Abstract—We propose a novel formulation for joint maximization of total weighted sum-spectral efficiency and weighted sum-harvested energy to study Simultaneous Wireless Information and Power Transfer (SWIPT) from a pricing perspective. Specifically, we consider that a transmit source communicates with multiple destinations using Orthogonal Frequency Division Multiplexing (OFDM) system within a dual-hop relay-assisted network, where the destination nodes are capable of jointly decoding information and harvesting energy from the same radio-frequency (RF) signal using either the time-switching (TS) or power-splitting (PS) based SWIPT receiver architectures. Computation of the optimal solution for the aforementioned problem is an extremely challenging task as joint optimization of several network resources introduce intractability at high numeric values of relays, destination nodes and OFDM sub-carriers. Therefore, we present a suitable algorithm with sub-optimal results and good performance to compute the performance of joint data processing and harvesting energy under fixed pricing methods by adjusting the respective weight factors, motivated by practical statistics. Furthermore, by exploiting the binary options of the weights, we show that the proposed formulation can be regulated purely as a sum-spectral efficiency maximization or solely as a sum-harvested energy maximization problem. Numerical results illustrate the benefits of the proposed design under several operating conditions and parameter values.

I. INTRODUCTION

Recent advancements and trends in the wireless communications sector indicate towards rapid growth in the number of involved devices, which is expected to cross 50 billion by the year 2020 [1]. On one hand, this has raised concerns over the ways and means to meet the ever-growing performance and capacity requirements in future. While on the other hand, increasing complex circuitry of the devices and hardware designs have imposed high energy demands, which does not only require energy-efficiency algorithms, but also limits them to provide nuclear or poor performance in-turn. With an interesting idea of utilizing the same Radio-Frequency (RF) signals as for the information exchange, Energy Harvesting (EH) techniques have evolved as potential compensators for this while addressing the future energy requirements [2].

Simultaneous Wireless Information and Power Transfer (SWIPT) is a recent paradigm introduced to ensure that both data processing and EH capabilities coexist within a device [3], [4]. In this regard, the time-switching (TS) and power-splitting (PS) based SWIPT receiver architectures have been adopted widely in the literature [2], [3]. In this paper, we investigate SWIPT from a pricing perspective using the TS- and PS-based receiver models for joint optimization of overall throughput and harvested energy.

The throughput at the end-users can be significantly improved by incorporating cooperative relaying [5] and orthogonal frequency division multiplexing (OFDM) [6], [7]. When combined with SWIPT and cooperative systems, OFDM does not only retain its existing advantages, but it also helps to provide extended coverage via relays to facilitate SWIPT. In particular, the power transfer distance is largely limited by the power sensitivity of the energy harvester, which considering the current state-of-the-art technology is of -10 dBm, significantly tighter than the -60 dBm assumed for effective information receivers [8].

In this paper, we focus on the Amplify-and-Forward (AF) relay protocol, where relays simply forward the received signal after amplification. In addition, we assume a multi-user OFDM system for facilitating SWIPT to multiple destination nodes. There have been several works in the literature focusing on (i) resource allocation for OFDM in both conventional and relay-aided communication systems (without SWIPT) [9], [10], (ii) SWIPT with OFDM [11], and (iii) relay selection for SWIPT in a single user scenario (without OFDM) [12], [13]. Some of these works, however, either do not consider the energy harvesting constraints [9], [10] or focus only on increasing the spectral efficiency (or data rate) under energy harvesting constraints and vice-versa, separately. Herein, we present a novel problem formulation to study joint optimization of total sum-spectral efficiency and sum-harvested energy under constraints on both minimum spectral efficiency and minimum harvested energy demand at each destination node.

The main contributions of this paper are listed as follows.
1) Firstly, we present a novel relay selection and resource allocation technique to optimize the network resources in a dual-hop scenario where multiple half-duplex AF relays assist a transmitter to transfer both information and energy to multiple destinations which are equipped with a either a TS or a PS-based SWIPT receiver architecture.
2) Secondly, we formulate an optimization problem for relay selection, carrier assignment in the two hops, sub-carrier pairing, sub-carrier power allocation, and the SWIPT splitting factor at each destination node in order to jointly maximize the total of weighted sum-spectral efficiency and weighted sum-harvested energy of the end-users subject to minimum spectral efficiency and minimum harvested energy.
energy demand constraint at each destination and power constraints at the source and relays.

