ABCDE - Short Summer Tutorial

Default Entailment Course

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In a nutshell

Human-level AI requires justifiable commonsense reasoning
→ in particular: need for formal, normative accounts of
  • **Defaults**: implications with exceptions
  • **Default inference**: plausible reasoning with defaults
    From A and *if A then normally B* plausibly infer B

**NMR-Tutorial:**
  • Selected topics/lessons from 40 years of DR research
  • Focus on theoretical/semantic issues
Contents

- From classical to generalized reasoning
- Defaults and default reasoning
- Context-based default reasoning
- Preferential model theory
- Qualitative default entailment
- Ranking measure semantics
- Rkm-based default entailment
- Ranking-construction paradigm
- System JZ
- (Probabilistic default entailment)
Abstract logic

A 2-valued logic $\mathcal{L} = (L, \vdash)$ is characterized by

- a language (type) $L$ together with
- an inference relation $\vdash \subseteq 2^L \times L$, or
  an inference operator $C : 2^L \rightarrow 2^L$ with $C(\Phi) = \{ \psi \mid \Phi \vdash \psi \}$

**Classical:** propositional/1st/2nd-order, modal/conditional, ...

**Alternative:** intuitionistic/constructive, resource-bounded, ...

**Generalized:** inductive/abductive, paraconsistent/ampliative, ...
Classical inference

In classical logic two standard ways to specify an inference rel. ⊨

• *Syntactic, proof-theoretic:* \( \Phi \vdash_{\mathcal{R}} \psi \) (\( \mathcal{R} \) rule base)
  iff there is an \( \mathcal{R} \)-derivation of \( \psi \) from \( \Phi \)

• *Semantic, model-theoretic:* \( \Phi \models \psi \)
  iff every model satisfying \( \Phi \) also satisfies \( \psi \)

**Classical task:** For a given semantic entailment \( \models \),
find a semi-decidable (ideally decidable) \( \vdash_{\mathcal{R}} = \models \)
Tarskian inference

\( \vdash \), \( \models \) are Tarskian inference relations (finitary):

- **Inclusion**: \( \Phi \vdash \psi \) for each \( \psi \in \Phi \)
- **Cut**: If \( \Phi \vdash \varphi_1, \ldots, \varphi_n, \Phi \cup \{\varphi\} \vdash \psi \), then \( \Phi \vdash \psi \)
- **Monotony**: If \( \Phi \vdash \psi \), then \( \Phi \cup \Psi \vdash \psi \)

Default inference is not Tarskian: **Incl+Cut** ok, never **Mon**

**Notation:**

- Tarskian inference: \( \vdash \) and \( Cn \)
- Generalized inference: \( \models \) and \( C \) or \( C\models \)
Nonmonotonic reasoning I

Real-world agents: must deal with incomplete, uncertain, erroneous, inconsistent, changing, and intractable info
→ need for plausible guesses, withdrawable given new evidence/assumptions
→ need for nonmonotonic reasoning: exploiting rules of thumb, heuristics, implicit assumptions, meta-level/self-reflective considerations ...

Goal: Enrich classical, monotonic core logic with reasonable formal accounts of nonmonotonic reasoning exploiting various concepts of rationality

Remind: Most reasoning outside of math is (also) nonmonotonic!
Nonmonotonic reasoning II

Nonmonotonic reasoning in practice:
- Inductive (prior/model/parameter choice, direct inference)
- Legal (norms ordered by recency, specificity, authority)
- Commonsense (cognitive heuristics, generics, implicatures)

To distinguish:
- Historically grounded, domain-specific reasoning conventions, e.g. in language and law
- Nonmonotonic reasoning concerned with underlying general theoretical and conceptual reasoning methods
Types of nonmonotonic reasoning

Generalized inference may violate any Tarskian principle:

- **Inconsistency repair**: Cut
  e.g. \( \{ \varphi, \neg \varphi, \psi \} \vdash_{inc} \psi \) but \( \{ \varphi, \neg \varphi, \psi, \neg \psi \} \not\vdash_{inc} \psi \)

- **Resource-bounded reasoning**: Incl

- **Probabilistic threshold reasoning**: Incl

- **Inductive reasoning**: Incl

- **Default reasoning**: Incl + Cut

For NMR: alternative finer-grained principles (see later)
Informal defaults

Default: standard/generic assumption, overridable by more concrete information

e.g. default/prototypical values in data bases, prima facie assumptions, legal conventions (presumption of innocence), implications/rules with exceptions, generic quantification, ...

Three common, overlapping readings:

- Plausibilistic/ontic: plausible/normal implications
- Auto-epistemic/context-dependent: classical implications/rules with autoepistemic or defeasible assumptions
- Normative: prima facie norms, amendable laws

Our focus: epistemic/plausibilistic/ontic interpretations
Formal defaults

Default: A default over a base language $L$ is an expression

$$\varphi \rightsquigarrow \psi$$ read as “if $\varphi$, then by default $\psi$”

where $\varphi, \psi \in L$ and $\rightsquigarrow$ denotes a defeasible implication

Strict implication: necessary implications without exceptions

$$\varphi \rightarrow \psi$$ read as “$\varphi$ strictly implies $\psi$”

not to be confused with material implication $\varphi \rightarrow \psi$ over $L$

Conditional language:

$L(\rightsquigarrow, \rightarrow) = \{\varphi \rightsquigarrow \psi, \varphi \rightarrow \psi \mid \varphi, \psi \in L\}$

Note: Defaults typically encode contingent information
Propositional and first-order defaults

Propositional defaults:
- Tweety is a bird plausibly implies that Tweety can fly
  \[ \text{Bird(Tweety)} \leadsto \text{Canfly(Tweety)} \]

First-order defaults:
- Birds normally can fly
  \[ \text{Bird}(x) \leadsto \text{Canfly}(x) \text{ (open/schematic defaults)} \]
  \[ \text{Bird}(x) \leadsto_{x} \text{Canfly}(x) \text{ (default quantifier, more expressive)} \]

Most work on DR is essentially propositional \( \rightarrow \text{our focus} \)
Default inference

**Default inference:** defeasible consequence relation exploiting defeasible and strict implication

**Standard default inferential task:** for $\Sigma \subseteq L, \Delta \subseteq L(\sim, \rightarrow)$

$$\Sigma \cup \Delta \vdash \psi \text{ or } \Sigma \vdash_{\Delta} \psi$$

In addition one may also consider appropriate monotonic inference relations $\vdash \subset \vdash$ extending the basic logical inference on $L$:

$$\Sigma \cup \Delta \vdash \psi \text{ or } \Sigma \vdash_{\Delta} \psi$$
Examples

A prototypical domain for benchmarks ...

