Counting non-isomorphic maximal independent sets of the n-cycle graph

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Abstract

It is known that the number of maximal independent sets of the n-cycle graph C_n is given by the nth term of the Perrin sequence. The action of the automorphism group of C_n on the family of these maximal independent sets partitions this family into disjoint orbits, which represent the non-isomorphic (i.e., defined up to a rotation and a reflection) maximal independent sets. We provide exact formulas for the total number of orbits and the number of orbits having a given number of isomorphic representatives. We also provide exact formulas for the total number of unlabelled (i.e., defined up to a rotation) maximal independent sets and the number of unlabelled maximal independent sets having a given number of isomorphic representatives. It turns out that these formulas involve both Perrin and Padovan sequences.

Keywords: Maximal independent set, cycle graph, combinatorial enumeration, dihedral group, group action, cyclic and palindromic composition of integers, Perrin and Padovan sequences.

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