3) Thirdly, we propose a novel Maximum Performance-based Resource Allocation (MPRA) algorithm with polynomial computational time-complexity and good performance, yielding sub-optimal results to address the aforementioned optimization problem. The corresponding weights ensure dimensional tractability for joint study of sum-rate and sum-harvested energy.

4) Finally, we show using the binary nature of the weights that the same formulation [as mentioned above, in 2)] can be projected into a complete sum-spectral efficiency maximization problem or only as a sum-harvested maximization problem under the aforementioned constraints. We demonstrate the effectiveness of the proposed method where significant gains are observed in comparison with a Semi-Fixed Resource Allocation (SFRD) approach [12], [14], [15] where the impact of varying the key system parameters is observed via numerical results.

Further sections of this paper are organized as follows. Section II provides an introduction to the system model. The problem formulation and the proposed solution are presented in Section III and Section IV, respectively. Brief analysis on the execution time-complexities of proposed methods is provided in Section V. Numerical results are shown in Section VI, followed by concluding remarks in Section VII.

II. SYSTEM MODEL

We consider a scenario where single transmit source $S$ broadcasts information and energy to $L$ destination nodes ($D_1, \ldots, D_L$) with the help of $K$ relay nodes ($R_1, \ldots, R_K$), such that each relay operates on the AF protocol and $K \geq L$ in general. Each device within the network is equipped with a single antenna. The overall SWIPT process takes place in two hops where in the first hop, the source transmits the relevant information intended for all the end-users to the relays over $N$ OFDM sub-carriers while in the second hop, the chosen relays forwards an amplified version of the desired signal to the destination nodes over another set of $N$ OFDM sub-carriers. From a practical view-point, the devices operating as the receiving end-nodes in such a set-up usually receive negligible signal intensity from the direct source transmission during the first hop due to severe attenuation and long distance, and thus we discard the contribution from the direct source-destination link in further analysis. It is also important to note that unlike its standard operation, each AF-relay performs sub-carrier switching to demultiplex, frequency convert, and multiplex again while taking channelization into account.

Denote $n \in \mathbb{Z}$ and $m \in \mathbb{Z}$ as the first hop and second hop sub-carrier indices such that $1 \leq n \leq N$ and $1 \leq m \leq N$, respectively. For smooth communication operation in the dual-hop system, a unique sub-carrier pairing $(n,m)$ is formed at each relay such that the indices $n$ and $m$ may be equal or not. Each relay is coupled with one destination only, which is exclusive to itself, thus forming a relay-destination coupling. This consideration is taken to reduce the control process overhead and to ease the synchronization process among the relay nodes. In addition, each relay is confined to have singular sub-carrier pairs such that each sub-carrier is tethered with one and only one relay. Thus, each relay has an exclusive set of sub-carrier pair(s) to serve its desired end-users. An example to realize the system set-up is depicted in Fig. 1 with $K = 3$, $L = 2$, and $N = 6$.

Each destination node is capable of performing data processing as well as energy harvesting simultaneously using a TS or PS architecture [12]. In this regard, we define a time-switching ratio at the $\ell$-th destination node as $\tau_\ell$, where $0 \leq \tau_\ell \leq 1$ and $\ell = 1, 2, \ldots, L$, such that the energy is harvested from the received signal for the first $\tau_\ell T$ seconds, where a given block transmission is of duration $T$ seconds, while information decoding takes place for the remaining duration of $(1-\tau_\ell)T$ seconds. In case of PS, a power splitter is employed so that a fraction $\phi_\ell$, where $0 \leq \phi_\ell \leq 1$, of the received signal power is used for energy harvesting, while the remaining is sent to the information decoder.

We denote the channel gain coefficient between $S$ and $R_k$ on the $n$-th OFDM sub-carrier in the first hop as $\psi_{1,n,k}$ and the channel gain coefficient between $R_k$ and $D_\ell$ on the $m$-th sub-carrier in the second hop as $\psi_{2,m,k,\ell}$. Define $\rho_{1,n}$ as the transmit power at $S$ on the $n$-th sub-carrier in the first hop. In order to re-transmit the signal, the following amplification coefficient is used at the $k$-th relay

$$\delta_{(n,m),k} = \sqrt{\frac{\rho_{2,m,k}}{\rho_{1,n} |\psi_{1,n,k}|^2 + \sigma_k^2}},$$

which makes sure that on the $m$-th sub-carrier, $R_k$ transmits with a power $\rho_{2,m,k}$, where the noise power at $R_k$ is denoted by $\sigma_k^2$. The total available power limits at $S$ and $R_k$ are denoted by $P_S$ and $P_{R_k}$, respectively.

