 0, 0)$ providing for inactivity of the firm in period t. If in addition $q^* = 0$, profits are maximized by *any* production plan of the form $(0, x/A_{t-1}, x)$.

plan $(0, 1/A_{t-1}, 1)$ or the production plan $(0, 0, 0)$. It would therefore be possible to rearrange profit-maximizing production plans across firms so that all firms plan to have output equal to either zero or one and moreover the aggregate impact of the intermediate-good sector on markets is unchanged.

The Aggregate Intermediate-Good Sector The set of all active firms in period t has Lebesgue measure n_t and generates

- an aggregate investment demand in period $t-1$ of $n_t i(q_t^*)$,
- a corresponding aggregate supply of bonds in $t-1$ of $(1+r_t)\, n_t i(q_t^*)$,
- an aggregate labor demand in period t of $n_t l_t$,
- an aggregate intermediate-good supply in period t of $n_t x_t = n_t$.

Zero-Profits In representing the set of all intermediate-good firms by $\Re_+$ with Lebesgue measure, I implicitly introduce a zero-profit condition. Given that labor supply in each period is bounded, in any equilibrium the set of intermediate-good firms employing more than some $\varepsilon > 0$ units of labor must have bounded measure and hence must be smaller than the set of all intermediate-good firms. Given that inactive intermediate-good firms must be maximizing profits just like the active ones, this implies that in any equilibrium for period t, $t = 1, 2, ...$, maximum profits of intermediate-good firms at equilibrium prices must be equal to zero.

Initial Conditions As to intermediate-good production in period $t = 1$ I assume that *none* of the intermediate-good firms active in this period has made an innovation investment prior to period 1. This reflects my intention to study the evolution of an economy that starts in an environment without technical change.

Economy-wide Productivity Indicators To conclude the account of the intermediate-good sector, I turn to the evolution of the economy-wide productivity indicators $A_1, A_2, ...$ As mentioned above I assume that all innovations are publicly available after one period. Anybody can then incorporate them into their production processes or take them as a basis for additional innovations. Proprietary use of innovations is thus limited to the period in which they occur. Given that for any t all firms that are active at t choose the same innovation rate q_t^* and attain the same labor productivity $a_t = A_{t-1}(1 + q_t^*)$, I identify A_t with a_t and write

$$A_{t+1} = A_t(1 + q_t^*) \qquad (2.19)$$

for $t = 1, 2, ...$, with $a_1 = \bar{A}_0 > 0$ given by initial conditions.

3. Intertemporal General Equilibrium

3.1. Definition

Turning to the behavior of the economy as a whole, I refer to a sequence $\{p_t, w_t, r_t\}$ of real prices for the intermediate good, real wages, and real interest rates for periods $t = 1, 2, ...$ as a *price system.* By an *allocation* I understand a sequence $\{C_t, L, B_t, Y_t, K_t, X_t, n_t, q_t, l_t\}$ that comprises a strategy $\{C_t, L, B_t\}$ for the household, a strategy $\{Y_t, K_t, X_t\}$ for the final-good sector, and, for each t, a measure n_t of intermediate-good firms active at t producing the capacity output $x_t = 1$ with input choices (q_t, l_t).

An *equilibrium* will correspond to a price system, an allocation, and a sequence $\{\Pi_t, A_t\}$ of distributed aggregate profits, and productivity indicators that satisfy the following conditions:

(E1) Given the initial bond endowment B_1 and the sequence $\{w_t, r_t, \Pi_t\}$ of real wages, interest rates, employment, and dividend distributions Π_t, the household sector saves according to (2.2) and supplies L units of labor in all periods.

(E2) For any t, the profit distribution Π_t which the household sector expects to receive at t is equal to the actual aggregate of the profits that accrue at t in the final-good sector and the intermediate-good sector, i.e.,

$$\Pi_t = Y_t - (1 + r_t)K_t - p_t X_t + n_t \left[p_t - w_t l_t - (1 + r_t)i\left(q_t\right)\right].$$

(E3) Given p_t and r_t for all $t \geq 1$ the final-good sector produces according to (2.3). Its profit-maximizing behavior is characterized by conditions (2.6) and (2.7).

(E4) Given the productivity indicator A_{t-1}, the real output price p_t, the real wage rate w_t, and the real interest rate r_t, for any $t \geq 1$, the input choice $(q_t, l_t, 1)$ minimizes the unit cost of production of an intermediate-good firm active at t. By assumption $q_1 = 0$.

(E5) Given the productivity indicator A_{t-1}, the real output price p_t, the real wage rate w_t, and the real interest rate r_t, for any $t \geq 1$,

$$\pi^* \leq 0, \tag{3.1}$$

with a strict inequality only if $n_t = 0$.

(E6) (final-good market) For any t,

$$Y_t = C_t + K_{t+1} + n_{t+1} i\left(q_{t+1}\right). \tag{3.2}$$

(E7) (intermediate-good market) For any t,

$$X_t \leq n_t, \tag{3.3}$$

with a strict inequality only if $p_t = 0$.

(E8) (bonds market) For any $t \geq 1$,

$$B_{t+1} = (1 + r_{t+1}) (K_{t+1} + n_{t+1} i (q_{t+1})) .$$

(E9) (labor market) For any t,

$$n_t l_t \leq L, \tag{3.4}$$

with a strict inequality only if $w_t = 0$.

(E10) For any t, the indicators A_t satisfy the updating condition (2.19).

