Generalized H_∞ observers design for systems with unknown inputs

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***Abstract*— A generalized H_∞ observers design is proposed for linear systems with unknown inputs. It generalizes the existing results concerning the proportional observer (PO) design and the proportional integral observer (PIO) design. The approach is based on the solutions of the algebraic constraints obtained from the unbiasedness conditions of the estimation error. The observer design is obtained from the solutions of linear matrix inequalities (LMIs). A numerical example is given to illustrate our approach.**

I. INTRODUCTION AND PROBLEM FORMULATION

The problem of estimating the state of a dynamical system from outputs and inputs has received many attentions since the first result obtained by Luenberger and Kalman for the deterministic and stochastic systems appeared [1] and [2]. This is due to the fact that the state variables are often not available or only a partial part of them are accessible. The observer has many applications from the feedback control to the fault detection and failure diagnosis. In the last few decades, the observer design has been extended to systems with unknown inputs (see [3], [4], [5] and references therein). All these results use the proportional observer (PO). Proportional integral observer (PIO) has been introduced by duality to PI controller which is generally used to achieve steady-state accuracy and is also useful for the control of unknown systems. The PIO constitutes an extension of the Luenberger observers (called also PO). The first result on the PIO was presented by [11] for simple input simple output systems (SISO), and extension to multivariable systems was presented in [12] and [14]. In [7] the authors showed the performances of the PIO, compared to PO, in presence of disturbances and uncertainties. Recently many results on the PIO for systems with unknown inputs were presented in [8]-[10]. On the other hand a new structure of the observer, called dynamic observer, was developed by [13] and [15]. It presented an alternative state estimation structure which can be considered as more general than PO and PIO, these latter two can be only considered as particular cases of this structure. The idea of including additional dynamic in the observer was presented in [16]. The structure of the proposed dynamic observer presented in [13] is dual of the dynamic feedback control structure.

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The present paper concerns a new form of a dynamic observer, which is more general than those presented in [13] and [15]. This observer is formed by a dynamical part and a static part, and it is used to estimate the state of the system in presence of unknown inputs and disturbances. It can be shown that the existing dynamic observers, the PO and PIO are only particular cases of the structure of our observer. A numerical example is presented to illustrate our approach.

Consider the following linear system with unknown inputs

$$\begin{aligned} \dot{x} &= Ax + Bw + Gd \\ y &= Cx + Dw \end{aligned} \tag{1}$$

with the initial state $x(0) = x_0$, where $x \in R^n, y \in R^p, w \in R^q$ and $d \in R^m$ are the state vector, the measurement output, the disturbance vector and the unknown input respectively. Matrices A, B, G, C and D are known and of appropriate dimensions.

We consider the generalized full-order linear time invariant observer, called generalized observer (GO), of the form

$$\begin{aligned} \dot{z} &= Nz + Jy + Ev \\ \dot{v} &= Tz + My + Hv \\ \hat{x} &= z + Fy \end{aligned} \tag{2}$$

where $z \in R^n, v \in R^t, \hat{x} \in R^n$ are the state vector of the observer, an auxiliary state vector and the estimate of x respectively. Matrices N, J, E, T, M, H and F are unknown and of appropriate dimensions which can be determined such that:

1.the estimation error $e(t) = \hat{x}(t) - x(t)$ and the auxiliary vector $v(t)$ converge to 0 for $w(t) = 0$;

2.for $w(t) \neq 0$ we must minimize the worst case estimation error $\|e\|_{L_2}$ for all bounded energy disturbances $w(t)$, i.e minimize

$$\sup_{w \in L_2 - \{0\}} \frac{\|e\|_{L_2}}{\|w\|_{L_2}}$$

II. H_∞ OBSERVERS DESIGN

In this section we shall present the conditions of the existence of the GO given by (2) for system(1). Then a new method for the design of GO will be given from the solution of the linear matrix inequality optimization problem. From system (1) and observer (2) we can give the following lemma:

Lemma 1 *The dynamics of $e(t)$ and $v(t)$ are independent of $x(t)$, $d(t)$ and $\dot{w}(t)$ if the following conditions are satisfied*

- $FD = 0$
- $\Psi G = 0$
- $MC - T\Psi = 0$

- $\Psi A - N\Psi + JC = 0$

where $\Psi = FC - I$.