The effective signal-to-noise ratio (SNR) observed in the $S \to R_k \to D_\ell$ link, over the $(n,m)$ sub-carrier pair at the decoding branch of $D_\ell$ for TS or PS scheme is respectively given by

$$\Upsilon_{(n,m),k,\ell}^{TS} = \frac{\rho_{1,n}|\psi_{1,n,k}\delta_{(n,m),k}\psi_{2,m,k,\ell}|^2}{\sigma_k^2 |\psi_{2,m,k,\ell}\delta_{(n,m),k}|^2 + \sigma_k^2},$$

$$\Upsilon_{(n,m),k,\ell}^{PS} = \frac{(1-\phi_\ell)\rho_{1,n}|\psi_{1,n,k}\delta_{(n,m),k}\psi_{2,m,k,\ell}|^2}{(1-\phi_\ell)\sigma_k^2 |\psi_{2,m,k,\ell}\delta_{(n,m),k}|^2 + \sigma_k^2},$$

where the down-conversion procedure at $D_\ell$ introduces a noise power of $\sigma_{\eta_\ell}^2$. These expression can respectively be re-written as follows
\[
Y_{1,n,k}^{TS} = \frac{Y_{1,n,k} Y_{2,m,k}}{1 + Y_{1,n,k} + Y_{2,m,k}},
\]
\[
Y_{2,m,k}^{PS} = \frac{(1 - \phi_k) Y_{1,n,k} Y_{2,m,k}}{1 + Y_{1,n,k} (1 - \phi_k) Y_{2,m,k}},
\]

where \( Y_{1,n,k} = \frac{\rho_{1,n} |\psi_{1,n,k}|^2}{\sigma^2_k} \), and \( Y_{2,m,k} = \frac{\rho_{2,m} |\psi_{2,m,k}|^2}{\sigma^2_k} \).

The factor 1/2 is introduced to compensate for the two time slots of the considered relay assisted communication.

Regarding the harvested energy, the energy yield over the \( S \rightarrow R_k \rightarrow D_\ell \) link for \((n,m)\) sub-carrier pair for TS or PS scheme is respectively given by,

\[
E_{1,n,k}^{TS} = \zeta \tau L \left[ \delta_{1,n,k} |\psi_{1,n,k}|^2 \left( \rho_{1,n} |\psi_{1,n,k}|^2 + \sigma^2_k \right) \right],
\]
\[
E_{1,n,k}^{PS} = \zeta \phi L \left[ \delta_{1,n,k} |\psi_{1,n,k}|^2 \left( \rho_{1,n} |\psi_{1,n,k}|^2 + \sigma^2_k \right) \right],
\]

where \( \zeta \) is the energy conversion efficiency of the receiver.

Further, these expression can be simplified using (1), as follows

\[
E_{1,n,k}^{TS} = \zeta \tau L \rho_{2,m,k} |\psi_{2,m,k}|^2,
\]
\[
E_{1,n,k}^{PS} = \zeta \phi L \rho_{2,m,k} |\psi_{2,m,k}|^2.
\]

Finally, we use the following triplet to simplify the notation

\[
(R_{1,n'}^{TS}, n', k'; E_{1,n'}^{TS}, n', k'; \tau_{1}) \Rightarrow \left( R_{1,n'}^{TS}, n', k'; E_{1,n'}^{TS}, n', k'; \tau_{1} \right)
\]

corresponding to the TS and PS schemes, respectively, with \( \tau_{1} = \{ \tau_L \text{ for TS; } \phi \text{ for PS} \} \).

### III. Problem Formulation

In this section, we formulate an optimization problem to maximize the total of weighted sum-spectral efficiency and weighted sum-harvested energy of the end-users subject to minimum spectral efficiency and minimum harvested energy demand constraint at each destination and power constraints at the source and relays. In this regard, we intend to optimize the relay selection and resource allocation along with computation of power in each sub-carrier for both the hops and the SWIPT splitting factor at each destination node.