In specifying a consistent circular flow of income, condition (E2) ensures that in equilibrium aggregate income equals total output in the final good sector. To see this use (E2), (E7) and (E9) for period t, and (E8) for $t - 1$ to find

$$\begin{aligned} w_t L + B_t + \Pi_t &= w_t L + B_t + Y_t - (1 + r_t) K_t - p_t X_t \\ &\quad + n_t [p_t - w_t l_t - (1 + r_t) i (q_t)] \\ &= Y_t + p_t [n_t - X_t] + w_t [L - n_t l_t] \\ &\quad + [B_t - (1 + r_t) (K_t + n_t i (q_t))] \\ &= Y_t. \end{aligned}$$

Two implications are immediate. First, given the assumption that intermediate-good firms active in $t = 1$ will not have innovated and with (2.8) stating the analogue of (E8) for $t = 0$ I find that aggregate savings as specified by (2.2) is

$$\frac{B_{t+1}}{1 + r_{t+1}} = sY_t \text{ for } t = 1, 2, ... \tag{3.5}$$

so that the bond market equilibrium condition (E8) can be stated as

$$sY_t = K_{t+1} + n_{t+1} i (q_{t+1}) \text{ for } t = 1, 2, ... \tag{3.6}$$

The second implication concerns Walras' Law. Indeed, from the household sector's budget constraint (2.1), (3.5), and (3.6) it follows that for all t (E6) holds if (E8) does. The equilibrium condition of the final-good market is therefore redundant.

3.2. The Dynamical System

I choose the capital intensity in the final-good sector as the state variable of the dynamical system and begin with the implications of the equilibrium conditions (E1) - (E10) for the price system $\{p_t, w_t, r_t\}$ and the state variable k_t.

Lemma 2 Let $\{p_t, w_t, r_t\}$, $\{C_t, L, B_t, Y_t, K_t, X_t, n_t, q_t, l_t\}$, $\{\Pi_t, A_t\}$ be an equilibrium. Then, for $t = 1, 2, ...$ $p_t > 0$, $w_t > 0$, $1 + r_t > 0$, and

$$k_t = \frac{K_t}{A_t L} \text{ for } t \geq 1 \tag{3.7}$$

with $k_1 = \bar{K}_1 / \ A_1 L > 0$ given by initial conditions.

Proof. See the Appendix.

Lemma 2 points to the close link between the labor market, the measure of active intermediate-good firms producing one unit of output, and the equilibrium in the intermediate-good market. Indeed, with each intermediate-good firm employing $l_t = 1/a_t = 1/A_t$ units of labor, I have in equilibrium for all t

$$A_t L = n_t = X_t \tag{3.8}$$

so that the capital intensity in the final-good sector is equal to capital per unit of efficient labor. Moreover, output of the final-good sector (2.3) has the familiar form

$$Y_t = F\left(K_t, A_t L\right).$$

I may use (3.8) to disentangle the notion of '*labor-saving* technical change' at the level of the individual firm from the notion of '*labor-augmenting* technical change' at the level of economic aggregates. When an intermediate-good firm innovates between period $t-1$ and t its labor productivity in t is $a_t = A_{t-1}\left(1 + q_t\right)$ and employment per firm shrinks by the factor

$$\frac{l_t}{l_{t-1}} = \frac{a_{t-1}}{a_t} = \frac{1}{1 + q_t}.$$

At the same time free entry in intermediate-good production assures full employment and with (3.8)

$$\frac{A_t L}{A_{t-1} L} = \frac{n_t}{n_{t-1}} = \frac{X_t}{X_{t-1}} = 1 + q_t.$$

Hence, at the level of economic aggregates technical change augments the measure of intermediate-good firms and the output of intermediate goods by a factor $1 + q_t$.

Next, I turn to the evolution of k_t for $t > 1$:

Proposition 1 Let $\{p_t, w_t, r_t\}$, $\{C_t, L, B_t, Y_t, K_t, X_t, n_t, q_t, l_t\}$, $\{\Pi_t, A_t\}$ be an equilibrium. Then, for $t = 2, 3, ...$ k_t evolves according to

$$k_t = \begin{cases} sf(k_{t-1}) & \text{if } q_t = 0, \\ \\ \frac{s}{1+q_t} f(k_{t-1}) - i(q_t) & \text{if } q_t > 0. \end{cases} \tag{3.9}$$

Proof. Consider (3.6) with (2.3):

$$sF(K_{t-1}, X_{t-1}) = K_t + n_t i(q_t).$$

Use Lemma 2, (E7) for periods $t-1$ and t, and the definition of k_t to write the latter as

$$k_t = \frac{n_{t-1}}{n_t} sf(k_{t-1}) - i(q_t).$$

From (3.8) I have for all t that $n_t = A_t L$. If $q_t > 0$, then $n_t = (1 + q_t) n_{t-1}$, and if $q_t = 0$, then $n_t = n_{t-1}$. In view of $i(0) = 0$ (3.9) follows.
QED.

The evolution of k_t depends on the amount of aggregate savings in $t-1$ and on the growth rate of labor productivity between $t-1$ and t. The term $sf(k_{t-1})$ is $(t-1)$-savings per unit of efficient labor in $t-1$. Without technical change the amount of efficient labor in the economy remains constant over time. Changes in k_t come about through a rise or a fall of the capital input employed in the final-good sector. Indeed, if $q_t = 0$ the capital intensity k_t evolves as in the neoclassical growth model and (3.9) is a special case of Solow's famous equation.

The presence of technical change between period $t-1$ and t modifies the expression for k_t in two ways. First, technical change 'augments' labor in t by a factor $1 + q_t$. Therefore, $sf(k_{t-1})$ must be divided by this factor. Second, $i(q_t)$ is simply innovation investment per unit of efficient labor in t, i.e., the amount of $(t-1)$-savings per unit of efficient labor in t which is no longer available as period-t capital in the final-good sector.

Proposition 1 treats q_t as a parameter. In order to endogenize it I have to embed the intermediate-good firms' innovation decision in the general equilibrium framework. The following proposition establishes that in equilibrium the chosen growth rate of labor productivity depends on k_t.