P,B,F for *Tweety is a penguin, a bird, can fly*  
*P, B, F* are assumed logically independent

- \( \{ P, P \rightarrow B, B \sim F \} \parallel F \)
- \( \{ P, P \rightarrow B, B \sim F \} \vdash P, B \)
- \( \{ P, \neg F, P \rightarrow B, B \sim F \} \not\parallel \mathbf{F} \) (exception tolerance)
- \( \{ P, P \rightarrow B, B \sim F, P \sim \neg F \} \parallel \neg F \) (specificity principle)
Reiter’s default rules

Reiter’s Default Logic (RDL) 1980 an influential NM formalism

RDL is based on context-dependent rules with autoepistemic assumptions, e.g. expressed as

\[ B : F / F \sim \text{If Tweety is a bird, and it is epistemically possible/consistent that he can fly, then assume that he can fly} \]

Reiter’s general default rules: over classical \((L, \vdash)\):

\[ \varphi : \eta_1, \ldots, \eta_n / \psi \ (\varphi, \eta_i, \psi \in L) \]

\(\varphi\) antecedent, \(\eta_i\) justifications, \(\psi\) consequent

“If \(\varphi\) given and each \(\eta_i\) is consistent, then conclude \(\psi\)”
Reasoning with Reiter’s rules

Reiter’s rules can be used to build defeasible proofs (arguments) producing maximal consistent speculative consequence sets - called \textit{extensions} - closed under the base logic. There may be

- **Multiple:** $W \cup D = \{\varphi\} \cup \{\varphi : \psi / \psi, \varphi : \neg \psi / \neg \psi\}$
  
  Applying one rule blocks the other one
  
  $\rightarrow$ the application order is relevant!

  2 extensions: $E_1 = \text{Cn}(\{\varphi, \psi\}), E_2 = \text{Cn}(\{\varphi, \neg \psi\})$

- **One:** $W \cup D = \{\varphi, \neg \chi\} \cup \{\varphi : \psi / \psi, \psi : \chi / \chi\}$

  1 extension: $E = \text{Cn}(\{\varphi, \psi, \neg \psi\})$

- **None:** Consider the paradoxical rule: $D = \{\text{T} : \neg \psi / \psi\}$

  If $\neg \psi$ is consistent with $E$, then $\psi \in E$

  If not, then $\psi$ is not derivable, and $\psi \not\in E$
Extension-based default reasoning

**Extensions:** acceptable speculative consequence sets $E = Cn(E)$, which can be defined in various ways (not restricted to RDL)

**Extension-based NMR:** $Ext : (W, D) \mapsto Ext(W, D) \subseteq 2^L$

**Skeptical inference:** $W \cup D \models^Ext \psi$ iff $\psi \in \bigcap Ext(W, D)$
Fixed point definition

Reiter’s rules refer to the set of expected consequences $S$ the justifications should be consistent with. Ideally, $S$ should be the actual constructed extension $E$

**Fixed point operator:** links expected with actual consequences:

$F_{(D,W)}(S)$ is the smallest $S' = Cn(S')$ with $W \subseteq S'$ and closed under all default rules $\varphi : \eta_1, \ldots, \eta_n / \psi$ in $D$ with $S \not\vdash -\eta_i$, i.e. whose justifications are consistent with $S$

**Reiter’s extensions:** $Ext_{rdl}(D, W) = \{ E | F_{(D,W)}(E) = E \}$

- Normal default theories ($\varphi : \psi / \psi$) always have extensions
- Prerequisite-free semi-normal ones ($T : \eta \land \psi / \psi$) may have none
- Extensions are mutually inconsistent (no $E \subset E'$)
Some links

Strong links with logic programming and formal argumentation, which differ by their restricted languages and extension concepts

Clauses in logic programs: \( a_i, b, s_j \) ground literals

\[
\begin{align*}
b \leftarrow a_1, \ldots, a_n, \text{not}(s_1), \ldots, \text{not}(s_m) & \sim \\
& a_1 \land \ldots \land a_n : \neg s_1, \ldots, \neg s_m / b
\end{align*}
\]

Stable sets \( \sim \) Reiter extensions restricted to ground atoms
Reiter’s default inference

Reiter defaults: usually interpreted as normal default rules

Translation:

\[ \tau : \varphi \rightsquigarrow \psi \leftrightarrow \varphi : \psi/\psi \quad \text{and} \quad \tau : \varphi \rightarrow \psi \leftrightarrow \varphi : \mathbf{T}/\psi \]

\[ \Sigma \cup \Delta \models^{rdl} \psi \iff \psi \in \bigcap \text{Ext}_{rdl}(\Delta^\tau, \Sigma) \]

Existence of extensions is here guaranteed! \((E \vdash F\) possible)

Alternative default implementations possible:

\[ \varphi \rightsquigarrow \psi \leftrightarrow \mathbf{T} : \varphi \rightarrow \psi/\varphi \rightarrow \psi \quad \text{resp.} \quad \mathbf{T} : \varphi \land \psi/\varphi \rightarrow \psi \]

but no clear advantages - only some trade-offs
Digression - KLM principles

In the 80s/90s: proliferation of DR and NMR formalisms

→ e.g.: to repair perceived inadequacies of earlier proposals

→ but: iterating, no end in sight ...

