Abstract:

Let $R$ be an infinite commutative integral domain with identity and let $n \geq 2$ be an integer. A function $f: R^n \to R$ is said to be associative if it solves the following system of $n - 1$ functional equations:

$$f(x_1, \ldots, f(x_i, \ldots, x_{i+n-1}), \ldots, x_{2n-1}) = f(x_1, \ldots, f(x_{i+1}, \ldots, x_{i+n}), \ldots, x_{2n-1}), \quad i \in \{1, \ldots, n - 1\}.$$ 

In this case, the pair $(R, f)$ is called an $n$-ary semigroup.

We provide a complete classification of all the $n$-ary semigroup structures defined by polynomial functions over $R$ (i.e., the $n$-ary semigroup structures polynomial-derived from $R$), thus generalizing Glazek and Gleichgewicht’s classification of the corresponding ternary semigroups.