To proceed, we define a binary variable \( \alpha_{k,\ell} = \{0, 1\} \) to form a relay–user coupling, where \( \alpha_{k,\ell} = 1 \) indicates the selection of \( R_k \) for \( D_\ell \) while \( \alpha_{k,\ell} = 0 \) implies \( k \)-th relay is not allocated to \( \ell \)-th user. It is explicit that one relay is coupled with single user only, and therefore we have,

\[
\sum_{k=1}^{K} \alpha_{k,\ell} = 1, \forall \ell; \quad \sum_{\ell=1}^{L} \alpha_{k,\ell} \leq 1, \forall k.
\]

We denote \( \beta_{n,\ell} \in \{0, 1\} \) as the binary variable to link the first hop sub-carrier \( n \) to \( D_\ell \), such that \( \beta_{n,\ell} = 1 \) indicates that \( n \) is used in the first hop to carry the relevant data for \( D_\ell \), and \( \beta_{n,\ell} = 0 \) otherwise. In this context, the following assignment rule must be satisfied,

\[
\sum_{\ell=1}^{L} \beta_{n,\ell} = 1, \forall n.
\]

Correspondingly, in order to pair the sub-carriers in the two hops, let us define \( \gamma_{(n,m)} \in \{0, 1\} \) as the respective indicator for sub-carrier pairing. Hence, \( \gamma_{(n,m)} = 1 \) implies that the sub-carrier \( n \) in the first hop is paired with sub-carrier \( m \) of the second hop and vice-versa when \( \gamma_{(n,m)} = 0 \). In this regard, the following must hold,

\[
\sum_{n=1}^{N} \gamma_{(n,m)} = 1, \forall m; \quad \sum_{m=1}^{N} \gamma_{(m,m)} = 1, \forall n.
\]

It is important to note that (C2) and (C3) automatically fixes the sub-carrier \( m \) in the second hop for \( D_\ell \).

The overall sum-spectral efficiency obtained at the \( \ell \)-th destination with a constraint, is given by

\[
\hat{R}_\ell = \frac{1}{2} L \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{m=1}^{N} \alpha_{k,\ell} \beta_{n,\ell} \gamma_{(n,m)} R_{(n,m),\ell} \geq \kappa_\ell,
\]

where \( \kappa_\ell \) is the minimum sum-spectral efficiency demanded by \( D_\ell \).

Considering all the intended sub-carriers at \( D_\ell \), the overall energy harvested at \( D_\ell \) with a constraint for minimum harvested energy demand of \( \xi_\ell \), is defined as follows

\[
\hat{E}_\ell = \sum_{i=1}^{L} \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{m=1}^{N} \alpha_{k,\ell} \gamma_{(n,m)} E_{(n,m),\ell} \geq \xi_\ell.
\]

The limitations on overall power at the transmitter and the relays are respectively represented as

\[
\sum_{n=1}^{N} \rho_{1,n} \leq P_S,
\]
\[
\sum_{\ell=1}^{L} \sum_{m=1}^{N} \alpha_{k,\ell} \beta_{n,\ell} \gamma_{(n,m)} \rho_{2,m,k} \leq P_{R_k}, \forall k.
\]

Thus, the formulated optimization problem can subsequently be written in its mathematical form as

\[
(P_1) : \max_{\{\alpha, \beta, \gamma, \rho, \theta\}} \omega_R \sum_{\ell=1}^{L} \hat{R}_\ell + \omega_E \sum_{\ell=1}^{L} \hat{E}_\ell
\]

subject to \((C1), (C2), (C3), (C4), (C5), (C6), (C7)\)

\[
(C8) : 0 \leq \theta \leq 1, \ell = 1, \ldots, L,
\]

where \( \alpha = \{\alpha_{k,\ell}\}, \beta = \{\beta_{n,\ell}\}, \gamma = \{\gamma_{(n,m)}\}, \rho = \{\rho_{1,n}, \rho_{2,m,k}\} \) and \( \theta = \{\theta_{\ell}\} \) denote the variables to be optimized for corresponding relay selection, carrier-destination assignment, sub-carrier pairing, sub-carrier power allocation in the two hops, and the SWIPT splitting factors, respectively. We refer the objective in \((P_1)\) as the fixed-price performance tuning function where the weight selections are expounded as fixed-price variables while the optimization of the network resources and the use of some regulatory sub-weight terms account for maximization of total sum-spectral efficiency and sum-harvested energy.

