Rapid testing of stabilised finite element formulations for the Reissner-Mindlin plate problem using the FEniCS project

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The FEniCS project allows for rapid numerical testing of some unusual weak form and function space constructions because of its advanced basis function tabulation (FIAT) and code generation (FFC) capabilities. Undertaking numerical testing of new element designs that might have been tedious, lengthy and error prone with a traditional FE package where 'elements' (function spaces) and 'physical models' (weak forms) are typically pre-defined for the user become relatively straightforward when using FEniCS.

We have been working to develop new meshfree approaches to solving the Reissner-Mindlin plate equations based on the application of a mixed weak form. Existing approaches have issues such as producing nearly-singular matrices or delivering sub-optimal convergence rates. It is well known that naïve numerical discretisations of the Reissner-Mindlin problem result in shear-locking, a strictly numerical phenomenon that occurs as the plate's thickness approaches zero which produces incorrect results.

A classic remedy for this problem is to move to a mixed formulation, where the transverse shear stresses are treated as independent variational quantities in the weak form. Whilst this type of problem is typically well-behaved as the plate becomes thin, it is not necessarily stable. The stability of a numerical method is governed by the celebrated Ladyzhenskaya-Babuška-Brezzi (LBB) conditions. In particular, simultaneous satisfaction of the first Brezzi condition (ellipticity on the kernel) and the second Brezzi condition (the inf-sup condition) makes the design of stable elements particularly difficult for the Reissner-Mindlin problem. Inspired by the work of Arnold and Brezzi we bypass the difficulty of satisfying the first Brezzi condition by stabilising the mixed weak form using a local discretisation-dependent parameter. We have designed what we believe to be the first meshless formulation for the Reissner-Mindlin plate model based directly upon a mixed weak form.

We have used FEniCS to explore the capabilities and behaviours of some element designs suggested by authors such as Arnold, Brezzi and Lovadina with the aim of making sound design choices for our meshfree method. We will demonstrate that by using FEniCS project questions about the effect of the stabilisation parameter on the solution as well as the performance and stability can be answered rapidly for a range of elements with minimal coding.