Determination of weights of interactive criteria from a reference set

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Abstract

We propose a model allowing to identify the weights (2-order fuzzy measure) of interactive criteria from a partial preorder over a reference set of alternatives, a partial preorder over the set of values related to each criterion, a partial preorder over interactions between pairs of criteria, and the knowlegde of the sign of some interactions between pairs of criteria. Formally, the input data of the problem can be summarized as follows:

- The set A of alternatives and the set N of criteria,
- A table of scores (utilities) $\{x_i(a)\,|\,i \in N, a \in A\}$,
- A partial preorder $\succeq_A$ on A (ranking of alternatives),
- A partial preorder $\succeq_N$ on N (ranking of criteria),
- A partial preorder $\succeq_P$ on the set of pairs of criteria (ranking of interaction indices),
- The sign of some interactions $a(i,j)$: positive, nul, negative (translating synergy, independence or redundancy).

All these data can be formulated with the help of linear equalities or inequalities. Strict inequalities can be converted into vague inequalities by introducing a positive slack variable ε.

Thus, the problem of finding a 2-order fuzzy measure can be formalized with the help of a linear program:

$$\max z = \varepsilon$$

subject to

$$\left.\begin{array}{ll} C(a) - C(b) \geq \delta + \varepsilon & \text{if } a \succ_A b \\ -\delta \leq C(a) - C(b) \leq \delta & \text{if } a \sim_A b \end{array}\right\} \text{partial semiorder with threshold } \delta$$

$$\left.\begin{array}{ll} I(i) - I(j) \geq \varepsilon & \text{if } i \succ_N j \\ I(i) = I(j) & \text{if } i \sim_N j \end{array}\right\} \text{ranking of criteria}$$

$$\left.\begin{array}{ll} a(i,j) - a(k,l) \geq \varepsilon & \text{if } \{i,j\} \succ_P \{k,l\} \\ a(i,j) = a(k,l) & \text{if } \{i,j\} \sim_P \{k,l\} \end{array}\right\} \text{ranking of pairs of criteria}$$

$$\left.\begin{array}{ll} a(i,j) \geq \varepsilon \text{ (resp. } \leq -\varepsilon) & \text{if } a(i,j) > 0 \text{ (resp. } < 0) \\ a(i,j) = 0 & \text{if } a(i,j) = 0 \end{array}\right\} \text{sign of interactions}$$

$$\left.\begin{array}{ll} \sum_i I(i) = 1 & \\ a(i) \geq 0 & \forall i \in N \\ a(i) + \sum_{j \in T} a(i,j) \geq 0 & \forall i \in N,\ \forall T \subseteq N \setminus i \end{array}\right\} \text{boundary and monotonicity conditions}$$

$$\left.\begin{array}{ll} I(i) = a(i) + \frac{1}{2} \sum_{j \in N \setminus i} a(i,j) & \forall i \in N \\ C(a) = \sum_{i \in N} a(i)\, x_i(a) + \sum_{\{i,j\} \subseteq N} a(i,j)\, [x_i(a) \wedge x_j(a)] & \forall a \in A \end{array}\right\} \text{definitions}$$

It seems natural to assume that the ranking over A is translated into a partial semiorder over the set of the global evaluations given by the Choquet integral. This partial semiorder has a fixed threshold δ, which can be tuned as wished.

Keywords: multicriteria decision making, interactive criteria, Choquet integral.

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