

# Characterization of some aggregation functions stable for positive linear transformations

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## Abstract

Synthesizing judgments is an important part of multiple criteria decision making methods. The most typical situation concerns individuals who form quantifiable judgments about a measure of an object (weight, length, area, height, volume, importance or other attributes, for instance in the framework of a hierarchy)(see [3,4]) or quantifiable judgments on pairs of alternatives along each criterion. In the latter case, the judgments are very often expressed with the help of fuzzy preference relations (see [8,9]).

In order to reach a consensus (overall opinion) on these judgments, classical aggregation functions have been proposed: arithmetic means, geometric means, root-power means and many others. Of course, given such an aggregation function, we can ask for a motivation of its use, i.e. for natural, reasonable assumptions which lead to this function. Conversely, we can specify some assumptions (called axioms or properties) and determine all the aggregation functions satisfying these. This is the topic with which we deal here.

This paper aims at describing the family of all aggregation functions fulfilling three specific properties. The first two are increasing monotonicity and stability for the same transformations of interval scales in the sense of the theory of measurement (see [15]), i.e. stability for positive linear transformations (we refer to the corresponding functional equation in [5,6] where the arithmetic mean is characterized). The third property is chosen among well-known algebraic properties such as associativity, decomposability and bisymmetry.

We make a distinction between aggregation functions having a fixed number of arguments (aggregation  $m$ -functions) and aggregation functions defined for all number of arguments (aggregation operators or aggregators).

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