Proof: The estimation error e is given by

$$\begin{array}{rcl} e & = & \hat{x} - x \\ & = & z + F(Cx + Dw) - x \\ & = & z + \Psi x + FDw \end{array}$$

its dynamic is given by:

$$\begin{array}{rcl} \dot{e} & = & \dot{z} + \Psi\dot{x} + FD\dot{w} \\ & = & Nz + J(Cx + Dw) + Ev \\ & & +\Psi(Ax + Bw + Gd) + FD\dot{w} \\ & = & Ne + (\Psi B + JD - NFD)w + Ev \\ & & +(\Psi A + JC - N\Psi)x + \Psi Gd + FD\dot{w} \end{array} \quad (3)$$

Also the dynamic of $v(t)$ can be written as

$$\begin{array}{rcl} \dot{v} & = & Te + Hv + (MC - T\Psi)x \\ & & +(MD - TFD)w \end{array} \quad (4)$$

Now, these dynamics are independent of x, d and $\dot{w}$ if items $1) - 4)$ of the lemma are satisfied, this proves the lemma. ■

If the 4 items of lemma 1 are satisfied, then equations (3) and (4) become

$$\dot{u} = Qu + Sw \quad (5)$$

where $u = \begin{pmatrix} e \\ v \end{pmatrix}$, $Q = \begin{pmatrix} N & E \\ T & H \end{pmatrix}$ and $S = \begin{pmatrix} \Psi B + JD \\ MD \end{pmatrix}$.

Now, we can give the following lemma which can be used in the sequel of the paper (see reference [17]).

Lemma 2 *Let matrices* $\mathbb{B} \in R^{n\times m}$, $\mathbb{C} \in R^{k\times n}$ *and* $\mathbb{Q} = \mathbb{Q}^T \in R^{n\times n}$ *be given. Then the following statements are equivalent:*

- *There exists a matrix* $\mathscr{Y}$ *satisfying*

$$\mathbb{B}\mathscr{Y}\mathbb{C} + (\mathbb{B}\mathscr{Y}\mathbb{C})^T + \mathbb{Q} < 0 \quad (6)$$

- *The following two conditions holds*

$$\mathbb{B}^{\perp}\mathbb{Q}\mathbb{B}^{\perp T} < 0 \quad (7)$$

$$\mathbb{C}^{T\perp}\mathbb{Q}\mathbb{C}^{T\perp T} < 0 \quad (8)$$

where $\mathbb{B}^{\perp}$ *is the orthogonal complement of* $\mathbb{B}$. *In this case, all matrices* $\mathscr{Y}$ *are parameterized as follows*

$$\mathscr{Y} = \mathbb{B}_R^{\dagger}\mathbb{K}\mathbb{C}_L^{\dagger} + \mathbb{Z} - \mathbb{B}_R^{\dagger}\mathbb{B}_R\mathbb{Z}\mathbb{C}_L\mathbb{C}_L^{\dagger} \quad (9)$$

where

$$\mathbb{K} = -\mathbb{R}^{-1}\mathbb{B}_L^T\mathbb{S}_1\mathbb{C}_R^T\left(\mathbb{C}_R\mathbb{S}_1\mathbb{C}_R^T\right)^{-1}$$

$$+\mathbb{S}^{1/2}\mathbb{L}\left(\mathbb{C}_R\mathbb{S}_1\mathbb{C}_R^T\right)^{-1/2}$$

$$\mathbb{S}_1 = \left(\mathbb{B}_L\mathbb{R}^{-1}\mathbb{B}_L^T - \mathbb{Q}\right)^{-1} > 0$$

$$\mathbb{S} = \mathbb{R}^{-1} - \mathbb{R}^{-1}\mathbb{B}_L^T\left(\mathbb{S}_1 - \mathbb{S}_1\mathbb{C}_R^T(\mathbb{C}_R\mathbb{S}_1\mathbb{C}_R^T)^{-1}\mathbb{C}_R\mathbb{S}_1\right)\mathbb{B}_L\mathbb{R}^{-1}$$