→ seeking rationality principles to evaluate and classify the beasts

In fact: it is easier to discuss and analyze abstract principles than examples tainted by diverging implicit world knowledge!

→ Principles for nonmonotonic inference relations - on L, typically keeping defaults fixed [Gabbay 85, Kraus et al. 90, Makinson 94], with representation theorems based on possible worlds semantics
KLM-principles for RDL I

- **Supraclassicality (SC):** \( \Sigma \vdash \psi \) implies \( \Sigma \models \neg \Delta \psi \)
- **Left Logical Equivalence LLE:**
  \( \Sigma \models \Sigma' \) and \( \Sigma \models \neg \Delta \psi \) implies \( \Sigma' \models \neg \Delta \psi \)
- **Right Weakening RW:**
  \( \Sigma \models \neg \Delta \psi \) and \( \psi \vdash \psi' \) implies \( \Sigma \models \neg \Delta \psi' \)
- **Right Conjunction AND:**
  \( \Sigma \models \neg \Delta \psi \) and \( \Sigma \models \neg \Delta \psi' \) implies \( \Sigma \models \neg \Delta \psi \land \psi' \)
- **Cautious Monotony CM:**
  \( \Sigma \models \neg \Delta \varphi \) and \( \Sigma \models \neg \Delta \psi \) implies \( \Sigma \cup \{ \varphi \} \models \neg \Delta \psi \)
- **Cautious Transitivity CUT:**
  \( \Sigma \models \neg \Delta \varphi \) and \( \Sigma \cup \{ \varphi \} \models \neg \Delta \psi \) implies \( \Sigma \models \neg \Delta \psi \).
- **Reasoning by Cases OR:**
  \[ \Sigma \cup \{ \varphi \} \vdash_\Delta \psi \text{ and } \Sigma \cup \{ \varphi' \} \vdash_\Delta \psi \implies \Sigma \cup \{ \varphi \lor \varphi' \} \vdash_\Delta \psi \]

- **Rational Monotony RM:**
  \[ \Sigma \vdash_\Delta \psi \text{ and } \Sigma \not\vdash_\Delta \neg \varphi \implies \Sigma \cup \{ \varphi \} \vdash_\Delta \psi \]

- **Consistency Preservation CP:**
  \[ \Sigma \vdash_\Delta \mathbf{F} \implies \Sigma \vdash \mathbf{F} \]
KLM-principles for RDL

⊢_{rdl} satisfies SC, LLE, RW, AND, CUT

Note that CUT is a prerequisite for incremental reasoning.

⊢_{rdl} violates Cautious monotony:
Let \( \Delta = \{ \phi \rightarrow \psi, \psi \rightarrow \chi, \chi \rightarrow \neg \psi \} \), then
\[
\{ \phi \} \cup \Delta \vdash_{rdl} \psi, \chi \text{ but } \{ \chi, \phi \} \cup \Delta \not\vdash_{rdl} \psi
\]
because \( Cn(\{ \phi, \chi, \neg \psi \}) \) is an extension.

⊢_{rdl} also violates OR:
\[
\{ \phi \lor \neg \phi \} \cup \{ \phi \rightarrow \psi, \neg \phi \rightarrow \psi \} \not\vdash \psi \text{ (no triggering)}
\]
Nonmonotonic modal logics

Nonmonotonic modal logics: represent $W \cup D$ in a modal logic AEL, GK, ... [McDermott, Doyle 80, Moore 83, Lin Shoham 92]

$\rightarrow$ more expressivity, flexibility, transparency

[Tru 91]: translate Reiter defaults using a knowledge modality $K$:

- $\varphi \in L \mapsto K(\varphi)$
- $\varphi : \eta/\psi \mapsto K(\varphi) \land K(\neg K(\neg \eta)) \rightarrow K(\psi)$

Extension concept $Ext^X$ for any modal logic $X$ ($\Phi \subseteq L(K)$):

$$Ext^X(\Phi) = \{ E \subseteq L(K) \mid E = \{ \psi \in L(K) \mid \Phi \cup \neg K(L(K) - E) \vdash_X \psi \}$$

$Ext_{rdl}(\Phi) = Ext^X(\Phi)$ for $T \subseteq X \subseteq S4$ restricted to $L$ [Tru 91]

Logic of defaults: allows to prove the equivalence of default bases
Specificity principle

**Intuition:** if defaults conflict, prefer the most specific one

If $\varphi$ subsumes $\varphi'$ and $\psi, \psi'$ conflict
i.e. $\varphi \vdash \varphi'$ or $\varphi \rightarrow \varphi' \in \Delta$, and $\psi \vdash \neg \psi'$:

$$
\varphi' \sim \psi' \preceq_{\text{spec}} \varphi \sim \psi \quad (\text{... is at least as specific as ...})
$$

Also desirable for defeasible subsumption: $\varphi \sim \varphi' \in \Delta$

*Student, adults, jobs:* \{s, s \sim a, a \sim j, s \sim \neg j\} $\models a, \neg j$

**But:** how to prioritize given longer conflicting chains of defaults?

Early try: theories of inheritance nets [Touretzky 86, Hory 94]

**However:** low expressivity, purely syntactic, clash of intuitions
The specificity issue for RDL

No specificity in RDL: it fails in its simplest form

\[ \{p, p \rightarrow b, b \sim f, p \sim \neg f\} \not\models^{rdl} f, \neg f \]

Two extensions \( Cn(\{p, b, f\}), Cn(\{p, b, \neg f\}) \)

Repair by encoding specificity with semi-normal rules? But:

- semi-normal default theories may have no extensions
- very cumbersome, possible side-effects
- \( \rightarrow \) maybe better with explicit preferences

Defeasible specificity hard to characterize, but we may try ...
Preferences for default reasoning

Which preferences? What’s their meaning? How to exploit them?