Proposition 2 Suppose (E3) - (E5) hold and let

$$\lim_{k_t \to \infty} \frac{f - k_t f'}{f'} > i'(0). \tag{3.10}$$

Then, there is $k_c > 0$ and a map $g : \Re_+ \to \Re_+$ such that

$$q_t^* = g(k_t) \text{ with } \begin{cases} g(k_t) = 0 & \text{for } k_t \leq k_c, \\ g(k_t) > 0 & \text{for } k_t > k_c. \end{cases} \tag{3.11}$$

Moreover, the equilibrium wage is

$$w_t = A_{t-1}(1+g)\left(f - k_t f' - f' i(g)\right), \tag{3.12}$$

where the argument of f and g is k_t.

Proof. See the Appendix.

Proposition 2 shows that the equilibrium conditions of both production sectors imply a relationship between the rate of productivity growth chosen by intermediate-good firms and the capital intensity in the final-good sector. This relationship is summarized in the function $g(k_t)$ which is piecewise defined. There is a critical level k_c so that in period t the economy must be in one of two regimes depending on how k_t relates to k_c. If $k_t > k_c$ I say that the economy is in the *innovation regime* because intermediate-good firms choose a strictly positive growth rate of labor productivity. If instead $k_t \leq k_c$ no innovation occurs and the economy is said to be in the *stationary regime.*

The intuition behind the proposition is the following. The technology of the final-good sector implies that the marginal productivity of capital falls in k_t whereas the marginal productivity of intermediate goods rises. Accordingly, the real interest rate falls and the price of the intermediate good rises in k_t. These price changes feed back onto wages through the zero-profit condition of the intermediate-good sector. The *expected relative factor price ratio* which I found to determine the intermediate-good sector's innovation decision can then be expressed as an increasing function of the capital intensity in the final-good sector. If the latter is sufficiently high the expected relative factor price will reach a level such that $q_t^* > 0$ (see (2.17)). Moreover, q_t^* will rise in k_t, i.e.,

$$\frac{dq_t^*}{dk_t} = \frac{dg(k_t)}{dk_t} \begin{cases} = 0 & \text{for } k_t \leq k_c \\ > 0 & \text{for } k_t \geq k_c. \end{cases} \tag{3.13}$$

I noticed following Lemma 1 that the intermediate-good firms' innovation decision relies on the *expected* relative factor price ratio. By Proposition 2, the relative factor price ratio depends on k_t which, from the vantage point of period $t-1$, is to be interpreted as the *expected* capital intensity. In equilibrium under perfect foresight actual and anticipated developments must coincide. This requirement leads to the characterization of the dynamical system in

Proposition 3 Denote

$$\underline{k}_c := f^{-1}\left(\frac{k_c}{s}\right). \tag{3.14}$$

The dynamical system of the economy for $t = 2, 3, ...$ is characterized by a continuous, monotonically increasing map $k_t = \Psi(k_{t-1})$ where $\Psi : \Re_+ \rightarrow \Re_+$ and satisfies

$$k_t = \begin{cases} sf(k_{t-1}) & \text{if } k_{t-1} \leq \underline{k}_c, \\ \frac{s}{1+g(k_t)} f(k_{t-1}) - i(g(k_t)) & \text{if } k_{t-1} > \underline{k}_c. \end{cases} \tag{3.15}$$

Proof. See the Appendix.

Proposition 3 shows the existence of a unique level of the capital intensity $\underline{k}_c$ that separates the stationary and the innovation regime from an ex ante point of view. The intuition is as follows. If $k_{t-1} \leq \underline{k}_c$, then $k_t \leq k_c$ even if none of the intermediate-good firms innovates. Next period's relative factor price ratio is too low for k_t to make innovation a profitable endeavor. On the other hand, if $k_{t-1} > \underline{k}_c$ then $k_t > k_c$ even if all intermediate-good firms active in t innovate so that the expectation of a high relative factor price ratio is fulfilled.

Observe that $\underline{k}_c$ reflects both the preferences and the technology of an economy. From (3.14) I readily verify that $\underline{k}_c$ is smaller the larger s and the smaller $i'(0)$. In other words, for an economy with a high propensity to save and/or an efficient innovation technology $\underline{k}_c$ is low.

3.3. The Equilibrium Path

This section studies the evolution of the economy that starts in the stationary regime. The equilibrium path can take two distinct forms. Figures 3.1 and 3.2 illustrate these cases.

Proposition 4 Let $0 < k_1 < k_c$. Then, the economy may evolve in two different ways:

- If $\underline{k}_c \geq k_c$ then the economy starts in the stationary regime and remains there forever. It converges towards the unique steady state defined by

$$k^* = sf(k^*). \tag{3.16}$$

- If $k_c > \underline{k}_c$ then the economy starts in the stationary regime, switches at some $t > 1$ into the innovation regime, and converges towards a unique steady state which satisfies