where matrices $\mathbb{R}$ *and* $\mathbb{L}$ *are arbitrary and of appropriate dimensions satisfying* $\mathbb{R} = \mathbb{R}^T > 0$ *and* $\|\mathbb{L}\| < 1$. *Matrices* $\mathbb{B}_L$, $\mathbb{B}_R$, $\mathbb{C}_L$ *and* $\mathbb{C}_R$ *are any full rank matrices such that* $\mathbb{B} = \mathbb{B}_L\mathbb{B}_R$ *and* $\mathbb{C} = \mathbb{C}_L\mathbb{C}_R$.

If the 4 items of lemma 1 are satisfied and by using (5) we obtain.

$$\begin{array}{rcl} \dot{u} & = & Qu + Sw \\ e & = & Lu \end{array} \quad (10)$$

where $L = \begin{bmatrix} I_n & 0 \end{bmatrix}$. Then the problem of the observer design is reduced to find matrices N, J, E, T, M, H and F such that :

- Matrix Q is a stability matrix for $w(t) = 0$;
- for $w(t) \neq 0$,

$$\sup_{w\in L_2-\{0\}} \frac{\|e\|_{L_2}}{\|w\|_{L_2}}$$

is minimized or equivalently $\|T_{we}\|_\infty$ is minimized .

where T_{we} represents the transfer function from the disturbance w to the estimation error e. The solution to this problem can be obtained from the bounded real lemma (see [17]) and is given by the following lemma:

Lemma 3 *System* (10) *is asymptotically stable for* $w = 0$ *and* $\|T_{we}\|_\infty < \gamma$ *for* $w \neq 0$, *if and only if there exists a matrix* $X > 0$ *such that the following LMI is satisfied*

$$\begin{bmatrix} XQ + Q^TX + L^TL & XS \\ S^TX & -\gamma^2 I \end{bmatrix} < 0 \quad (11)$$

The problem formulated in this lemma is not easy to solve by using the form (11) where the entries of matrices Q and S are subjected to the constraints of lemma 1. The following section will provide us with the parameterization of all these entries.

A. Parameterization of the observer matrices

Before presenting the observer design method we shall present the parameterization of the solutions to the 4 conditions of lemma 1.

Now from conditions 1) and 2) of lemma 1 and by using the definition of matrix Ψ we have the following equation

$$FF_2 = F_1 \quad (12)$$

where $F_2 = \begin{bmatrix} D & CG \end{bmatrix}$ and $F_1 = \begin{bmatrix} 0 & G \end{bmatrix}$. The necessary and sufficient condition for (12) to have a solution is that

$$rank\begin{bmatrix} D & CG \\ 0 & G \end{bmatrix} = rank\begin{bmatrix} D & CG \end{bmatrix} \quad (13)$$

Remark 1 *The condition* (13) *is dependent on the matrix* D, *one can see that if* $D = 0$, *we obtain the following condition* $rank\begin{bmatrix} CG \\ G \end{bmatrix} = rank(CG)$, *which is equivalent to* $rank\begin{bmatrix} I & -C \\ 0 & I \end{bmatrix}\begin{bmatrix} CG \\ G \end{bmatrix} = rank G = rank(CG)$. *This is the condition generally used in the observers for systems with unknown inputs as can be seen from reference [4].*

In the sequel of this paper, we shall make the following assumption:

Assumption 1 *We assume that condition* (13) *is satisfied.*

Under Assumption 1 a solution to (12) is given by

$$F = F_1 F_2^+ \tag{14}$$

where F_2^+ is any generalized inverse matrix satisfying the condition $F_2 F_2^+ F_2 = F_2$.

Now from condition 3) of lemma 1 and by using the definition of matrix Ψ we obtain

$$T = -ZC \tag{15}$$

where $Z = M - TF$.

On the other hand from condition 4) we obtain

$$N = N_1 - KC \tag{16}$$

where $N_1 = A - FCA = A - F_1 F_2^+ CA$ and $K = J - NF$.

