Analysis of the salary trajectories in Luxembourg

A finite mixture model approach

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joint work with

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Outline

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  - and/or Develop complementary systems (mix of funded and unfunded system)

Our research project

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- description of the salary trajectories in Luxembourg
- analysis of the sustainability of the current system
- a theoretical model based on a diversification principle

The data

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- age in the first year of professional activity

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Nagin's semiparametric finite mixed model (Carnegie Mellon University) :

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- observation of a set of individual trajectories
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- estimation of a mean trajectory for every subgroup

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$$L = \frac{1}{\sigma} \prod_{i=1}^{N} \sum_{j=1}^{r} \pi_j \prod_{t=1}^{T} \phi \left(\frac{y_{i_t} - \beta^j x_{i_t}}{\sigma} \right). \tag{1}$$

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Software:

SAS-based Proc Traj procedure
by Bobby L. Jones (Carnegie Mellon University).

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DATA TEST;
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PROC TRAJ DATA=TEST OUTPLOT=OP OUTSTAT=OS OUT=OF
OUTEST=OE ITDETAIL;
      ID ID; VAR O1-O20; INDEP T1-T20;
      MODEL CNORM; MAX 8000; NGROUPS 6; ORDER 4 4 4 4 4 4;
RUN;
```

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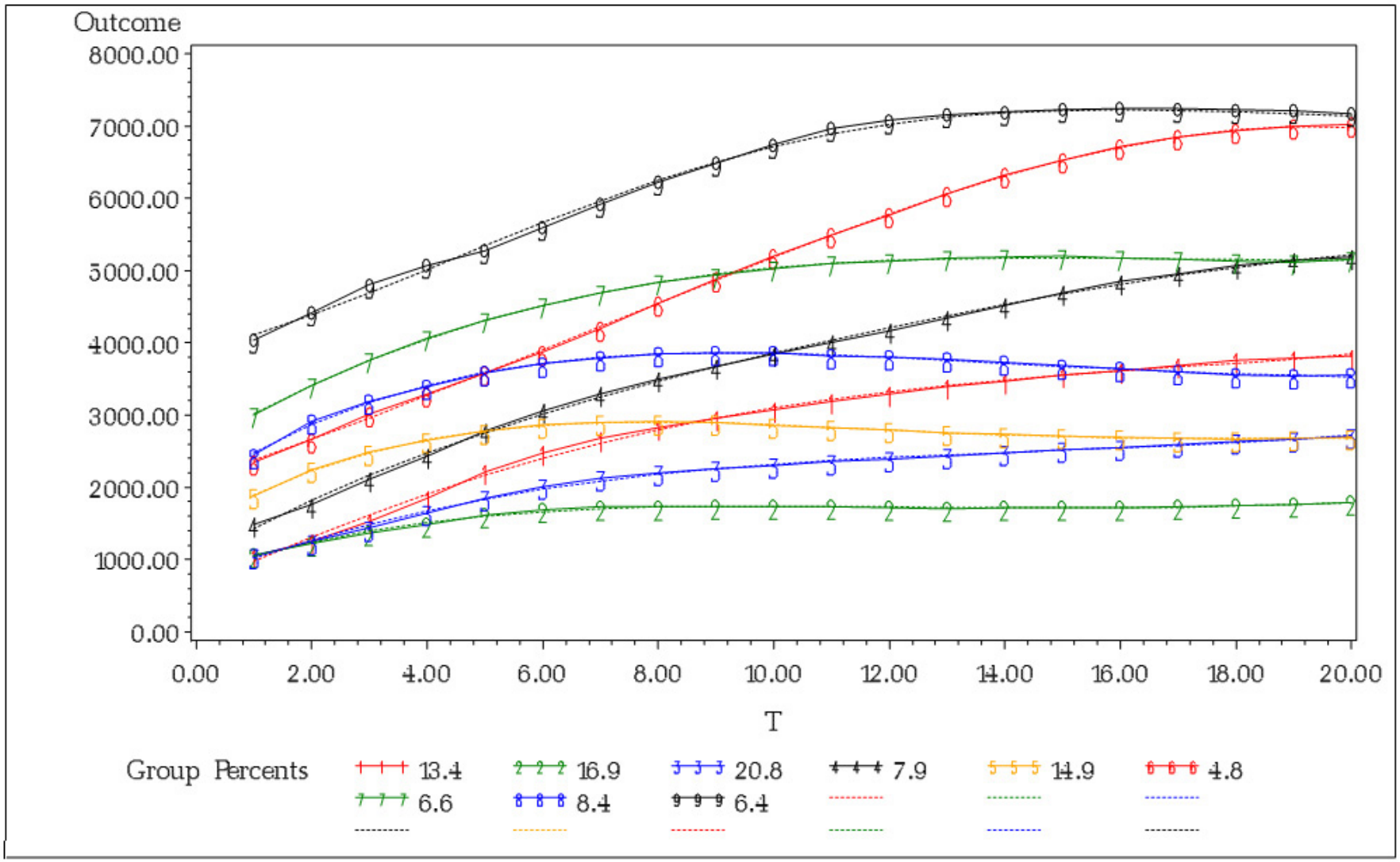

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Classification with only the use of the socioeconomic variables and a few years of salary is an interesting statistical problem.

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- Hypothesis 2. People work for 40 years and have a life expectancy of 20 years afterwards.
- Hypothesis 3. Let d denotes the intergenerationnal demographical rate, i.e. at time t, if N_0 denotes the number of people beginning to work and N_t the number of people working for t years, then

$$N_t = \frac{N_0}{(1+d)^t}.$$

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τ_1 = sum of all salaries earned by active workers / sum of all pensions paid to retirees at time t

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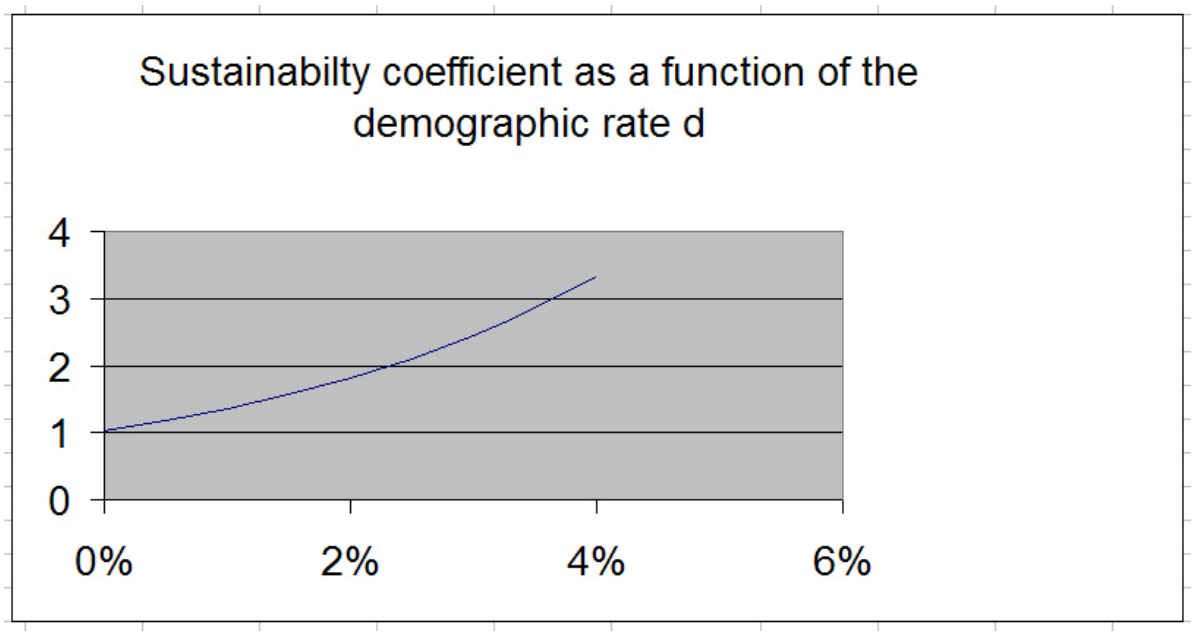

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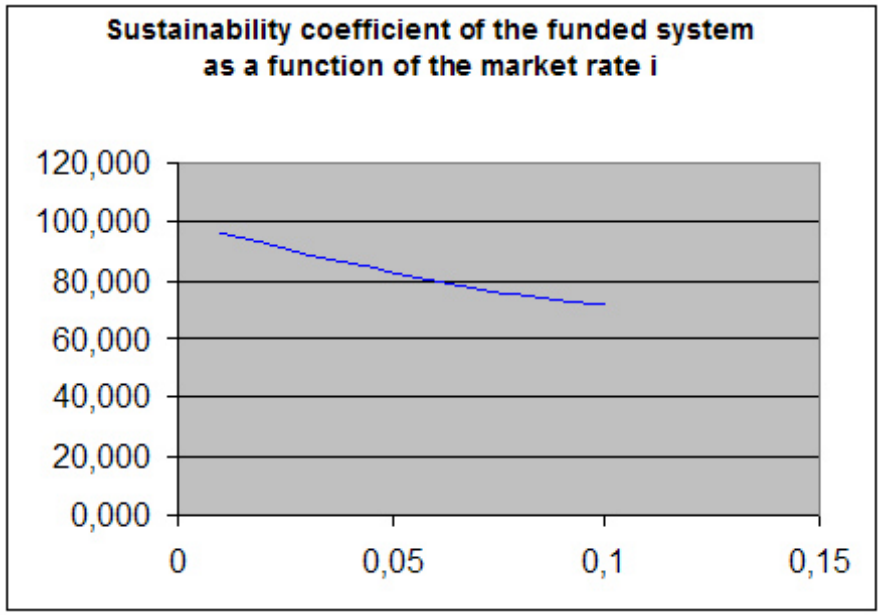

