Equilibrated patch recovery for accurate evaluation of upper error bounds in quantities of interest

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Octavio A. González-Estrada[1,4], Juan J. Ródenas *[,5], Enrique Nadal[5] , Stéphane Bordas[1,2] and Pierre Kerfriden[1,3]

[1]Cardiff School of Engineering. Institute of Modelling and Simulation in Mechanics and Materials.
Cardiff University, Queen's Buildings, The Parade, Cardiff CF24 3AA Wales, UK.
[2]Professor in Engineering Royal Academy of Engineering/Leverhulme Senior Research Fellow.
Email: stephane.bordas@alum.northwestern.edu
[3]Lecturer in Engineering. Email: pierre.kerfriden@gmail.com
[4]Research Associate in Engineering. Email: estradaoag@cardiff.ac.uk
[5]Centro de Investigación en Tecnología de Vehículos (CITV),
Universidad Politécnica de Valencia, E-46022-Valencia, Spain.
Email: jjrodena@mcm.upv.es; ennaso@upvnet.upv.es

ABSTRACT

Lately, there is an increasing interest on the use of goal-oriented error estimates which help to measure and control the local error on a linear or non-linear quantity of interest (QoI) which is relevant for design purposes (such as the mean stress value in a particular area, displacements, the stress intensity factor for fracture problems,...). In general, residual-based error estimators have been used to obtain upper and lower bounds of the error in quantities of interest for finite element approximations.

In this work, we propose an alternative *a posteriori* procedure based on recovery techniques to evaluate an upper error bound of the QoI. It can be shown that the following expression yields an upper bound of the QoI provided that the recovered stress fields for the primal, *p*, and the dual, *d*, problems, $\boldsymbol{\sigma}_p^*$ and $\boldsymbol{\sigma}_d^*$ (evaluated from the corresponding finite element solutions, $\boldsymbol{\sigma}_p^h$ and $\boldsymbol{\sigma}_d^h$) are statically admissible:

$$Q(\mathbf{e}) = B(\mathbf{e}_p, \mathbf{e}_d) = B(\mathbf{u}_p - \mathbf{u}_p^h, \mathbf{u}_d - \mathbf{u}_d^h) \leq \int (\boldsymbol{\sigma}_p^* - \boldsymbol{\sigma}_p^h)^t D^{-1} (\boldsymbol{\sigma}_d^* - \boldsymbol{\sigma}_d^h) d\Omega \quad (1)$$

We have used a recovery procedure based on the superconvergent patch recovery (SPR) technique to obtain the recovered stress fields. This recovery technique was previously used in [1] and [2] to obtain upper bounds of the error in energy norm and has been used in this paper to obtain nearly statically admissible recovered stress fields $\boldsymbol{\sigma}_p^*$ and $\boldsymbol{\sigma}_d^*$ and thus a computable version of the bound presented in (1). A Goal Oriented Adaptivity (GOA) methodology has been developed based on the use of equation (1).

REFERENCES

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