$$k^{**} = \frac{s}{1 + g(k^{**})} f(k^{**}) - i(g(k^{**})). \tag{3.17}$$

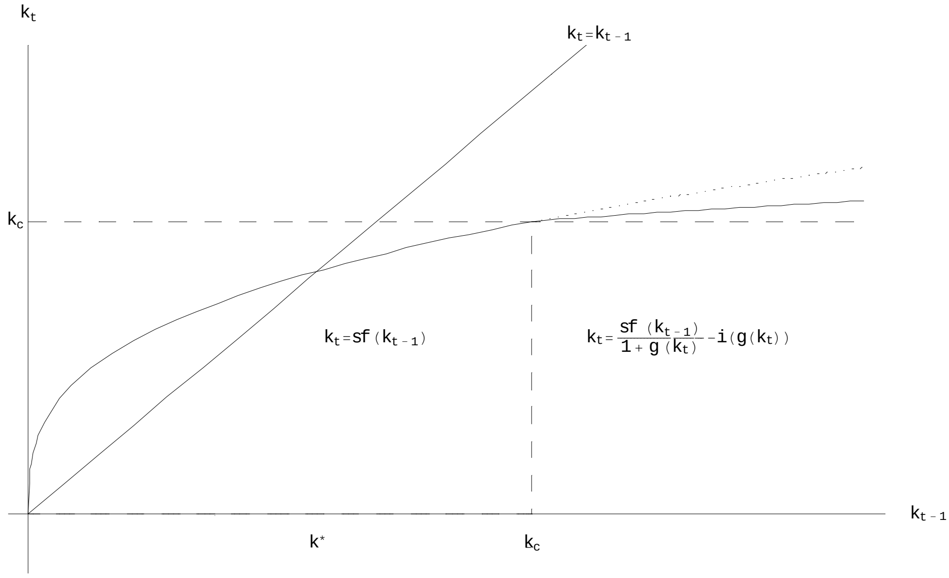

Figure 3.1: $k_c \leq \underline{k}_c$: Steady State in the Stationary Regime.

Proof. See the Appendix.

As $0 < k_1 < k_c$, the economy starts in the stationary regime in $t = 1$. This is consistent with the previous assumption that none of the intermediate-good firms active in $t = 1$ will have innovated. Then, the equilibrium paths depends on how the critical values $\underline{k}_c$ and k_c relate to each other. It is not difficult to see that $\underline{k}_c$ is more likely to exceed k_c if s is low and $i'(0)$ is large. Therefore, an economy for which $\underline{k}_c \geq k_c$ saves little in relation to the efficiency of its innovation technology. According to Proposition 4 such economy cannot reach the innovation regime. The convergence towards the steady state is as in the neoclassical model without technical change. The steady state is stationary in the sense that per-capita output and consumption is constant.

For $k_c > \underline{k}_c$ I observe "growth through stages." The economy starts in the stationary regime, accumulates a sufficient amount of capital which induces at some t expectations about relative factor prices that lead to innovation. Following the switch into the innovation regime, there is both capital accumulation and innovation. More precisely, for $\underline{k}_c < k_{t-1} < k^{**}$, the capital intensity grows and from (3.7) it must hold that

$$\frac{K_t}{K_{t-1}} > \frac{A_t}{A_{t-1}} = (1 + q_t)\,,$$

i.e., capital grows faster than labor productivity. Moreover, from (3.13) it follows that q_t rises over time. Productivity growth is endogenous and responds to a rising capital intensity in final-good production. Indeed, the *expected factor price ratio* can be written as

$$\frac{w_t}{1+r_t} = A_{t-1}(1+g)\left(\frac{f-k_t f'}{f'} - i(g)\right) \tag{3.18}$$

and increases in k_t.[8] Hence, the innovation regime is characterized by extensive and intensive growth: inputs expand and output per unit of labor increases.

Observe that the evolution of k_t is governed by diminishing returns. When $k_{t-1} > \underline{k}_c$, then (3.15) defines $k_t = \Psi(k_{t-1})$ implicitly through

$$(1+g(k_t))[k_t + i(g(k_t))] = sf(k_{t-1}), \tag{3.19}$$

which in conjunction with the Inada condition $\lim_{k\to\infty} f'(k) = 0$ implies that in the innovation regime growth of k_t cannot be sustained forever. Therefore, in the long run, capital per unit of effective labor approaches a constant, i.e. capital and labor productivity grow at the same rate which satisfies

$$q^{**} = g(k^{**})$$

and is susceptible to changes in parameters like s.

Corollary 1 It holds that

$$\frac{dk^{**}}{ds} > 0, \frac{dq^{**}}{ds} > 0. \tag{3.20}$$

Proof. See the Appendix.

As expected, increasing the propensity to save raises the steady-state capital intensity so that the relative price of labor and the growth rate of labor productivity rises.

4. Discussion and Extensions

4.1. Subsidizing Capital in the Final-Good Sector

Suppose the final-good sector receives a subsidy $\sigma > 0$ per unit of capital K_t employed in t which is financed by a tax on the household sector's labor income. An immediate implication is that the analogue of the first-order condition (2.6) is now

$$1 + r_t = f'(k_t) + \sigma. \tag{4.1}$$

[8] Using (6.4) in the proof of Proposition 2 the derivative of the right-hand side of (3.18) with respect to k_t is $-A_{t-1}(1+g) f'' f/(f')^2 > 0$.

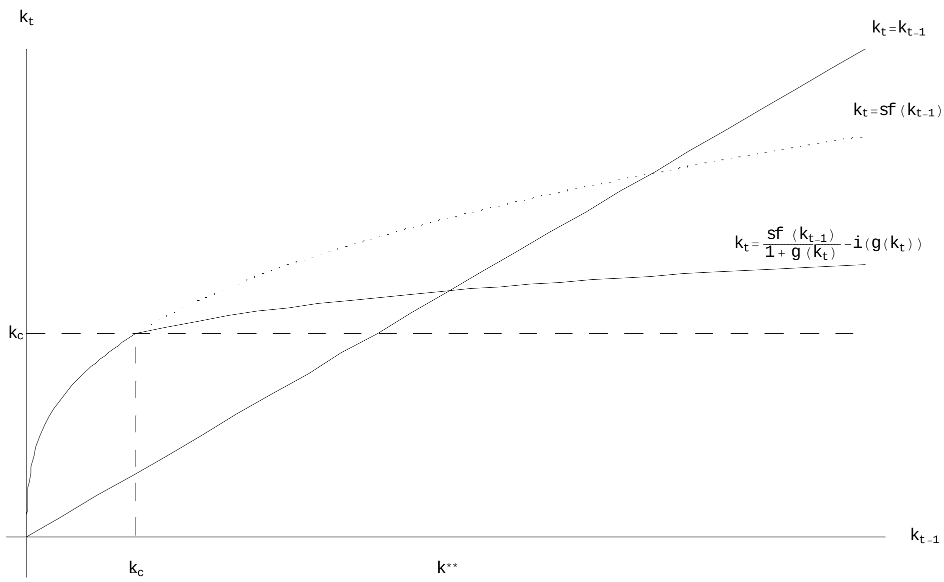

Figure 3.2: $k_c > \underline{k}_c$: Steady State in the Innovation Regime.