Using these values, matrices Q and S can be written as

$$\begin{aligned} Q &= \begin{bmatrix} N & E \\ T & H \end{bmatrix} \\ &= \begin{bmatrix} N_1 - KC & E \\ -ZC & H \end{bmatrix} \\ &= Q_1 - YQ_2 \end{aligned} \tag{17}$$

and

$$S = \begin{bmatrix} \Psi B + JD \\ MD \end{bmatrix} = S_1 - YS_2 \tag{18}$$

where $Q_1 = \begin{bmatrix} N_1 & 0 \\ 0 & 0 \end{bmatrix}$, $Y = \begin{bmatrix} K & E \\ Z & H \end{bmatrix}$, $Q_2 = \begin{bmatrix} C & 0 \\ 0 & -I \end{bmatrix}$, $S_1 = \begin{bmatrix} (FC - I_n)B \\ 0 \end{bmatrix}$ and $S_2 = \begin{bmatrix} -D \\ 0 \end{bmatrix}$.

B. Observer design

From the obtained results of section A and from the results of lemma 2 we can give the observer design from the solution of a linear matrix inequality formulation.

Now, define the following matrices $\mathbb{B} = \begin{bmatrix} -I \\ 0 \end{bmatrix}$ and $\mathbb{C} = \begin{bmatrix} Q_2 & S_2 \end{bmatrix}$, then we have the theorem as follows:

Theorem 1 *Under assumption 1, system* (10) *is asymptotically stable for $w = 0$ and $\|T_{we}\|_\infty < \gamma$ for $w \neq 0$, if and only if there exists a matrix $X > 0$ such that the following LMI is satisfied*

$$\mathbb{C}^{T\perp} \mathbb{Q} \mathbb{C}^{T\perp T} < 0 \tag{19}$$

In this case, all matrices Y are parametrized as follows

$$Y = X^{-1} \mathscr{Y} \tag{20}$$

where

$$\mathscr{Y} = -\mathbb{R}^{-1} \mathbb{B}_L^T \mathbb{S}_1 \mathbb{C}_R^T \left(\mathbb{C}_R \mathbb{S}_1 \mathbb{C}_R^T \right)^{-1}$$

$$+ \mathbb{S}^{1/2} \mathbb{L} \left(\mathbb{C}_R \mathbb{S}_1 \mathbb{C}_R^T \right)^{-1/2}$$

$$\mathbb{S}_1 = \left(\mathbb{B}_L \mathbb{R}^{-1} \mathbb{B}_L^T - \mathbb{Q} \right)^{-1} > 0$$

$$\mathbb{S} = \mathbb{R}^{-1} - \mathbb{R}^{-1} \mathbb{B}_L^T \left(\mathbb{S}_1 - \mathbb{S}_1 \mathbb{C}_R^T (\mathbb{C}_R \mathbb{S}_1 \mathbb{C}_R^T)^{-1} \mathbb{C}_R \mathbb{S}_1 \right) \mathbb{B}_L \mathbb{R}^{-1}$$

with $\mathbb{Q} = \begin{bmatrix} XQ_1 + Q_1^T X + L^T L & XS_1 \\ S_1^T X & -\gamma^2 I \end{bmatrix}$, matrices $\mathbb{R}$ and $\mathbb{L}$ are arbitrary and of appropriate dimensions satisfying $\mathbb{R} = \mathbb{R}^T > 0$ and $\|\mathbb{L}\| < 1$.

Proof: From lemma 3 the observer error (3) is asymptotically stable, for $w(t) = 0$, and the H_∞-norm bound $\|T_{we}\|_\infty < \gamma$ for $w(t) \neq 0$, if and only if there exists a positive matrix X such that (11) is satisfied, by using (17) and (18) we obtain the following LMI

$$\mathbb{Q} + \mathbb{C} \mathscr{Y} \mathbb{B} + (\mathbb{C} \mathscr{Y} \mathbb{B})^T < 0 \tag{21}$$

where $\mathscr{Y} = XY$.