In order to examine \((P_1)\) from a pricing perspective, consider \( \omega_R = \hat{\omega}_R \theta_R \), where \( \hat{\omega}_R = 0.08 \) currency units/bps/Hz.
[16] and $\varrho_R$ is the demanded harvesting energy regulation term defined as $\xi_R$ normalized by per $\mu J$ unit of energy; and

$$\omega_E = \omega_E \bar{\omega}_E,$$

where $\bar{\omega}_E = 1$ currency units/$\mu J$ [17] and $\varrho_E$ is the demanded spectral efficiency regulation term defined as $\kappa_S$ normalized by per bps/Hz unit of spectral efficiency. The fixed pricing of the sub-weights $\omega_R$ and $\omega_E$ are motivated by statistics from [16] and [17]. However, there is no clear pricing metric for harvesting energy in practice or any related work in the literature. But since most of the research works based on SWIPT [2] indicate that the harvested energy usually ranges between $\mu J$ and $n J$, therefore we envision that the pricing metric for harvested energy will be nearly close to our assumption of $\bar{\omega}_E$ in future. On the same trend, it is noteworthy that (P1) can be casted only as the sum-spectral efficiency maximization problem for $\omega_R = 1$ units/bps/Hz and $\omega_E = 0$ units/J, while the same problem can be projected as only a sum-harvested energy maximization problem when $\omega_R = 0$ units/bps/Hz and $\omega_E = 1$ units/J. In this context, detailed discussion is provided in the forthcoming sections.

For any non-zero values or binary alterations of $\omega_E$ and $\omega_R$, obtaining the optimal solution is a cumbersome process due to joint optimization of network resources in (P1) involving mixed-integer variables. An exhaustive search over the feasible space with $(K \cdot L)^{NL}$ combinational possibilities of $\alpha$, $\beta$, and $\gamma$ would yield an optimal solution, which however, becomes impossible to realize practically at very high values of $K$, $L$, and $N$. Therefore, we provide in the succeeding section a novel sub-optimal solution with polynomial execution time-complexity in this regard, and analyze the problem (P1) from three different perspectives.

**IV. MAXIMUM PERFORMANCE-BASED RESOURCE ALLOCATION (MPRA) METHOD**

In this section, we present the Maximum Performance-based Resource Allocation (MPRA) technique to address the problem in (P1). Correspondingly, we propose in the following sub-sections (i) an MPRA algorithm to jointly optimize $\alpha$, $\beta$, and $\gamma$, (ii) sub-carrier power allocation in the two hops, and (iii) computation of SWIPT splitting factor at each destination.