**Extrinsic preferences**: external attributions
- authority of the source (e.g. for default norms)
- utility, usefulness (for practical reasoning)
- application order (procedural, execution needs)

**Our topic**: **Intrinsic preferences**: fixed by the defaults
- strict/defeasible specificity
- reliability, strength
- aggregated preference structure
Default reasoning with preferences

Simplest: meta-level preferences over proper defaults $\Delta \cap L(\sim)$

$\rightarrow$ preferences guide the default inference process: many ways

Standard default inference $\sim$ parametrized by a transitive $\prec$:

$$\Sigma \cup \Delta \mid \sim \prec \psi$$

Several approaches based on Reiter’s account: e.g. [Brewka 94]

(NMR tradition: $\delta < \delta'$ means “$\delta$ preferred to $\delta'$”)
Prioritized default logics

Prioritized default theory: \((W, D, <)\), where

- \((W, D)\) is a default theory
- \(D = D_s \cup D_n\) collects strict resp. normal default rules
- \(< \subseteq D^2_n\) is a strict, well-founded partial order
  i.e., every subset has \(<\)-minimal element(s) (true for finite \(<\))

**Why partiality?** \(<_{\text{spec}}\) may be partial, or total + partially known

**Why well-foundedness?** Bottom-up construction of extensions

**Outcome:** handles transparent inheritance/specificity scenarios
Prioritized extensions - an example

Greedy quasi-inductive definition with priorities:

$E$ is a prioritized extension of $(W, D, <)$, or $E \in Ext_{pdl}$, iff

there is a well-ordering $\prec$ of $D_n$ extending $< (\subseteq \prec)$ s.t.

$E = \bigcup E^{\prec}_\alpha$ where $E^{\prec}_0 = Cn_{D_s}(W)$,

$E^{\prec}_{\alpha+1} = Cn_{D_s}(E^{\prec}_\alpha \cup \{ cons(\delta) \})$,

if there is a $\prec$-minimal default $\delta \in D$ active in $E^{\prec}_\alpha$

$(\varphi : \psi / \psi$ is active in $X$ iff $X \vdash \varphi$ and $X \not\vdash \neg \psi, \psi)$

otherwise $E^{\prec}_{\alpha+1} = E^{\prec}_\alpha$
Some problems

- **Risk of incoherence:** by complex, meaning-blind interactions between defaults, preferences and the logical structure (especially for intrinsic preferences) ignoring each other

- **Complex specificity:** Specificity orderings may achieve logical coherence, but viable transparent notions of defeasible specificity are elusive as the theory of inheritance nets has shown

- **RDL legacy:** PDL inherits several deficiencies of RDL

- **Greedy approach:** possible tensions between the application order and the chosen preference order
  
e.g. $a \rightsquigarrow b < T \rightsquigarrow \neg b < T \rightsquigarrow a : Cn(\{a, \neg b\})$ or $Cn(\{a, b\})$?

**Alternative definitions:** either similar issues, or no extensions
Qualitative plausibility models

Default conditionals:

$$\varphi \rightsquigarrow \psi : \varphi \text{ plausibly/normally implies } \psi$$

Idea: Defaults seen as constraints over epistemic orders

Models: Preferred model structures over $$\mathcal{L} = (L, \models)$$:

- $$\langle W, \leq, w_0 \rangle$$ with $$w_0 \in W \subseteq [T]_\mathcal{L}$$, and
- $$\preceq$$ a preorder over $$W$$ ($$\preceq = \leq \cap \nleq$$)
  
  ($$v \preceq w$$: $$v$$ is at least as preferred/plausible as $$w$$)

Satisfaction relation: $$\models_{pr}$$ for $$L \cup L(\rightarrow) \cup L(\rightsquigarrow)$$.

- $$\langle W, \leq, w_0 \rangle \models_{pr} \varphi$$ iff $$w_0 \models \varphi$$
- $$\langle W, \leq, w_0 \rangle \models_{pr} \varphi \rightarrow \psi$$ iff $$[\varphi] \cap W \subseteq [\psi]$$
Preferential conditional semantics

Naively: \((W, \preceq, w_0) \models_{min} \varphi \rightsquigarrow \psi\) iff \(\text{Min}_{\preceq}([\varphi]) \subseteq [\psi]\)

But: There may be no minima - and imposing them artificially (stopperedness, smoothness) is neither natural nor necessary

Example: \(W = \{w, w_1, w_2, \ldots\}\) with \(w \models \neg \varphi, w_i \models \varphi,\) and let \(\preceq\) be an infinite descending chain \(\ldots < w_3 < w_2 < w_1.\) Then \((W, \preceq, w_0) \models_{min} T \rightsquigarrow \neg \varphi\) - despite arbitrarily preferred \(\varphi\)-worlds

Generalized semantics: \((W, \preceq, w_0) \models_{min} \varphi \rightsquigarrow \psi\) iff
for each \(w \models \varphi,\) there is \(w \succeq v \models \varphi\) s.t. for all \(v \succeq v' \models \varphi, v' \models \psi\)

Now: \((W, \preceq, w) \models_{min} \varphi \rightarrow \psi\) iff \((W, \preceq, w) \models_{min} \varphi \land \neg \psi \rightsquigarrow F\)
Preferential conditional logic I

$\models_{min}$ defines a monotonic entailment relation $\vdash_{pcl}$:

$$\Sigma \cup \Delta \vdash_{pcl} \gamma \ \text{iff} \ \left[ \Sigma \cup \Delta \right]_{pcl} \subseteq \left[ \gamma \right]_{pcl} \ (\text{for} \ \gamma \in L \cup L(\neg, \rightarrow))$$

Axioms of preferential conditional logic $\vdash_{pcl}$:

- $\varphi, \varphi \rightarrow \psi / \psi$ (Modus Ponens rule)
- $\varphi \sim \varphi$ (Reflexivity)
- If $\vdash \varphi \leftrightarrow \varphi'$ then $\varphi \sim \psi / \varphi' \sim \psi$ (Left logical equivalence)
- If $\vdash \psi \rightarrow \psi'$ then $\varphi \sim \psi / \varphi \sim \psi'$ (Right weakening)
- $\varphi \sim \psi, \varphi \sim \psi' / \varphi \sim \psi \land \psi'$ (Right conjunction)
Preferential conditional logic II

