Error estimation and error bounding in quantities of interest based on equilibrated recovered displacement fields

E. Nadal[†*], O.A. González-Estrada[‡], J.J. Ródenas[†], S.P.A. Bordas[‡], P. Kerfriden [‡], F.J. Fuenmayor[†]

[†]Centro de Investigación de Tecnología de Vehículos (CITV), Universitat Politècnica de València,
Camino de Vera s/n. E-46022-Valencia, Spain.
jjrodena@mcm.upv.es, ennaso@upv.es, ffuenmay@mcm.upv.es

[‡]Institute of Mechanics & Advanced Materials, Cardiff University, School of Engineering,
Queen's Building, The Parade, Cardiff CF24 3AA Wales, UK.
estradaoag@cardiff.ac.uk, bordass@cardiff.ac.uk, pierre.kerfriden@gmail.com

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ABSTRACT

Over the last ten years there has been an increase on the use of goal-oriented error estimates aimed to quantify the local error on a (non)linear quantity of interest (QoI) that might result relevant for design purposes. Residual-based error estimators have been used recursively to obtain upper and lower bounds of the error in quantities of interest for finite element approximations. In this paper, we present a recovery technique for 2D linear elasticity problems, based on the superconvergent patch recovery (SPR), which provides recovered displacement and stress fields that are then utilised to evaluate practical upper and lower error bounds of QoI.

The recovery technique based on displacements considers the fulfilment of boundary and internal equilibrium equations, Dirichlet constraints and, for singular problems, the splitting of the displacement and stress fields into singular and smooth parts (as described in [1]). Similar recovery techniques considering stresses were previously used to obtain upper bounds of the error in energy norm [2, 3]. However, enforcing continuity over the locally equilibrated stress fields evaluated on patches introduced a lack of equilibrium which have to be accounted for using correction terms. The evaluation of these correction terms required approximations of the exact error in the displacements that were obtained using projection techniques which leaded to a higher computational effort. To overcome this difficulty, we utilise a recovery based on displacements to directly obtain an estimation of the exact error in the displacements, thus easing the evaluation of the correction terms.

The recovered displacements are first used to obtain nearly statically admissible recovered fields for the primal and dual problems. After that we also obtain compatible stress fields coming from enhanced continuous displacement fields that fulfil the essential boundary conditions. With these two recovered stress fields we then evaluate lower error bounds (adapting the procedure presented in [4]), computed versions of the upper error bounds, as well as very accurate error estimates, for different quantities of interest.

One particular feature of the considered approach is that the error bounds for the QoI are given in terms of the errors in energy norm for the primal and dual solutions, following the ideas presented by Oden and Prudhomme [5]. After obtaining the solutions for the primal and dual problems we obtain recovered fields for both problems using the equilibrated recovery technique. For the dual problem we must define analytical expressions that describe the body loads and boundary tractions required by the equilibrated

recovery process. Once the recovered fields are obtained we evaluate correction terms and the error estimates in energy norm for the dual and primal solutions. Then, using the same methodology in [5], we evaluate lower and upper bounds for the quantity of interest at hand.

Numerical tests using 2D benchmark problems with exact solution are used to investigate the quality of the proposed technique. Results for different quantities of interest show that the technique provides practical error bounds and sharp error estimates that can be used in goal oriented adaptive procedures.

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