The solvability conditions of (21) are

$$\mathbb{B}^\perp \mathbb{Q} \mathbb{B}^{\perp T} < 0 \tag{22}$$

$$\mathbb{C}^{T\perp} \mathbb{Q} \mathbb{C}^{T\perp T} < 0 \tag{23}$$

[17]. Now we have $\mathbb{B}^\perp = \begin{bmatrix} 0 & I \end{bmatrix}$, than inequality (22) is always satisfied and the solvability conditions reduce to (23) which is exactly (19). On the other hand since $\mathbb{C}$ and $\mathbb{B}$ are of full row and full column ranks respectively , equation (9) of lemma 2 becomes $\mathscr{Y} = \mathbb{K}$. The rest of the proof stems from lemma 2. ■

Remark 2 *The generalized observer design can be also directly obtained from lemma 3, in fact that inequality* (11) *or inequality* (21) *can be rewritten as follows*

$$\begin{bmatrix} \Pi & XS_1 - \Omega S_2 \\ S^T X - S_2^T \Omega^T & -\gamma^2 I \end{bmatrix} < 0 \tag{24}$$

where $\Pi = XQ_1 + Q_1^T X - \Omega Q_2 - Q_2^T \Omega^T + L^T L$. In this case the parameter matrix $Y = X^{-1}\Omega$.

This formulation is simple and easy to solve, however it does not give the general solution to this design compared to that given in theorem 1. In fact, one can obtain the evident solution corresponding to the PO which leads to matrices $E = 0$, $T = 0$ and H a stability matrix.

Design algorithm summary

The results of section 2 can be summarized in the following design algorithm:

- step 1: Check the condition (13), then compute matrices N_1, S_1, S_2, Q_1 and Q_2;
- step 2: Calculate $\mathbb{C}$ and solve the LMI (19) to find matrix X, then find matrix $\mathscr{Y}$ of theorem 1 by choosing matrices $\mathbb{R} = \mathbb{R}^T > 0$ and $\|\mathbb{L}\| < 1$ satisfying the conditions given in theorem 1, and compute matrix Y given by (20);
- step 3: From the obtained values of step 2 compute the different matrices of the observers, N given by (16), T given by (15) and matrices J and M given by $J = K + NF$ and $M = Z + TF$.

III. NUMERICAL EXAMPLE

To illustrate our approach, let us consider a linear system with unknown input in the form of (1) where matrices

$$A = \begin{bmatrix} -1+\Delta & 0 & 0 \\ 0 & -10+\Delta & 0 \\ 0 & 0 & -1+\Delta \end{bmatrix},\ B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix},$$

$$G = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} and\ D = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

where $\Delta = 0.1\sin t$ is an uncertainty. The initial conditions are $x_1(0) = 10, x_2(0) = 5, x_3(0) = 1$.

By applying our approach to design a generalized observer for this system we obtain the following matrices: $X =$

$$\begin{bmatrix} 0.6440 & -0.0082 & -0.5018 & 0 & 0 & 0 \\ -0.0082 & 0.1100 & -0.0082 & 0 & 0 & 0 \\ -0.5018 & -0.0082 & 0.6440 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.1458 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.1458 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.1458 \end{bmatrix},$$

$$Y = \begin{bmatrix} 13.7141 & -4.7127 & -0.2005 & -0.2005 & -0.2005 \\ 3.5385 & 3.5326 & -0.3529 & -0.3529 & -0.3529 \\ -3.7411 & 12.7424 & -0.2005 & -0.2005 & -0.2005 \\ 0.8643 & 0.8638 & -17.5861 & -0.1309 & -0.1309 \\ 0.8643 & 0.8638 & -0.1309 & -17.5861 & -0.1309 \\ 0.8643 & 0.8638 & -0.1309 & -0.1309 & -17.5861 \end{bmatrix},$$

$$Z = \begin{bmatrix} 0.8643 & 0.8638 \\ 0.8643 & 0.8638 \\ 0.8643 & 0.8638 \end{bmatrix},\quad \mathbb{R} =$$

$$\begin{bmatrix} 0.0500 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0500 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0500 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0500 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0500 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0500 \end{bmatrix}$$ and