• $\varphi \leadsto \psi$, $\varphi' \leadsto \psi / \varphi \lor \varphi' \leadsto \psi$ (**Reasoning by cases**)
• $\varphi \leadsto \varphi'$, $\varphi \leadsto \psi / \varphi \land \varphi' \leadsto \psi$ (**Cautious monotony**)
• $\varphi \leadsto \varphi'$, $\varphi \land \varphi' \leadsto \psi / \varphi \leadsto \psi$ (**Cautious transitivity**)
• $\varphi \leadsto \text{F} / \neg \varphi$ (**Necessity**)
• $\varphi \rightarrow \psi$ if and only if $\varphi \land \neg \psi \leadsto \text{F}$ (**Strict implication**)

Object-level modus ponens fails: $\varphi, \varphi \leadsto \psi \not\vdash_{pcl} \psi$

because the actual world can be exceptional!
Preferential entailment

How to specify nonmonotonic reasoning with default conditionals?

Simplest approach: $\vdash^p \; (\text{Preferential entailment/System P})$

$\{\varphi_1, \ldots, \varphi_n\} \cup \Delta \vdash^p \psi$ iff $\Delta \vdash_{pcl} \varphi_1 \land \ldots \land \varphi_n \vdash \psi$

Basic defeasible modus ponens: $\{\varphi\} \cup \{\varphi \vdash \psi\} \cup \Delta \vdash^p \psi$

$\{\varphi, \neg \psi\} \cup \{\varphi \vdash \psi\} \cup \Delta \not\vdash^p \psi$ if $\{\varphi \vdash \psi\} \cup \Delta \not\vdash \varphi \rightarrow \psi$

Simple specificity: $\{s, s \vdash a, a \vdash j, s \vdash \neg j\} \vdash^p a, \neg j$

Defeasible monotony fails: $\varphi \vdash \psi \not\vdash_{pcl} \varphi \land \chi \vdash \psi$

hence $\{\varphi, \chi\} \cup \{\varphi \vdash \psi\} \not\vdash^p \psi$ (only if $\varphi \land \chi \not\vdash \psi$)
System Z I

In System P, irrelevant generic info $\chi$ can block an inference!

**Idea:** Inference based on plausibility maximization

Rational closure [Lehmann, Magidor 92], System Z [Pearl 90]

**Z-algorithm:** (our variant) for finite $\Sigma \cup \Delta$:

1. Translate $\varphi \rightarrow \psi$ into $\varphi \land \neg \psi \sim F$

2. Construct by induction $(\Delta_{\geq i} \mid 0 \leq i)$ and $(\rho_i \mid 0 \leq i)$ s.t.
   - $\rho_0 = T$, $\Delta_{\geq i} = \{ \varphi \rightarrow \psi \in \Delta \mid \varphi \vdash \rho_i \}$
   - $\rho_{i+1} = \bigvee \{ \varphi \land \neg \psi \mid (\varphi \rightarrow \psi) \in \Delta_{\geq i} \}$

For $i < j$ we have $\rho_j \vdash \rho_i$ and $\Delta_{\geq j} \subseteq \Delta_{\geq i}$.

For finite $\Delta$, there is a smallest $N$ s.t. $\Delta_{\geq N} = \Delta_{\geq N+1}$, $\rho_N \not\vdash \rho_{N+1}$
System Z II

Z-rank $\sim$ degree of exceptionality

Z-rank of defaults: $Z(\delta) = \text{maximal } n \text{ s.t. } \delta \in \Delta_{\geq n}$

Z-rank of worlds: $Z(w) = \text{max} \{n \mid w \models \rho_n\}$, i.e. largest $Z(\delta)$ s.t. $w$ violates $\delta$ ($Z(\varphi \leadsto \mathbf{F}) = \infty$)

There is a canonical ranked model $(W^Z_\Delta, \preceq^Z_\Delta)$ with

- $W^Z_\Delta = [\neg \rho_\infty] = \{w \in [\mathbf{T}]_\mathcal{L} \mid Z(w) < \infty\}$
- $v \preceq^Z_\Delta w$ iff $Z(v) \leq Z(w)$

Z-entailment (System Z):

$\{\varphi_1, \ldots, \varphi_n\} \cup \Delta \models^z \psi$ iff $(W^Z_\Delta, \preceq^Z_\Delta) \models_{\text{min}} \wedge \varphi_i \leadsto \psi$
Properties of System Z

$\vdash^\sim_\Delta$ verifies all the KLM-principles:

**SC, LLE, RW, AND, OR, CUM (= CUT+CM), RM**

**Defeasible transitivity:**

\[
\{s, s \rightsquigarrow a, a \rightsquigarrow j\} \vdash^\sim a, j
\]

\[
\{s, \neg j, s \rightsquigarrow a, a \rightsquigarrow j\} \vdash^\sim a, \neg j
\]

**Defeasible specificity:**

\[
\{s, s \rightsquigarrow \neg j, s \rightsquigarrow a, a \rightsquigarrow j\} \vdash^\sim a, \neg j \quad (\text{also } \{\ldots\} \not\vdash^\sim j)
\]

**LLE for defaults:** $\Delta \vDash_{pcl} \Delta'$ implies $\vdash^\sim_\Delta = \vdash^\sim_{\Delta'}$.
Problems for System Z

Simple exceptional inheritance fails:

\{\textit{dutch}, \neg \textit{tall}, \textit{dutch} \rightsquigarrow \textit{tall}, \textit{dutch} \rightsquigarrow \textit{loud}\} \not\models^z \textit{loud}

The Z-model of $\Delta$ is given by: $\textit{dtl} \prec \neg \textit{tl} \sim \textit{dtl} \sim \textit{dtl} \sim \textit{dtl} \sim \neg \textit{tl} \sim \textit{dnl}$
i.e. $Z(\textit{dtl}) = 0$ and $Z(\neg \textit{tl}) = Z(\textit{dtl}) = Z(\neg \textit{tl}) = 1$

Hence $\textit{dutch} \wedge \neg \textit{tall} \not\models^z \textit{loud}$: no exceptional inheritance

Replacing \textit{dutch} by \textbf{T}, we can also falsify the \textit{Irrelevance Principle}.