**A. Joint Optimization of $\alpha$, $\beta$, and $\gamma$**

We consider the joint optimization of $\alpha$, $\beta$, and $\gamma$ using a novel MPRA algorithm to obtain an adequate combination of these binary matrices in order to reduce the computational complexity to polynomial time. The overall MPRA algorithm is summarized at the top of the next column. To proceed, we first apply in Step 1, the well known Water-filling (WF) approach [18] to perform power allocation to each sub-carrier in the second hop and compute the corresponding spectral efficiency and harvested energy at each destination from each relay, ignoring the inter-carrier interference (ICI). We then form the objective function similar to the one in (P1), and compute $\alpha$ based on the maximum of the objective where each destination node is coupled with corresponding relay exclusive to itself. On the same trend, we apply in Step 2 the WF strategy to allocate power to all the sub-carriers in the first hop and compute the corresponding spectral efficiency from the source to each relay, ignoring the ICI. Using $\alpha$, the spectral efficiency metric for the first hop and the objective metric for the second hop, we allocate $L$ sub-carriers on the first hop to the destinations on a first-come-first-serve basis depending on the maximum spectral efficiency while tagging these sub-carriers individually with $L$ sub-carriers on the second hop depending on the maximum objective. This step partially fixes $\beta$ and $\gamma$ while guaranteeing that at least one sub-carrier on both the hops is fixed to serve each destination. In Step 3, we disable the non-chosen relays and allocate the remaining network resources in Step 4, following the similar mechanism as explained above. Finally, the algorithm returns

```plaintext
Algorithm: Maximum Performance-based Resource Allocation (MPRA)

1: Require:
   (i) Number of destination nodes: $L$
   (ii) Number of relays: $K$
   (iii) Number of sub-carriers: $N$
   (iv) Channel gains: $\{h_{1,n,k}\}$
   (v) Channel gains: $\{h_{2,m,k,\ell}\}$
   (vi) Weighting factor corresponding to rate: $\omega_R$
   (vii) Weighting factor corresponding to harvested energy: $\omega_E$

2: Initialize:
   $N_1 = \{1, 2, \ldots, N\}$
   $N_2 = \{1, 2, \ldots, N\}$
   $K = \{1, 2, \ldots, K\}$
   $L = \{1, 2, \ldots, L\}$
   $\alpha_{m,k} = 0, \forall k \in K, \forall \ell \in L$.
   $\beta_{n,k,\ell} = 0, \forall n \in N_1, \forall \ell \in L$.
   $\gamma_{n,m,k,\ell} = 0, \forall n \in N_1, \forall m \in N_2$.

3: Use Water-filling (WF) [18] approach to allocate power to each sub-carrier in the second hop from each relay to each destination.

4: Set $\theta_{k} = 0.5, \forall k$, and compute the corresponding sum-spectral efficiency ($\bar{R}_{m,k,\ell}$) and sum-harvested energy ($\bar{E}_{m,k,\ell}$) at each destination from each relay, ignoring the ICI.

5: Define: $O_{m,k,\ell} = \omega_R \bar{R}_{m,k,\ell} + \omega_E \bar{E}_{m,k,\ell}$.

6: while loop $\neq L + 1$ do
   7: $\{k^*, m^*\} = \arg \max \{O_{m,k,\ell}\}$.
   8: Assign $\alpha_{m,k^*,\ell} = 1$.
   9: Set: $\bar{O}_{m,k^*,\ell} = 0, \forall m, \forall \ell$; and $\bar{O}_{m^*,k,\ell} = 0, \forall k, \forall \ell$.
   10: loop = loop + 1.

end while

7: Use Water-filling (WF) [18] approach to allocate power to each sub-carrier in the first hop from the transmit source to each relay.

8: Compute the sum-spectral efficiency from $S$ to each relay: $R_{n,k}$.

9: for $\ell = 1$ to $L$ do
   10: Find $\kappa^*$ such that $\alpha_{\kappa^*,\ell} = 1$.
   11: $\{n^*\} = \arg \max \{R_{n,k}\}$; and $\{m^*\} = \arg \max \{\bar{O}_{m,k^*,\ell}\}$.
   12: $N_1 = N_1 \setminus \{n^*\}$; len = length($N_1$).
   13: Set: $R_{n^*,k} = 0, \forall k$; and $\bar{O}_{m^*,k,\ell} = 0, \forall k, \forall \ell$.
   14: Assign $\beta_{n^*,\ell} = 1$, and $\gamma_{n^*,m^*,\ell} = 1$.

end for

15: for $k = 1$ to $K$ do
   16: if $\sum(\alpha_{k,\ell}) = 0; \forall \ell$ then
      17: Set: $R_{n,k} = 0, \forall n, \forall \ell$; and $\bar{O}_{m,k,\ell} = 0, \forall m, \forall \ell$.

end if

end for

18: for $\ell = 1$ to $L$ do
   19: Find $\kappa^*$ such that $\alpha_{\kappa^*,\ell} = 1$.
   20: $\{n^*\} = \arg \max \{R_{n,k}\}$; and $\{m^*\} = \arg \max \{\bar{O}_{m,k^*,\ell}\}$.
   21: $N_1 = N_1 \setminus \{n^*\}$; len = length($N_1$).

end for

19: for $\ell = 1$ to $L$ do
   20: Find $\kappa^*$ such that $\alpha_{\kappa^*,\ell} = 1$.
   21: $\{n^*\} = \arg \max \{R_{n,k}\}$; and $\{m^*\} = \arg \max \{\bar{O}_{m,k^*,\ell}\}$.
   22: $N_1 = N_1 \setminus \{n^*\}$; len = length($N_1$).

end while

22: Return: Variables: $\alpha, \beta, \gamma$.
```
From a pricing perspective, the performance efficiency of the solution of $\theta$ with increasing values of $\ell$ should serve as the optimal lower bound in (15) or (16) should serve as the optimal values of $\phi$ for obtaining upper limit of $\varphi$. 