This affects the equilibrium incentives to innovate in the following way.

Proposition 5 Let

$$\lim_{k_t \to \infty} \frac{f - k_t f'}{f' + \sigma} > i'(0). \tag{4.2}$$

Then, in equilibrium there is $k_{c\sigma} = k_c(\sigma) > 0$ and a map $g_\sigma : \Re_+^2 \to \Re_+$ such that

$$q_t^* = g_\sigma \text{ with } \begin{cases} g(\sigma, k_t) = 0 & \text{for } k_t \leq k_{c\sigma}, \\ g(\sigma, k_t) > 0 & \text{for } k_t > k_{c\sigma}. \end{cases} \tag{4.3}$$

Moreover,

$$\frac{dk_{c\sigma}}{d\sigma} > 0 \text{ and, for } k_t > k_{c\sigma}, \frac{dg_\sigma}{d\sigma} < 0. \tag{4.4}$$

Proof. See the Appendix.

Proposition 5 extends Proposition 2 for $\sigma > 0$. The upshot is that a subsidy reduces the incentive to innovate. At a given level of k_t the interest rate rises due to (4.1) so that the price of an innovation investment in the intermediate-good sector increases. This has two implications stated in (4.4). First, increasing

σ raises $k_{c\sigma}$. Second, for $k_t > k_{c\sigma}$ the growth rate of labor productivity g_σ is lower the higher σ. The latter captures the fact that the final-good sector and the intermediate-good sector compete for scarce resources. Subsidizing one activity leads to an expansion of this activity at the expense of the other.

The dynamical system is easily derived from replacing g by g_σ in (3.15) and noting that for $\sigma > 0$ I have $\underline{k}_{c\sigma} > \underline{k}_c$ because $k_{c\sigma} > k_c$. Accordingly, the equilibrium path has the same properties as stated in Proposition 4 for the case $\sigma = 0$. Yet, a subsidy on capital may have drastic consequences for the evolution of the economy. First, as $\underline{k}_{c\sigma} > \underline{k}_c$ it postpones the switch into the innovation regime. Second, a simple geometrical argument shows that a subsidy may prevent the switch altogether. Indeed, an economy for which the underlying parameters imply $k_c > \underline{k}_c$ may experience a change in the relation of these critical values to $k_{c\sigma} \leq \underline{k}_{c\sigma}$ so that it converges towards the stationary steady state. A policy that fosters extensive growth may prevent intensive growth altogether.

As to the steady state in the innovation regime one finds[9]

Proposition 6 If the steady state is in the innovation regime then $k_\sigma^{**} = k^{**}(\sigma)$ and $q_\sigma^{**} = g(\sigma, k^{**}(\sigma))$ with

$$\frac{dk_\sigma^{**}}{d\sigma} > 0 \tag{4.5}$$

$$\frac{dq_\sigma^{**}}{d\sigma} < 0. \tag{4.6}$$

Proof. See the Appendix.

Hence, the subsidy raises the steady-state capital intensity. More interestingly, while economies with a high subsidy converge towards a steady state with a higher capital intensity in the final-good sector the steady-state growth rate of the labor productivity is lower. The latter is the result of two opposing effects:

$$\frac{dq_\sigma^{**}}{d\sigma} = \underset{(-)}{\frac{dg_\sigma}{d\sigma}} + \underset{(+)}{\frac{dg_\sigma}{dk_t}} \underset{(+)}{\frac{dk_\sigma^{**}}{d\sigma}} < 0.$$

First, there is a negative direct effect which captures competition among the two sectors for scarce resources. Second, there is an indirect effect through the complementarity of capital and labor in the reduced form of the final-good production function. A higher capital intensity in the aggregate raises the expected factor price ratio and thereby the incentive to innovate.[10]

[9] If the steady state is in the stationary regime it is unaffected by the subsidy as in equilibrium the tax on wage income is equal to the additional capital income that the household sector receives due to the rise in the equilibrium interest rate.

[10] Note that the sign of $dq_\sigma^{**}/d\sigma$ is actually sensitive to the underlying savings hypothesis. Had we endogenized the savings decision through an infinitely lived household and undertaken comparative statics along the Euler condition we would find $dq_\sigma^{**}/d\sigma > 0$.

4.2. Population Growth

Allow for the labor force to grow at a constant rate $\lambda > 0$. As population growth does not interfere with the production technology all prices expressed as functions of the state variable k_t remain unchanged and so does the incentive to innovate. However, aggregate $(t-1)$-savings per unit of efficient labor in t falls with λ. Following the same procedure that led to Proposition 3 one finds the dynamical system of the economy as in (3.15) with s replaced by $s' \equiv s/(1+\lambda)$ and $\underline{k}_c$ by

$$\underline{k}_{c\lambda} := f^{-1}\left(\frac{k_c}{s'}\right). \tag{4.7}$$

The following comparative static results obtain:

Proposition 7 It holds that

$$\frac{d\underline{k}_{c\lambda}}{d\lambda} > 0. \tag{4.8}$$

If $k_c > \underline{k}_{c\lambda}$ I have

$$\frac{dk_\lambda^{**}}{d\lambda} < 0, \; \frac{dq_\lambda^{**}}{d\lambda} < 0. \tag{4.9}$$

Proof. (4.8) follows from the monotonicity of $f(.)$. The results of (4.9) follow immediately from the definition of s' and Corollary 2.

