Optimal mix of funded and unfunded pension systems:

The case of Luxembourg

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- a theoretical model based on a diversification principle

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Software:

SAS-based Proc Traj procedure
by Bobby L. Jones (Carnegie Mellon University).

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PROC TRAJ DATA=TEST OUTPLOT=OP OUTSTAT=OS OUT=OF
OUTEST=OE ITDETAIL;
      ID ID; VAR O1-O20; INDEP T1-T20;
      MODEL CNORM; MAX 8000; NGROUPS 6; ORDER 4 4 4 4 4 4;
RUN;
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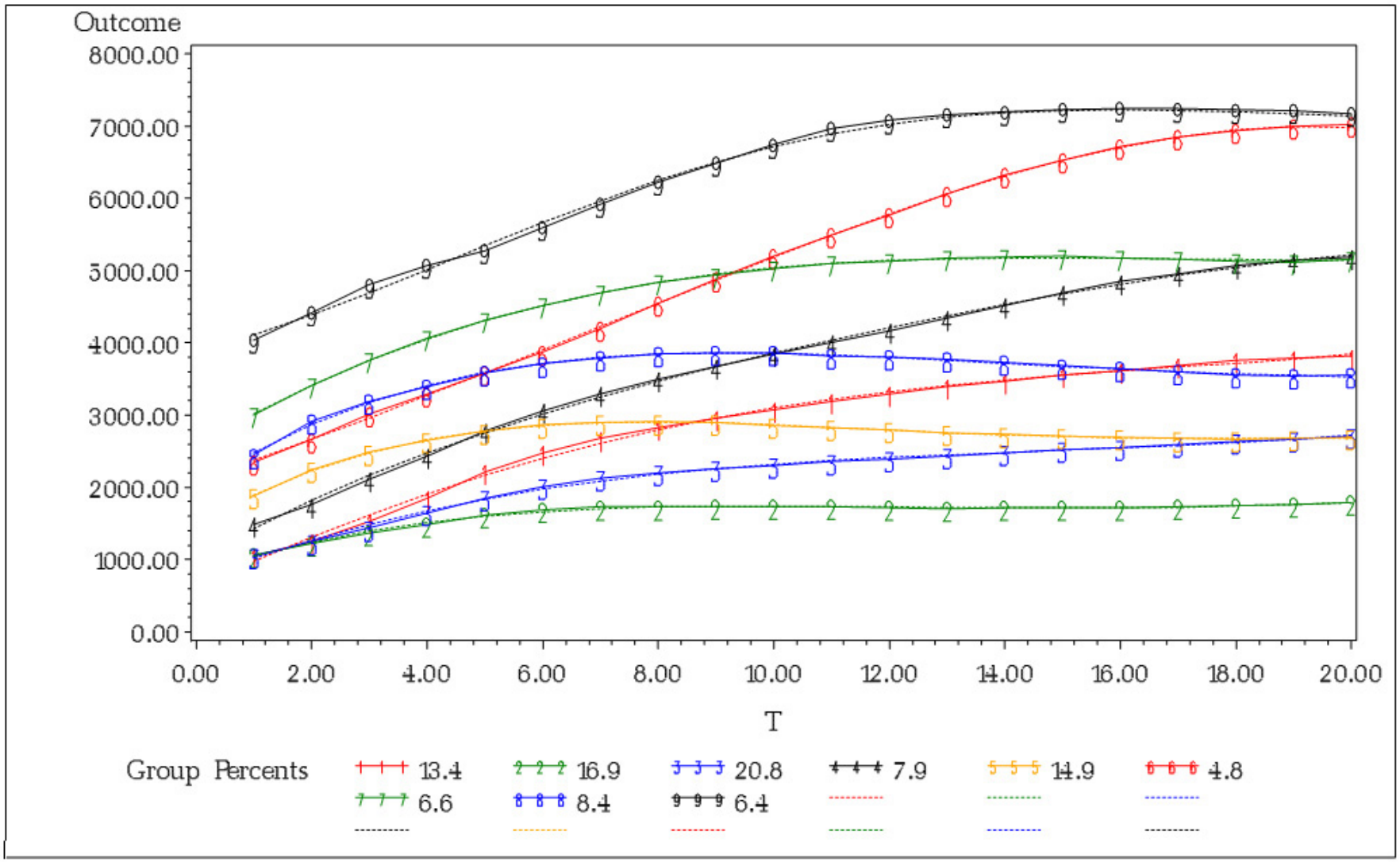