\textbf{IRR}: If $\Sigma \cup \Delta$ and $\Sigma' \cup \Delta'$ \not\models_{pcl} \textbf{F} have disjoint vocabularies, then $\Sigma \cup \Delta \models \psi$ iff $\Sigma \cup \Sigma' \cup \Delta \cup \Delta' \models \psi$
System LEX I

Idea: compare not just the highest-ranked violated defaults but also lower-ranked ones, as well as their number at the different ranks

Lexicographic entailment [Lehmann 1995]: \( \models^{lex} \)

Violation sequences for worlds: \( \text{lex}_\Delta(w) = (z_i(w) \mid i \leq \infty) \)
where \( z_i(w) = |\{\delta \in \Delta \mid w \models \varphi_\delta \land \neg \psi_\delta, Z(\delta) = i\}| \)
\( \Delta_{\text{dutch}}: \text{lex}(d \triangleright tl) = (1, 0, \ldots 0), \text{lex}(d \triangleright t \triangleright l) = (2, 0, \ldots 0) \)
\( \Delta_{\text{imp}} = \{p \leadsto F\}: \text{lex}(p) = (0, 0, \ldots 1), \text{lex}(\neg p) = (0, 0, \ldots 0) \)
System LEX II

Lex-ordering: \( v \preceq^{lex} w \) iff \( z_\infty(w) \neq 0 \), or
\( z_\infty(w) = 0 \) and for the highest \( i \) with \( z_i(v) \neq z_i(w) \), \( z_i(v) \leq z_i(w) \)

LEX: \( \Sigma \cup \Delta \models^{lex} \psi \) iff \((W_{\Delta}^{lex}, \preceq^{lex}_{\Delta}) \models_{min} \wedge \varphi_i \leadsto \psi\)

Winged birds example: \( \Delta = \{ p \rightarrow b, b \leadsto f, p \leadsto \neg f, b \leadsto w \} \)
\( \{ p \} \cup \Delta \models^{lex} \neg f, w \) because
\( \text{lex}(pb\neg f\neg w) = (2, 0, \ldots 0), \text{lex}(pbf\neg w) = (1, 1, \ldots 0), \)
\( \text{lex}(pb\neg fw) = (1, 0, \ldots 0), \text{lex}(pbfw) = (0, 1, \ldots 0) \)
\( pb\neg fw \) is obviously the most plausible world
Properties and problems

LEX extends Z: $\sim^z \subset \sim^{lex}$, hence it is more speculative.

**Static priorities:** Z-ranks of defaults are pre-computed, no inductive prioritization considering e.g. the fine-grained preference status of default antecedents.

**Radical ad hoc prioritization:** Violating a more specific conflicting defaults has automatically more weight than violating two independent less specific defaults - which is probabilistically unsound and in conflict with irrelevance considerations.
**Beyond plausibility orders**

Drawbacks of qualitative plausibility orders:

- For $w \prec w'$ and $v \prec v'$, the relative plausibility of $w'$ w.r.t. $w$ cannot be compared to that of $v'$ w.r.t. $v$
- No proper conditional independence notion
- Insufficient expressiveness/granularity
- Translation between/aggregation of plausibility contexts unclear
- Expected utility hard to model

**Idea:** Use plausibility valuations from world sets to an additive structure of ordered values

Fine-grained example: probability measures $P : Prop \rightarrow [0, 1]$
Plausibility valuations

General plausibility val. [Friedman, Halpern 96]:

\[ Pl : B \rightarrow (V, \bot, \top, \prec) \text{ with} \]
\[ Pl(\emptyset) = \bot, \quad Pl(W) = \top \quad \text{and} \quad Pl(A) \leq Pl(B) \text{ if } A \subseteq B \]

Desiderata

- Conditional plausibility + reasonable independence concept
- \( \prec \) total order (partial order = set of total ones)

**Simplest solution**: Ranking measures [Spohn 88, 12, Wey 95]

Ranking measures generalize

- Spohn’s rk-functions measuring the implausibility/surprise of propositions, used to model revisable graded plain belief
- Real-valued multiplicative possibility [Dubois, Prade 94]
Real-valued ranking measures

**Real-valued ranking measures** (integers not enough!)

Let $\mathbb{B}_\mathcal{L}$ be the boolean algebra of $\mathcal{L}$-propositions

$R : \mathbb{B} \to ([0, \infty], +, \leq)$ is a real-valued ranking measure (rmk) iff

- $([0, \infty], 0, +, \geq)$: ordered additive structure of pos. reals with $\infty$
- $R(W) = 0, R(\emptyset) = \infty$ (expresses impossibility)
- $R(A \cup B) = \min \{ R(A), R(B) \}$ for all $A, B \in \mathbb{B}$

Conditional ranking measure:

$R(B|A) = R(A \cap B) - R(A)$ for $R(A) \neq \infty$, else $R(B|A) = \infty$.