For the SWIPT splitting factors are obtained, respectively, by SWIPT splitting factor at each destination node for each of $\omega$. However, before we discuss about the optimization of the $\omega$ that by defining $\omega = 0$ units/J. Finally, we demonstrate that by defining $\omega = 0$ units/bps/Hz, (P1) can be projected as a maximization problem of sum-harvested energy only. However, before we discuss about the optimization of the SWIPT splitting factor at each destination node for each of the above cases, it is worth mentioning that the feasible set for the SWIPT splitting factors are obtained, respectively, by jointly taking into account (C4) and (C5) as follows

$$
\tau_\ell = \left[ \frac{\xi_\ell}{\sum_{k=1}^{K} \sum_{i=1}^{L} \sum_{n=1}^{N} \sum_{m=1}^{N} \alpha_{k,\ell} \gamma_{(n,m)} E_{(n,m),k,i}} - 2\kappa_\ell, \right]
$$

$$
\phi_\ell = \left[ \frac{\xi_\ell}{\sum_{k=1}^{K} \sum_{i=1}^{L} \sum_{n=1}^{N} \sum_{m=1}^{N} \alpha_{k,\ell} \gamma_{(n,m)} R_{(n,m),k,i}} - 1 \exp(2\kappa_\ell) - 1 \Delta, \right]
$$

where $\Delta$ is infinitesimally small positive number. The proof for obtaining upper limit of $\phi_\ell$ is provided in Appendix A.

1) Pricing View-point: Here, we assume non-zero positive values of $\omega_R$ and $\omega_E$, as described previously. Note that the objective in (P1) is a decreasing function of $\theta_\ell$ at the high SNR regime (as considered in this work). This means that the value of the fixed-price performance tuning function decreases with increasing values of $\theta_\ell$. Therefore, the corresponding lower bound in (15) or (16) should serve as the optimal solution of $\theta_\ell$ in order to maximize the objective of (P1). From a pricing perspective, the performance efficiency of the algorithm implemented to utilize maximum network resources would earn more profits to the operator at a fixed price. More discussion on this aspect is provided in numerical results section of this paper.

2) The Sum-Spectral Efficiency Maximization Problem: For $\omega_R = 1$ units/bps/Hz and $\omega_E = 0$ units/J, (P1) becomes a sum-spectral efficiency maximization problem with constraints on minimum harvested energy and minimum spectral efficiency at each destination. Apparently, the objective function in (P1) decreases with increasing values of $\theta_\ell$ and hence the corresponding lower bound in (15) or (16) satisfies the maximization of the objective under respective constraints.

3) The Sum-Harvested Energy Maximization Problem: Considering $\omega_R = 0$ units/bps/Hz and $\omega_E = 1$ units/J, leads to the projection of (P1) as a sum-harvested energy maximization problem. In this context, we observe that the objective function in (P1) increases with increasing values of $\theta_\ell$ and hence the upper bound in (15) or (16) maximizes the objective while respecting the minimum constraints on demanded spectral efficiency and demanded harvested energy.

V. EXECUTION TIME-COMPLEXITY ANALYSIS

In this section, we briefly present a comparative study of the proposed MPRA algorithm with (i) the exhaustive search, and (ii) a Semi-Fixed Resource Distribution (SFRD) technique, in terms of execution time-complexity. In SFRD, a random selection of $\alpha$, $\beta$ and $\gamma$ matrices is chosen, followed by the power allocation and computation of SWIPT splitting factors as in the proposed MPRA method. The comparison between the execution time complexities of the three algorithms viz-a-viz, exhaustive search, MPRA, and SFRD is summarized in TABLE I. Numerical analysis of these algorithms is presented in the subsequent section.

VI. SIMULATION RESULTS

In this section, we present some simulation results to test the efficacy of the proposed MPRA algorithm. We assume the $P_S = P_{R_k} = \ldots = P_{R_K} = P$, and $\zeta = 0.8$ throughout this paper. Each point depicted in all the emulations correspond to the relevant average of the objective function over 500 Monte-Carlo random channel realizations.

