The concept of system signature has been introduced in 1985 by F. Sammaniego in the reliability analysis of "coherent systems" and a few related papers appeared in the applied-probability literature in the subsequent years. Much more attention, in the last few years, has been paid to this concept, which is more and more revealing as a quite natural and powerful tool in the reliability analysis. Given a "coherent" system with $n$ components, its "signature" is nothing else than a probability distribution over $\{1, 2, ..., n\}$, having a very immediate meaning and role in the case when the components' lifetimes are independent, identically distributed (or exchangeable) random variables. Recently, a few contributions have been devoted to analyzing the role of signature in the analysis of symmetry properties of a system and to extending signature-related analysis to non-exchangeable cases. Despite its relevance for applications, basic computational problems related with signature are still open. Interesting results in this direction are expected to come from a specific combinatorial and algebraic analysis.

The present Mini-symposium aims to discuss recent work in these directions along with related interactions and developments.

**Programme**

10,00 - 11.00 Fabio Spizzichino (Università La Sapienza)
*The concept of system signature and related extensions of exchangeability*

BREAK

11,30 - 12,30 Jean-Luc Marichal (Université de Luxenbourg)
*Extensions of system signature and Barlow-Proshan importance index to dependent lifetimes*

BREAK

14,30 - 15,30 Pierre Mathonet (Université de Liège)
*Signatures, decompositions of reliability and approximation problems*

BREAK

16,00 - 17.00 Alessandro D'Andrea (Università La Sapienza)
*A combinatorial characterization of system signatures*

17,00 Discussion and Conclusions

Mini-symposium organized in the frame of 2009 La Sapienza University Project: Modelli e Algoritmi Stocastici
ABSTRACTS OF TALKS

Fabio Spizzichino (Università La Sapienza)
*The concept of system signature and related extensions of exchangeability*

*Abstract*: This talk will mainly have an introductory character and will be divided into two parts. In the first part, I will present the concept of signature, its meaning, main applications, and its special properties in the case of systems with exchangeable components. In the second part, some extensions of the concept of exchangeability, suggested by a signature-based analysis, will be discussed.

Jean-Luc Marichal (Université de Luxenbourg)
*Extensions of system signature and Barlow-Proschan importance index to dependent lifetimes*

*Abstract*: The concept of system signature was introduced by Samaniego for systems whose components have i.i.d. lifetimes. A measure of importance for components was introduced by Barlow and Proschan for systems whose components have independent lifetimes. We consider the extensions of these two concepts to the continuous dependent case and give explicit expressions for these extensions in terms of both the structure function and the distribution function of the component lifetimes. We then investigate the special case of independent continuous lifetimes and examine the subcase of independent Weibull lifetimes.

Joint work with Pierre Mathonet

Pierre Mathonet (Université de Liege)
*Signatures, decompositions of reliability and approximation problems*

*Abstract*: We consider coherent systems made of $n$ interconnected components. Assuming that these components have i.i.d. lifetimes, F. Samaniego introduced the concept of signature as the $n$-tuple whose $k$th coordinate is the probability that the $k$th component failure causes the system to fail. It turns out that in this particular situation the signature $s$ depends only on the structure of the system and not on the distribution of the lifetimes of the components. The natural extension of this definition of signature in the non-i.i.d. situation gives rise to a signature vector $p$ that depends in general both on the structure of the system and on the joint distribution of the lifetimes of the components.

In the first part of this presentation, we will focus on a signature-based decomposition of the system reliability in terms of reliabilities of $k$-out-of-$n$ systems, also introduced in the i.i.d. case by F. Samaniego. In the non-i.i.d. case, we will show that, at any time, this representation still holds true for every coherent system if and only if the component states are exchangeable. We will also discuss conditions for obtaining an alternative representation of the system reliability in which the signature $s$ is replaced by its non-i.i.d. extension $p$. Finally, we will discuss conditions for the system reliability to admit both the representations. In the second part of the presentation, we will explain how the signature vectors can be obtained by approximating the Boolean structure function of the system by a symmetric pseudo-Boolean function in the sense of (weighted) least-squares.

Joint work with Jean-Luc Marichal and Tamas Waldhauser.

Alessandro D'Andrea (Università La Sapienza)
*A combinatorial characterization of system signatures*

*Abstract*: Several computational problems related with the concept of system signature are still open, despite its relevant interest in the applications. In this talk I will address the problem of explicit characterizations of signatures. Let $X$ be a set with $n$ elements. With every non decreasing function $f:2^X \rightarrow \{0,1\}$ one may associate the corresponding signature $s(f) = (s_1,...,s_n)$. Thanks to a simple combinatorial result, I will show how to determine whether an $n$-tuple is of the form $s(f)$ for some (explicit) $f$. 