Systemic risk

Modelisation based on portfolio type risk management principles

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	Market risk	Demographic risk
Repartition	Negligeable	Extreme
Capitalization	Extreme	Negligeable

Global sustainabilty coefficient

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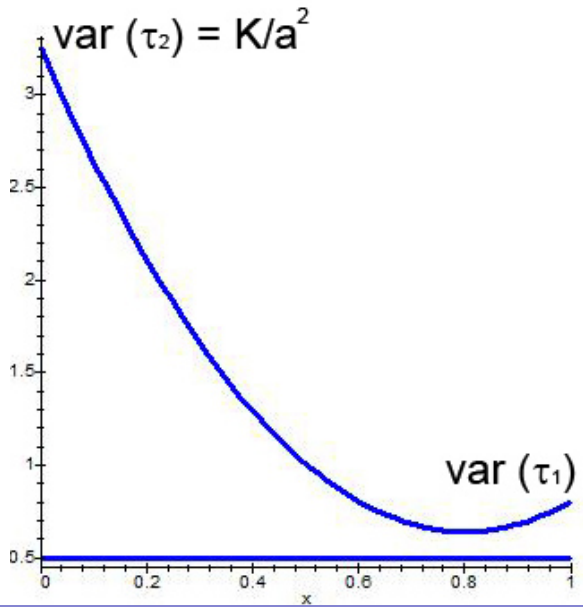

Gain of sustainabiliy and optimal saving amount

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We suppose that the utility function $U = U(a)$ of an active worker is decreasing in a.

Gain of sustainabiliy and optimal saving amount

Theorem: The value $x = x^*$ for which the utility function U atteins is maximum under the sustainabilty constraint

$$G(x) \geq G^*$$

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Moreover the individual needs a constant annual saving amount

$$a^* = \sqrt{\frac{G^* K}{var(\tau_1)(1 - G^*)}},$$

where $K = Var[\frac{S_j}{a_j(i-\lambda_j)} i \frac{(1+i)^T - (1+\lambda_j)^T}{(1+i)^T - 1}]$ depends on the salary trajectory.

Example

An individual worker wants to divide by 2 the variability of his PAYG sustainability constraint needs to save annually at least the following amount (depending on his salary evolution subgroup):

Group	G1	G2	G3	G4	G5	G6	G7	G8	G9
Annuity	4466	713	1448	5231	220	6364	2809	743	3140