$$\mathbb{L} = \begin{bmatrix} 0.1500 & 0.1500 & 0.1500 & 0.1500 & 0.1500 \\ 0.1500 & 0.1500 & 0.1500 & 0.1500 & 0.1500 \\ 0.1500 & 0.1500 & 0.1500 & 0.1500 & 0.1500 \\ 0.1500 & 0.1500 & 0.1500 & 0.1500 & 0.1500 \\ 0.1500 & 0.1500 & 0.1500 & 0.1500 & 0.1500 \\ 0.1500 & 0.1500 & 0.1500 & 0.1500 & 0.1500 \end{bmatrix}.$$ By using the Yalmip toolbox of Matlab, we find the optimal $\gamma = 1$ and the corresponding generalized observer is then given by

$$\dot{z} = \begin{bmatrix} -13.7141 & 0 & 3.7127 \\ -3.5385 & -10.0000 & -3.5326 \\ 3.7411 & 0 & -13.7424 \end{bmatrix} z +$$

$$\begin{bmatrix} -0.0000 & 9.0013 \\ -0.0000 & 7.0711 \\ 0.0000 & 9.0013 \end{bmatrix} y + \begin{bmatrix} -0.2005 & -0.2005 & -0.2005 \\ -0.3529 & -0.3529 & -0.3529 \\ -0.2005 & -0.2005 & -0.2005 \end{bmatrix} v$$

$$\dot{v} = \begin{bmatrix} -0.8643 & 0 & -0.8638 \\ -0.8643 & 0 & -0.8638 \\ -0.8643 & 0 & -0.8638 \end{bmatrix} z + \begin{bmatrix} -0.0000 & 1.7281 \\ -0.0000 & 1.7281 \\ -0.0000 & 1.7281 \end{bmatrix} y +$$

$$\begin{bmatrix} -17.5861 & -0.1309 & -0.1309 \\ -0.1309 & -17.5861 & -0.1309 \\ -0.1309 & -0.1309 & -17.5861 \end{bmatrix} v$$

$$\hat{x} = z + \begin{bmatrix} 1 & -1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} y$$

In order to compare our results with those obtained by PO, we also design a H_∞ PO with the same $\gamma = 1$ and we obtain the following observer:

$$\dot{z} = \begin{bmatrix} -1.11 & 0 & 0.11 \\ 0 & -10 & 0 \\ 0.11 & 0 & -1.11 \end{bmatrix} z + (1.0e-04) * \begin{bmatrix} 0 & -0.3387 \\ 0 & 0 \\ 0 & -.3387 \end{bmatrix} y$$

$$\hat{x} = z + \begin{bmatrix} 1 & -1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} y$$

Simulation results are presented in figures Fig.1- Fig.7. The performances of GO and PO are compared. Fig.1, Fig.3 and Fig.5 present the original states x_1, x_2 and x_3 and their estimations by using our GO and by using the PO, respectively. Fig.2, Fig.4 and Fig.6 present the estimation errors e $(e = \hat{x}_1 - x_1)$ obtained by the GO and by the PO, respectively. Fig.7 presents the unknown input and disturbance. It can be seen from these results that the GO presents the better performances in presence of disturbance. These simulation results prove the effectiveness of our proposed design approach.

IV. CONCLUSION

In this paper we have presented a novel H_∞ dynamic observers design for systems with unknown inputs. This design has been obtained from the parameterization of the solutions of the LMI. The presented observer GO constitutes a generalization of the existing static and dynamic observers. Contrary to the existing results for the observers design in presence of unknown inputs, the GO can be used in presence of disturbance. A numerical example has been presented and shows the better performances of the GO compared to the Luenberger observer (called also PO) .

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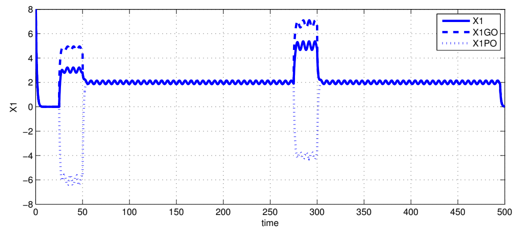