$R_0$ is the uniform rkm with $R_0(A) = 0$ for $A \neq \emptyset$. $R(\varphi) := R([\varphi])$
Ranking epistemology

Ranking measure values $\sim$ degrees of implausibility/surprise

Ranking measures may model belief states (Spohn):

Belief strength in $\varphi$ is $r$ iff $R(\neg \varphi) = r$

Conventional threshold: $Bel(\varphi)$ iff $R(\neg \varphi) \geq 1$ ($R(\varphi) = 0$)

Belief is closed under conjunction (plain belief) because

$R(\neg(\varphi \land \psi)) = R(\neg \varphi \lor \neg \psi) = \min\{R(\neg \varphi), R(\neg \psi)\} \geq 1$

$R(\neg \varphi) = \infty$: $\neg \varphi$ is epistemically impossible, i.e. $\varphi$ a dogm

Probabilistic link: $R(A) = r \sim P(A) = O(\varepsilon^r)$,

for infinitesimals $0 < \varepsilon \ll 1$
Ranking measure semantics

Rkm semantics for default $\sim$ and strict implication $\rightarrow$:

$R \models_{rk} \varphi \sim \psi$ iff $R(\varphi \land \psi) + 1 \leq R(\varphi \land \neg \psi)$ iff $R(\neg \psi|\varphi) \geq 1$

$R \models_{rk} \varphi \rightarrow \psi$ iff $R(\varphi \land \psi) + \infty \leq R(\varphi \land \neg \psi)$ iff $R(\varphi \land \neg \psi) = \infty$

$[\Delta]_{rk} = \{R \mid R \models_{rk} \delta \text{ for all } \delta \in \Delta\}$

We sometimes need a weaker satisfaction $\models^>_{rk}$ using $R(\neg \psi|\varphi) > 0$

**Monotonic rk-entailment:** $\Delta \vdash_{rk} \delta$ iff $[\Delta]_{rk} \subseteq [\delta]_{rk}$

$\vdash_{rk}$ satisfies the rules for preferential conditional logic
Rkm-based default entailment

**Goal:** A rkm-based framework to specify default inference

**Idea:** Rkm semantics + preferred model theory for conditionals

**Preferred rkm choice function:** $\mathcal{I} : \Delta \mapsto \mathcal{I}(\Delta) \subseteq [\Delta]_{rk}$

**Rkm-based default entailment w.r.t. $\mathcal{I}$:** $\vdash^\mathcal{I}$:

$\Sigma \cup \Delta \vdash^\mathcal{I} \psi$ iff for all $R \in \mathcal{I}(\Delta)$ $R \models_{rk}^> \Sigma \Rightarrow \psi$

**Defeasible modus ponens:**

$\{\varphi\} \cup \{\varphi \Rightarrow \psi\} \cup \Delta \vdash^\mathcal{I} \psi$ ({$\varphi \Rightarrow \psi$} $\cup \Delta \vdash_{rk} \varphi \Rightarrow \psi$)

**Preferentiality:** $\vdash^\mathcal{I}_\Delta$ verifies SC, LLE, RW, AND, CUT, CM, OR
Rkm-based reconstructions

**Preferential entailment:** $\neg^p = \neg^p$ where $I_p(\Delta) = [\Delta]_{rk}$

System P is the weakest $\neg^I$ because $I_p(\Delta)$ is maximal

**Conditional monotonicity:** $\Sigma \cup \Delta \vdash^p \psi$ implies $\Sigma \cup \Delta \cup \Delta' \vdash^p \psi$

For finite $\Delta \not\vdash_{rk} F$, there is a unique rkm-mode $R^z_\Delta$ which attributes the lowest possible rkm-values to each proposition:

$$R^z_\Delta(A) = \inf_{\leq} \{ R(A) \mid R \vdash_{rk} \Delta \}$$

**System Z:** $\neg^z = \neg^I_z$ with $I_z(\Delta) = \{ R^z_\Delta \}$

**LLE for defaults:** for Systems P, Z we have

$[\Delta]_{rk} = [\Delta']_{rk}$ implies $\neg^I_\Delta = \neg^I_{\Delta'}$
Reconstruction of System LEX

Semantic problems with System LEX:

**Non-standard:** Reconstructing $\sim^{lex}$ requires infinite rkm-values:

$1 < 1+1 < \ldots N < \ldots N+N < \ldots N^2 < N^2+1 < \ldots N^2+N < \ldots N^2+N^2 < \ldots N^3 < \ldots < \infty$

$(1,0,\ldots,0) < (2,0,\ldots,0) < \ldots (0,1,\ldots,0) < \ldots (0,2,\ldots,0) < \ldots \infty$

$\rightarrow$ LEX imposes excessively high implausibility values

**Context dependence:** The relative rkm-values may depend on the rkm-value of a generic context $X$:

$$R^{lex,N}_{\{T\sim a\}}(\neg a) = 1 \quad \text{but} \quad R^{lex,N}_{\{X\sim a,T\sim \neg X\}}(\neg a \mid X) = N$$

Are there less extreme rkm-inference notions supporting inheritance?
**Ranking measure constructions**

How to find reasonable $R \models_{rk} \Delta$ respecting the structure of $\Delta$?

**Idea:** Focus on those $R$ obtained by iterated Spohn-style revision on the uniform rkm prior $R_0$ with $\varphi \rightarrow \psi$ for $\varphi \nabla \psi \in \Delta$

**Informally:** Specify ranking models by adding context-dependent penalties $\geq 0$ for each default a world violates

Minimal change when strengthening belief in $\varphi$: make $\neg \varphi$-worlds uniformly more implausible

**Ranking construction models:** Let $\Delta = \{\varphi_i \nabla \psi_i \mid i \leq n\}$

$$Constr(\Delta) = \{ R \models_{rk} \Delta \mid R = R_0 + \sum_{i \leq n} a_i [\varphi_i \land \neg \psi_i], a_i \in [0, \infty]\}$$

**Fact:** If $\Delta \not\models F$, then $Constr(\Delta) \neq \emptyset$
Construction-based default entailment I

Strategy: Default inference based on preferred rkm-constructions, i.e. $\sim^I$ with $I(\Delta) \subseteq Constr(\Delta)$

System J: $\sim^j = \sim^{Ij}$ where $I^j(\Delta) = Constr(\Delta)$

Exceptional inheritance: for logically independent $a, b$,

$\{\neg a, T \rightsquigarrow a, T \rightsquigarrow b\} \sim^j b$

$R \models_{rk} \neg a \rightsquigarrow b$ if $R = R_0 + x(T \land \neg a) + y(T \land \neg b)$ for $1 \leq x, y$

because then $R(\neg a \land b) + 1 = x + 1 \leq x + y = R(\neg a \land \neg b)$
Construction-based default entailment II