QED.

Hence, I arrive at the interesting result that economies whose population grows faster reach the innovation regime later or may even fail to reach it if $\underline{k}_{c\lambda}$ becomes too high. The reason is simple. Population growth renders labor more abundant which diminishes the relative price of labor and thus the incentive to engage in labor-saving innovation. This force is also behind (4.9): higher population growth leads to a steady state with a lower capital intensity and a lower growth rate of labor productivity. Thus, besides the desirable absence of scale effects the model is broadly consistent with the empirical fact that countries with high population growth rates tend to be poorer on average (Jones (2002), p. 33).

5. Concluding Remarks

This paper presents the idea that phases of extensive growth may be a precursor to phases of intensive growth. This occurs as capital accumulation along neoclassical lines leads to a rise in the relative price of labor that in turn induces labor-saving innovation investments. The analysis emphasized two arguments why extensive growth does not have to imply a stationary state in the long run.

First, a sufficient amount of capital is necessary to induce a switch into the innovation regime. Second, as factor prices are determined by the evolution of economic aggregates capital accumulation must be maintained in the innovation regime as it feeds back on the incentives to innovate. Clearly, the second argument supports the view expressed by Howitt and Aghion (1998) that at the level of economic aggregates capital accumulation and innovation are complementary factors in long-run growth. Yet, at the level of individual firms both sectors compete for foregone consumption as an input. Therefore, subsidizing capital accumulation may reduce long-run growth.

The present paper also contributes to the understanding of large observed international differences in per-capita output that persist or even grow over time. On the one hand, I find that only small differences in preference parameters like the propensity to save, demographical parameters like the population growth rate or technological parameters may postpone or prevent an economy from reaching the innovation regime. Similar effects may be attributed to inadequate policy measures such as a subsidy on capital. On the other hand, these parameters also account for differences in steady-state growth rates.

Finally, let us summarize the main differences between the neoclassical growth model with exogenous technical change and the innovation regime of the present model. First, the switch into the innovation regime is brought about by an endogenous movement of factor prices. Second, the growth of labor productivity accelerates along the transition to the steady state. Third, with growth being endogenous, changes in the savings rate or the population growth rate do not only affect the level but also the long-run growth rate of per-capita output.

6. Appendix

6.1. Proof of Lemma 2

From (2.6), (2.7), and the fact that the function $f(k_t)$ satisfies standard Inada conditions it follows that $p_t > 0$ and $1 + r_t > 0$. If $w_t \leq 0$, intermediate-good firms chose $q_t^* = 0$, $x_t = 1$, and earned strictly positive profits at $p_t > 0$ thus violating (E5).

With all equilibrium prices being positive consider (E9) for any t, condition (2.13) in conjunction with the updating condition (2.19) and (E7) to find successively that

$$L = n_t l_t = n_t / A_{t-1}(1 + q_t) = n_t / A_t = X_t / A_t \Leftrightarrow X_t = A_t L. \tag{6.1}$$

Then, the assertion follows from the definition of k_t and the fact that $\bar{K}_1$ and $A_1 = \bar{A}_0$ are given by initial conditions.

QED.

6.2. Proof of Proposition 2

Condition (E5) requires zero-profits of all active intermediate-good firms, i.e.

$$p_t - \frac{w_t}{A_{t-1}(1 + q_t)} - (1 + r_t)\, i(q_t) = 0. \tag{6.2}$$

By (2.6) and (2.7) r_t and p_t depend on k_t. Then (6.2) determines w_t as a function of (A_{t-1}, q_t, k_t) :

$$w_t = A_{t-1}(1 + q_t)\left(f - k_t f' - f' i(q_t)\right). \tag{6.3}$$

If not indicated otherwise the argument of f is k_t.

By (E4) the choice of labor productivity must minimize unit costs. Hence, for any t, q_t must satisfy (2.15). With (2.6), (2.7), and (6.3) this requires

$$\frac{f - k_t f'}{f'} \leq \left[i'(q_t)(1 + q_t) + i(q_t)\right] \quad \text{with strict inequality } \textit{only} \text{ if } q_t = 0. \tag{6.4}$$

Due to the concavity of f, the left-hand side of (6.4) is strictly increasing in k_t. Moreover, Inada conditions imply that $\lim_{k_t \to 0} (f - k_t f') / f' = 0$. From (2.11), the right-hand side of (6.4) is strictly increasing in q_t. Moreover, $\lim_{q_t \to 0} i'(1 + q_t) + i = i'(0) > 0$. Hence, (6.4) defines a map $g : \Re_+ \to \Re_+$ that associates a unique value $q_t \geq 0$ with each $k_t \geq 0$. Equation (3.10) is sufficient to guarantee the existence of a critical level $k_c > 0$ defined by

$$\frac{f(k_c) - k_c f'(k_c)}{f'(k_c)} = i'(0) \tag{6.5}$$

so that $q_t^* = g(k_t)$ has the properties stated in the proposition.

Insertion of $q_t^* = g(k_t)$ for q_t into (6.3) gives (3.12).

QED.