Fig. 1. original state x1 and its estimate

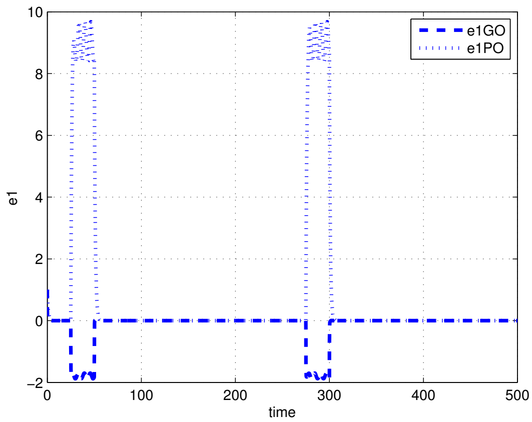

Fig. 2. estimation error e1 of GO and PO

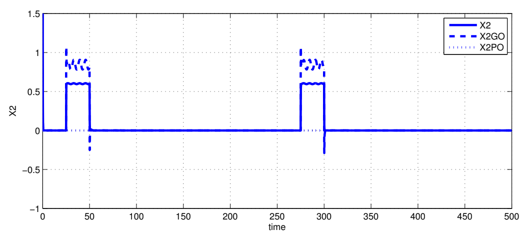

Fig. 3. original state x2 and its estimate

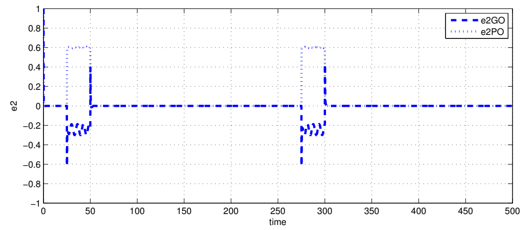

Fig. 4. estimation error e2 of GO and PO

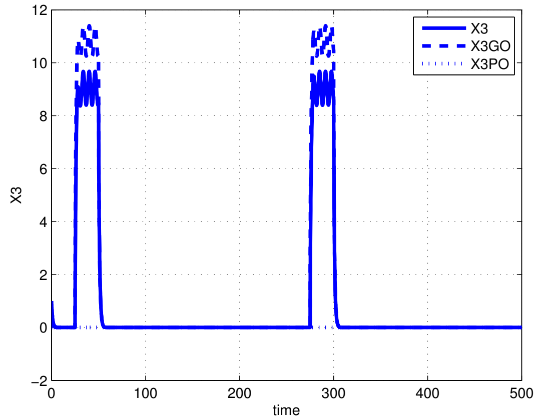

Fig. 5. original state x3 and its estimate

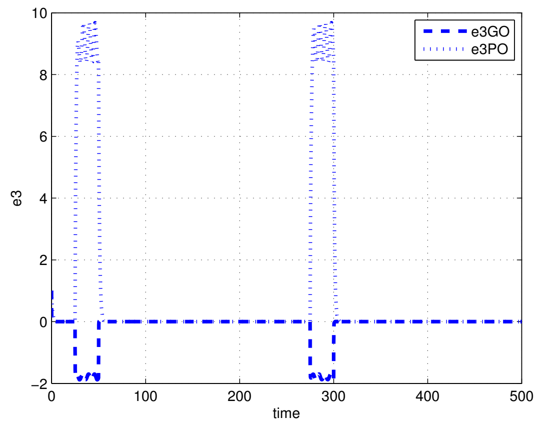

Fig. 6. estimation error e3 of GO and PO

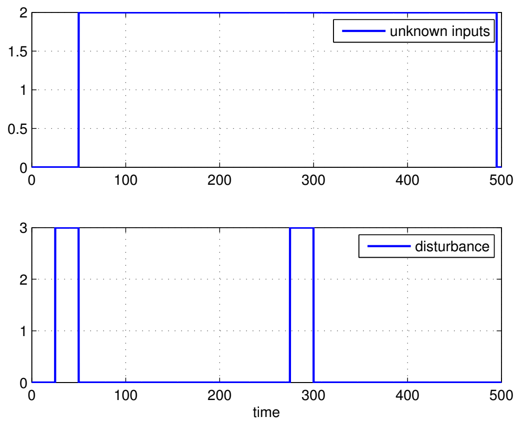

Fig. 7. unknown input and disturbance