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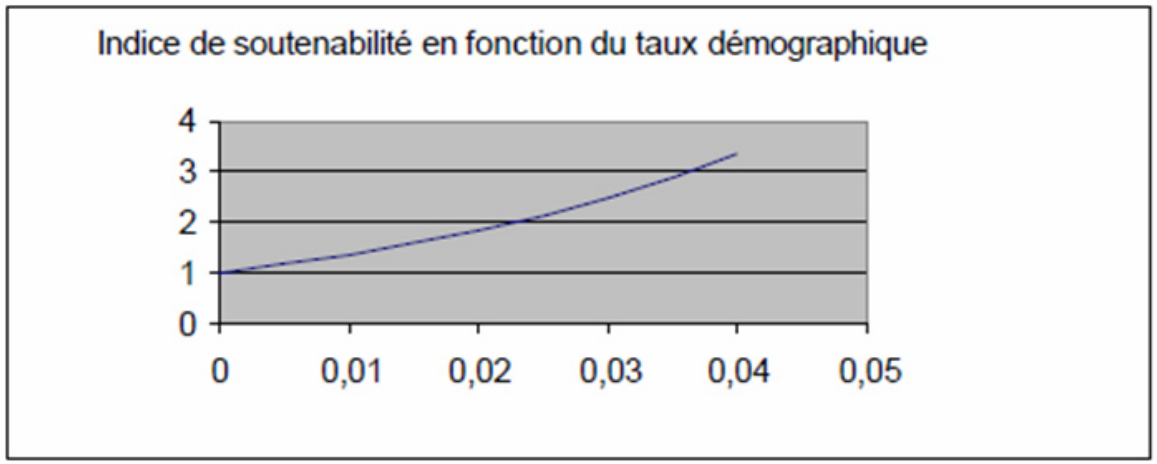

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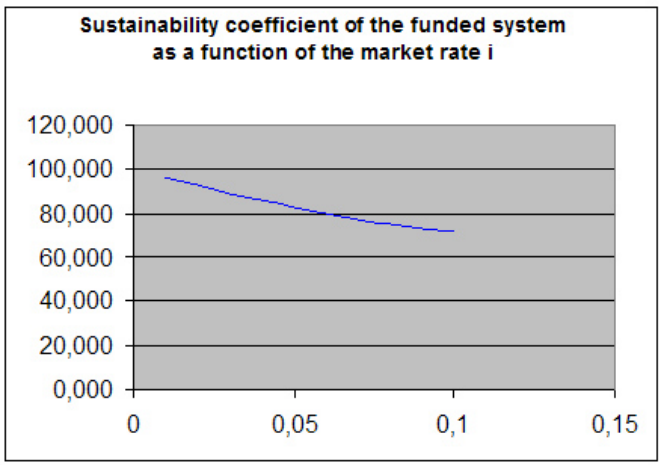

Systemic risk

Modelisation based on portfolio type risk management principles

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	Market risk	Demographic risk
Repartition	Negligeable	Extreme
Capitalization	Extreme	Negligeable

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is the number of euros necessary to pay 1 euro for the pension.

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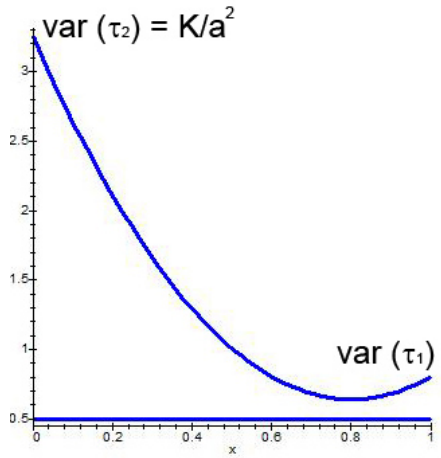

Gain of sustainabiliy and optimal saving amount

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Moreover the individual needs a constant annual saving amount

$$a^* = \sqrt{\frac{G^* K}{var(\tau 1)(1 - G^*)}},$$

where $K = Var[\frac{S_j}{a_j(i-\lambda_j)} i \frac{(1+i)^T - (1+\lambda_j)^T}{(1+i)^T - 1}]$ depends on the salary trajectory.

Example

An individual worker wants to divide by 2 the variabbility of his PAYG sustainability constraint needs to save annualy at least the following amount (depending on his salary evolution subgroup):

Groupe	G1	G2	G3	G4	G5	G6	G7	G8	G9
Annuité	4466 €	713 €	1448 €	5231 €	220 €	6364 €	2809 €	743 €	3140 €