We consider a scenario as depicted in Fig. 2, where a transmit source is placed at $(0,15)$ m, $K$ relays are randomly located inside the blue region of area $100$ m$^2$, and $L$
destination nodes are spatially distributed randomly as well, located within the gray region of area 300 m². The ITU Radiocommunication Sector (ITU-R) P.1238 channel model is employed where we simulate a wireless broadband network with central frequency at 1.9 GHz in the considered frequency-selective channel. For the specified room dimension stated above, the root mean square (rms) delay of 36.3078 ns is considered for 5 multi-path arrivals whose mean is obtained by using the Poisson process. The K-factor corresponding to the Rician distribution (chosen for both the hops) is assumed to be 3.5. The effect from shadowing is discarded in the system set-up where each OFDM sub-carrier undergoes flat-fading under an overall system bandwidth of 20 MHz. We assume \( \sigma_{nl}^2 = \sigma_L^2 = -80 \) dBm.

We demonstrate the effect of increasing the harvested energy demand on the fixed-price performance tuning function for different values of \( P \), with \( K = 3, L = 2, N = 32, \) and \( \kappa_L = 1.5 \) bps/Hz for the TS in Fig. 3(a) and for the PS in Fig. 3(b). Based on the discussion in Section III, we choose \( \tilde{\omega}_R = 0.08 \) currency units/bps/Hz and \( \tilde{\omega}_E = 1 \) currency unit/J. The minimum spectral efficiency demand will automatically be satisfied based on the proposed solution for the TS and PS, provided the intervals in (15) and (16) hold, respectively. In this context, we observe from Fig. 3(a) and Fig. 3(b) that the fixed-price performance tuning function increases with increasing values of harvested energy and also with the increasing transmit power values. In addition, MPRA is found to outperform the SFRD technique with PS providing an additional performance advantage over TS for both the methods. We may interpret these observations from the perspective of two operators, say operator 1 employs the MPRA scheme and operator 2 uses SFRD method to serve their individual customers. Then, for a fixed price of \( \tilde{\omega}_R \) and \( \tilde{\omega}_E \) with different set of offers and various harvested energy demand options, operator 1 is capable of providing far better services in comparison to operator 2. Both the operators, however, can further improve their individual services by using the maximum transmit power value within the allowed safety regulation guidelines of their region. Better services from operator 1 would in-turn attract more customers towards leveraging its services, thereby increasing the operator’s profit.

Next, we illustrate the impact of frequency diversity on the system with \( K = 5, L = 4, \) and \( P = 1 \) W, where we set \( \omega_R = 1 \) units/bps/Hz and \( \omega_E = 0 \) units/J in Fig. 4(a) to
study the sum-spectral efficiency function under harvesting energy demand constraint at each destination node, and \( \omega_R = 0 \) units/bs/Hz and \( \omega_E = 1 \) units/J in Fig. 4(b) to analyze the sum-harvested energy function under a minimum spectral efficiency demanded at each destination node. We find that the performance of the MPRA algorithm increases significantly on increasing the number of OFDM sub-carriers. Moreover, the traditional rate-energy (R-E) trade-off [3] can be deduced on increasing the number of OFDM sub-carriers. Moreover, it is observed that the PS receiver architecture outperforms the TS-based SWIPT receiver architecture.

VII. CONCLUSION

In this paper, we proposed a novel formulation to jointly optimize the total weighted sum-spectral efficiency and weighted sum-harvested energy of the end-users in a system comprising of single transmit source, multiple AF-relays, and multiple destination nodes with a OFDM-based transmit scheme; under a fixed pricing metric. Assuming known channel conditions, we proposed Maximum Performance-based Resource Allocation (MPRA) algorithm with a low-computational complexity in contrast to an exhaustive search method for high parameter values. Furthermore, we showed using the binary conditioning options at the corresponding weights that the proposed formulation can be re-casted into a complete sum-spectral efficiency maximization or only as a sum-harvested energy maximization problem. We provided adequate closed-form solution to the SWIPT splitting factors for the aforementioned cases, respectively.

APPENDIX A

UPPER BOUND FOR THE PS FACTOR AT EACH USER

Herein, we determine the upper bound for the PS ratio at all the destination nodes. To proceed, we first reduce (5) using the following approximation in the high-SNR regime for the \( \ell \)-th user

\[
\phi_\ell = 1 - \Delta \frac{\exp(2\phi_\ell) - 1}{\sum_{i=1}^{L} \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{m=1}^{N} \alpha_k,\ell \beta_n,\ell \gamma(n,m) T_{PS}^{TS}(n,m),k,i).}
\]

Hence, proved.

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REFERENCES