Error estimation and bounding in energy norm based on a displacement recovery technique

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ABSTRACT

Traditionally, recovery based error estimators have considered the evaluation of an enhanced stress field obtained from the raw Finite Element (FE) stress solution in linear elasticity. Instead of that, one can also obtain a recovered displacement field form the FE displacements [1]. Herein, we describe a super-convergent patch recovery of the displacement field which considers the local fulfilment of boundary and internal equilibrium equations, Dirichlet constraints and, for singular problems, the splitting of the displacement and stress fields into singular and smooth parts following the ideas presented in [2]. The development of this displacement recovery technique was motivated by the need to calculate the error in the displacement field required for the evaluation of upper bounds of the error in energy norm using a stress recovery technique [3, 4].

The accuracy of the proposed displacement recovery technique when used for error estimation in energy norm has been shown to be similar to that of the previously developed stress recovery technique. Although its computational cost is slightly higher than that of the stress recovery technique, the displacement recovery technique provides important advantages:

- The recovered displacement solution can be projected to more refined meshes to obtain the initial displacements vector used by iterative solvers, thus reducing the number of iterations needed to reach the solution.
- Practical upper error bounds can be directly obtained without using the projection techniques proposed in [3, 4].
- Adapting the ideas described in [5], we have also been able to evaluate accurate lower bounds of the error in energy norm.
- The lower and practical upper bounds of the error in energy norm provided by the displacement recovery technique allowed for the development of error bounding techniques in quantities of interest as described in a paper presented in this session.

Numerical tests based on the use of problems with known analytical solution have been used to validate the proposed techniques for error estimation and error bounding in energy norm.

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