Disaggregation of bipolar-valued outranking relations and application to the inference of model parameters

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The goal of this presentation it to show how performances of alternatives and weights related to criteria can be determined from three different *bipolar-valued outranking relations* which are given beforehand. We furthermore detail how these results can be used in real-world decision problems in a *disaggregation* phase which helps to determine the different parameters linked to the representation of the decision maker's preferences by an outranking relation.

Let $X = \{x, y, z, \ldots\}$ be a set of p alternatives and $N = \{1, \ldots, n\}$ be a set of n criteria. Each alternative of X is evaluated on each of the criteria of N. Let us write $g_i(x)$ for the performance of alternative x on criterion i of N. In this work, we will regard, without any loss of generality, such a *performance function* g_i $(i \in N)$ as having its values in $[0, 1]$ s.t.:

$$\forall x, y \in X,\ g_i(x) \geq g_i(y) \Rightarrow x \text{ is } \textit{at least as good as } y \text{ on criterion } i. \quad (1)$$

With each criterion i of N we associate its *weight* represented by a rational number w_i from the interval $[0, 1]$ such that

$$\sum_{i=1}^{n} w_i = 1.$$

To enrich the model which can be based on Formula (1), it is possible to associate different thresholds (weak preference, preference, weak veto, veto; see, e.g., [BMR07]) with the criteria functions which allow to represent more precisely a decision maker's (DM's) local "*at least as good as*" preferences.

Let S be a binary relation on X. Classically, the proposition "*x outranks y*" (xSy) $(x, y \in X)$ is assumed to be validated if there is a sufficient majority of criteria which supports an "*at least as good as*" preferential statement and there is no criterion which raises a *veto* against it [Roy85].

With this outranking relation we associate a bipolar valuation $\widetilde{S}$ which represents the *credibility of the validation or non-validation* of an outranking situation observed between each pair of alternatives [BMR07]. The maximum value

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1 of $\widetilde{S}$ is reached in the case of *unanimous concordance*, whereas the minimum value -1 is obtained in the case of *unanimous discordance*. $\widetilde{S}$ is called the *bipolar-valued characterisation* of the outranking relation S, or, for short, the *bipolar-valued outranking relation*.

In this work, given $\widetilde{S}$, we detail how the performances $g_i(x)$ ($\forall i \in N, \forall x \in X$) of the alternatives and the weights w_i ($\forall i \in N$) associated with the criteria can be determined. We present three different definitions of the outranking relation, where the first model takes only into account a preference threshold, the second one considers also a weak preference threshold, and finally, the third one adds also two veto thresholds.

From a practical point of view, the determination of the performances of the alternatives on the criteria may be questionnable, as in general, in a decision problem, these evaluations are given beforehand. Nevertheless, from an experimental point of view, the determination of a performance table from a given valued outranking relation can be of some help. Furthermore, we show that particular cases of our developments might be applicable in real world Multiple Criteria Decision Analysis for the disaggregation of outranking relations in order to tune the parameters underlying the decision maker's preferences.

References

[BMR07] R. Bisdorff, P. Meyer, and M. Roubens. Rubis: a bipolar-valued outranking method for the best choice decision problem. *4OR, Quaterly Journal of the Belgian, French and Italian Operations Research Societies*, 2007. in press, `doi:10.1007/s10288-007-0045-5`.

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