6.3. Proof of Proposition 3

The following two cases must be considered:

1. If intermediate-good firms expect some $k_t \leq k_c$ then from Proposition 2 none of them innovates in $t - 1$ and from (3.9) and (3.11) k_t is

$$k_t = \Psi(k_{t-1}) = s f(k_{t-1}). \tag{6.6}$$

The expected and the actual level of k_t coincide if and only if

$$k_c \geq k_t = sf\left(k_{t-1}\right) \Leftrightarrow k_{t-1} \leq f^{-1}\left(\frac{k_c}{s}\right) = \underline{k}_c. \tag{6.7}$$

If instead $k_{t-1} > \underline{k}_c$ the economy cannot be in the Solow regime in t because (6.6) yielded $k_t > k_c$ which is incompatible with initial expectations.

2. If intermediate-good firms expect some $k_t > k_c$ then from Proposition 2 all of them innovate. From (3.9) with (3.11) Ψ is then implicitly defined by

$$k_t = \frac{s}{1+g\left(k_t\right)} f\left(k_{t-1}\right) - i\left(g\left(k_t\right)\right). \tag{6.8}$$

Perfect foresight requires

$$\Psi\left(k_{t-1}\right) > k_c. \tag{6.9}$$

From total differentiation of (6.8) I obtain

$$\frac{dk_t}{dk_{t-1}} = \frac{sf'}{g'\left(k_t+i\right)+\left(1+g\right)\left(1+i'g'\right)} > 0, \tag{6.10}$$

where the argument of g is k_t, the argument of i is $g\left(k_t\right)$, and the argument of f is k_{t-1}.
As

$$\frac{s}{1+g\left(k_c\right)} f\left(k_{t-1}\right) - i\left(g\left(k_c\right)\right) = sf\left(k_{t-1}\right),$$

the map $\Psi\left(k_{t-1}\right)$ is continuous at k_c. Hence, with (6.10) it follows that (6.9) holds for $k_{t-1} > \underline{k}_c$.
QED.

6.4. Proof of Proposition 4

By Proposition 2 the economy starts in the stationary regime because $0 < k_1 < k_c$.

Case 1: $\underline{k}_c \geq k_c$

I have $\underline{k}_c > k_c > k_1 > 0$. By Proposition 3 the economy evolves for at least one period according to the difference equation $k_t = sf\left(k_{t-1}\right)$. From $\underline{k}_c > k_c$ it follows that $\underline{k}_c > sf\left(\underline{k}_c\right) = k_c$. Moreover, $sf\left(k_{t-1}\right)$ is continuous, strictly increasing and concave with $f\left(0\right) = 0$ and $f'\left(0\right) = \infty$. Therefore, it has two steady states on $[0, \underline{k}_c]$ which satisfy (3.16). The trivial steady state $k_1^* = 0$ is unstable whereas the second $k_2^* > 0$ is globally stable in the sense that for all $k_1 \in (0, k_c]$ and $k_1 \neq k^*$, $k_t \to k^*$ for $t \to \infty$. This follows from the fact that to the left (right) of k^* I have $sf\left(k_{t-1}\right) > k_t$ $\left(sf\left(k_{t-1}\right) < k_t\right)$.

Case 2: $k_c > \underline{k}_c$

If $k_c > \underline{k}_c$ then $sf\left(\underline{k}_c\right) = k_c > \underline{k}_c$ and there is no steady state with $k^* > 0$ on $[0, \underline{k}_c]$. Hence, the economy reaches the innovation regime at some period $t' > 1$. The transition is described by

$$k_{t'} = \frac{s}{1+g\left(k_{t'}\right)} f\left(k_{t-1}\right) - i\left(g\left(k_{t'}\right)\right).$$

From period $t'+1$ onwards the economy remains in the innovation regime and converges for $t \to \infty$ towards the steady state that satisfies (3.17). The steady state is globally stable in the sense that for all $k_1 \in (0, k_c]$, $k_t \to k^{**}$ for $t \to \infty$. This follows from (6.10) and the fact that $0 < dk_t/dk_{t-1} < sf'\left(k_{t-1}\right)$ for all $k_{t-1} > 0$.
QED.

6.5. Proof of Corollary 1

Total differentiation of (3.17) gives

$$\frac{dk^{**}}{ds}=\frac{f}{g'\left(k^{**}+i\right)+\left(1+g\right)\left(1+i'g'\right)-sf'}, \tag{6.11}$$

where the argument of f and g is k^{**} and the argument of i is g. From the stability of the steady state and (6.10) it follows that

$$\left.\frac{dk_t\left(k_{t-1}\right)}{dk_{t-1}}\right|_{k_{t-1}=k_t=k^{**}}=\frac{sf'}{g'\left(k^{**}+i\right)+\left(1+g\right)\left(1+i'g'\right)}<1$$

so that the denominator of (6.11) is positive. Hence, the first claim in (3.20) follows. With (3.13) I find the second claim:

$$\frac{dq^{**}}{ds}=\frac{dg\left(k^{**}\right)}{dk}\frac{dk^{**}}{ds}>0.$$

QED.

6.6. Proof of Proposition 5

From the zero-profit condition of an intermediate-good firm, (4.1) and (2.7) I obtain the break-even wage as

$$w_t=A_{t-1}\left(1+q_t\right)\left(f-k_tf'-\left(f'\left(k_t\right)+\sigma\right)i\left(q_t\right)\right). \tag{6.12}$$

This leads to the analogue of (6.4):

$$\frac{f-k_tf'}{f'+\sigma}\leq\left[i'(q_t)\left(1+q_t\right)+i\left(q_t\right)\right] \tag{6.13}$$

with strict inequality *only* if $q_t=0$

where the argument of f is k_t. (6.13) defines the map g_σ which associates a unique $q_t\geq 0$ with each $(\sigma,k_t)\geq 0$. Condition (4.2) is sufficient to guarantee the existence of a critical level $k_{\sigma c}=k_c\left(\sigma\right)>0$ defined by

$$\frac{f\left(k_{\sigma c}\right)-k_{\sigma c}f'\left(k_{\sigma c}\right)}{f'\left(k_{\sigma c}\right)+\sigma}=i'(0)$$

which is generalizes (6.5) for $\sigma>0$. Upon total differentiation I obtain the first comparative-static result in (4.4). Similarly, total differentiation of (6.13) at (σ,k_t) for $k_t>k_{\sigma c}$ gives

$$\frac{dg_\sigma}{d\sigma}=\frac{-\frac{f-k_tf'}{(f'+\sigma)^2}}{2i'+\left(1+g_\sigma\right)i''}<0 \tag{6.14}$$

which shows the second comparative-static result in (4.4).

QED.

6.7. Proof of Proposition 6

The steady state in the innovation regime satisfies[11]

$$(1 + g(\sigma, k^{**}))(k^{**} + i(g(\sigma, k^{**}))) = sf(k^{**}) \tag{6.15}$$

which defines $k_{\sigma}^{**} = k^{**}(\sigma)$ with

$$\frac{dk_{\sigma}^{**}}{d\sigma} = -\frac{dg_{\sigma}}{d\sigma} \frac{i'(1 + g_{\sigma}) + (k + i)}{(1 + i' g'_{\sigma})(1 + g_{\sigma}) + g'_{\sigma}(k + i) - sf'} > 0 \tag{6.16}$$

at (k^{**}, σ). Here, $g'_{\sigma} := dg_{\sigma}/dk_t$. From the stability of the steady state the denominator is positive. With (6.14) the sign in (4.5) follows.

Turning to the steady-state growth rate of labor productivity I note that

$$q_{\sigma}^{**} = g_{\sigma}(k_{\sigma}^{**})$$

so that at $(k_{\sigma}^{**}, \sigma)$

$$\frac{dq_{\sigma}^{**}}{d\sigma} = \frac{dg_{\sigma}}{d\sigma} + \frac{dg_{\sigma}}{dk_t} \frac{dk_{\sigma}^{**}}{d\sigma}.$$

One readily verifies with (6.14) and (6.16) that the right-hand side is smaller than zero iff

$$(1 + g(k_{\sigma}^{**}, \sigma)) > sf'(k_{\sigma}^{**}). \tag{6.17}$$

From (6.15) I know that at the steady state

$$(1 + g(k_{\sigma}^{**}, \sigma)) = \frac{sf(k_{\sigma}^{**})}{k_{\sigma}^{**} + i(g(k_{\sigma}^{**}, \sigma))}$$

so that (6.17) is satisfied at $(k_{\sigma}^{**}, \sigma)$ iff

$$\begin{aligned} \frac{sf}{k_{\sigma}^{**} + i(g)} &> sf' \\ \Leftrightarrow \frac{f - f' k_{\sigma}^{**}}{f'} &> i(g). \end{aligned}$$

Yet, from (6.13) at $(k_{\sigma}^{**}, \sigma)$ I have

$$\begin{aligned} \frac{f - f' k_{\sigma}^{**}}{f' + \sigma} &= i'(g)(1 + g) + i(g) \\ \Leftrightarrow \frac{f - f' k_{\sigma}^{**}}{f'} &= \left(1 + \frac{\sigma}{f'}\right)\left[i'(g)(1 + g) + i(g)\right] > i(g). \end{aligned}$$

QED.

[11] Existence and uniqueness of a steady state in the innovation regime follow from the same arguments that are given in Proposition 4 for the case $\sigma = 0$.

References

[1] **H. Bester and E. Petrakis,** *Wage and Productivity Growth in a Competitive Industry*, Journal of Economic Theory, 109, 52–69, (2003).

[2] **O. Galor and O. Moav**, *Das Human Kapital***,** mimeo, Hebrew University of Jerusalem (2001).

[3] **W. Fellner**, *Two Propositions in the Theory of Induced Innovations***,** Economic Journal, 71, 305–308, (1961).

[4] **M. Hellwig and A. Irmen**, *Endogenous Technical Change in a Competitive Economy*, Journal of Economic Theory, 101, 1–39, (2001).

[5] **P. Howitt and P. Aghion**, *Capital Accumulation and Innovation as Complementary Factors in Long-Run Growth*, Journal of Economic Growth, 3, 111–130, (1998).

[6] **Ch. Jones**, *Introduction to Economic Growth*, 2nd edition, Norton & Company, NY, (2002).

[7] **P. Krugman,** *The Myth of Asia's Miracle*, Foreign Affairs, 73, (1994), 62–78.

[8] **L. Lau and K. Kim,** *The Sources of Economic Growth of the East Asian Newly Industrializing Countries*, Journal of the Japanese and International Economies, 8, (1994), 235–271.

[9] **K. Matsuyama,** *Growing Through Cycles*, Econometrica, 67, (1999), 335–347.

[10] **H. Pack**, *Technological Change and Growth in East Asia: Macro versus Micro Perspectives*, in: Stiglitz, Joseph and Yusuf Shahid, *Rethinking the East Asian Miracle*, Oxford University Press, (2001).

[11] **L. Rivera-Batiz and P. Romer,** *Economic Integration and Endogenous Growth*, Quarterly Journal of Economics, 106, (1991), 531–555.

[12] **R. Solow,** *A Contribution to the Theory of Economic Growth*, Quarterly Journal of Economics, 70, 65–94, (1956).

[13] **T. Swan,** *Economic Growth and Capital Accumulation*, Economic Record, 32, (1956).

[14] **A. Young,** *The Tyranny of Numbers: Confronting the Statistical Realities of the East Asian Growth Experience*, Quarterly Journal of Economics, 110, (1995), 641–680.