Advantages: Simplicity, robustness, intuitive behaviour

System $J^+$: $\mathcal{I}^+(\Delta) = \text{rk-construction models with } a_i \geq 1$

- $\{
eg a\} \cup \{T \leadsto a, T \leadsto a \lor b\} \not\sim^j b$, but
- $\{
eg a\} \cup \{T \leadsto a, T \leadsto a \lor b\} \sim^{j^+} b$

Systems $J, J^+$ may be too cautious by not fully exploiting the idea of plausibility maximization
Minimal rkm-constructions

**Idea:** Maximizing plausibility by minimizing shifting

**JM:** $\mathcal{I}^{jm}(\Delta) = \text{set of } R \in Constr(\Delta) \text{ with pointwise minimal shifting vectors } \vec{a}_i$

**Non-uniqueness:**

$\mathcal{I}^{jm}(\{T \leadsto a, T \leadsto b, T \leadsto a \land b\})$ is uncountable

$\mathcal{I}^{jm}(\Delta) = \{R_0 + x[\neg a] + x[\neg b] + y[\neg a \lor \neg b] \mid x + y = 1\}$

Minimal rkm-constructions have not been born equal ...
Big Birds Hammer

Big Birds Hammer: Birds are normally small, birds can normally fly, exceptional birds (small or unable to fly) normally cannot fly. What about the flying abilities of big birds?

\[ \{b, \neg s\} \cup \{b \sim f, b \sim s, b \land \neg(s \land f) \sim \neg f\} \sim \neg f? \]

By specificity one may expect \( \{b, \neg s\} \cup \Delta \nsim \neg f \)

(\( \text{which holds for System Z, minimal information entailment} \))

But \( \mathcal{L}^{im}(\Delta) = \{(2-x)[b \land \neg s \land f] + x[b \land \neg s] + 1[b \land \neg f] \mid x \in [0, 2]\}, \)

\( R^{1}(b \land \neg s \land f) = 2 = R^{1}(b \land \neg s \land \neg f), \) hence \( \{b, \neg s\} \cup \Delta \not \models^{im} \neg f \)

The best fitting solution: \( R^{0} = 2[b \land \neg s \land f] + 1[b \land \neg f] \)
Justifiable constructibility

**Idea:** Ranking constraints should not be over-satisfied

**Justifiable constructibility:**

\[ R = \sum_{i \leq n} a_i[\varphi_i \land \neg \psi_i] \] is justifiably constructible model of \( \Delta \) iff proper shifting of \([\varphi_i \land \neg \psi_i]\), i.e. \( a_i > 0 \), implies satisfaction as an equality constraint: \( R(\varphi_i \land \psi_i) + 1 = R(\varphi_i \land \neg \psi_i) \)

**System JJ:** \( \mathcal{I}_{jj}(\Delta) \) = justifiably constructible rk-models of \( \Delta \)

**Fact:** \( \mathcal{I}_{jj}(\Delta) \subseteq \mathcal{I}_{jm}(\Delta) \)

**Big Birds Hammer:** JJ provides the unique correct solution:

\[ \mathcal{I}_{jj}(\Delta_{bbh}) = \{2[b \land \neg s \land f] + 1[b \land \neg f]\} \]
Non-uniqueness: If $\Delta = \{ T \leadsto a, T \leadsto b, T \leadsto a \land b \}$, then $\mathcal{I}^{jj}(\Delta) = \mathcal{I}^{jm}(\Delta)$ is again uncountable.
Canonical preferred ranking models

**Goal:** Specify for each $\Delta$ a canonical preferred rkm model improving on Systems Z/LEX. Two main strategies:

- **Ranking measure fusion:** “take the average”
- **Canonical rkm construction:** liike for System Z/LEX

**Ranking measure fusion:** For each rk-choice $\mathcal{I}$, let $\hat{\mathcal{I}}$ be s.t.

$$\hat{\mathcal{I}}(\Delta) = \{R^*_\Delta\} \text{ with } R^*_\Delta(A) = \inf_{\subseteq} \{R(A) | R \in \mathcal{I}(\Delta)\}$$

$R^*_\Delta$ is the most plausible lower bound of the $\mathcal{I}$-preferred $\Delta$-models

**Example:** for System Z, $\mathcal{I}^z(\Delta) = \hat{\mathcal{I}}^p = [\Delta]_{rk}$

**Fact:** $R^*_\Delta \in Mod_{rk}(\Delta)$, but $R^*_\Delta \not\in Constr(\Delta)$ is possible
**System JJR**

**JJR:** $\mathcal{I}^{jjr}(\Delta) = \hat{\mathcal{I}}^{jj}(\Delta) = \{R^{jjr}_\Delta\} - \text{the best of both worlds?}

If the justifiably constructible model is unique, it is the JJR-model

For $\Delta = \{T \leadsto a, T \leadsto b, T \leadsto a \wedge b\}$, $R^{jjr}_\Delta = 1[-a \lor -b]$

(it is the Z-model of $\Delta$ and justifiably constructible)

**Constructibility counterexample:** Nested crossing

$\Delta_{nc} = \{T \leadsto a, T \leadsto b, T \leadsto r, T \leadsto s, T \rightarrow (a \wedge b \leftrightarrow r \wedge s)\}$

$\mathcal{I}^{jj}(\Delta_{nc}) = \{x[-a] + x[-b] + (1 - x)[-r] + (1 - x)[-s] \mid x \in [0, 1]\}$

$R^{jjr}_{\Delta_{nc}} = 1[-(a \wedge b)] + 1[-(a \lor b \lor r \lor s)] + \infty[-(a \wedge b \leftrightarrow r \wedge s)]$

$\notin Constr(\Delta_{nc})$
Canonical preferred rk-constructions

**Goal:** Incremental construction of a canonical rkm-model $R^*_\Delta$ of $\Delta$ in the spirit of System Z

**Examples:** the only rk-constructible ones I am aware of ...

System JZ, JLZ [Wey 98, 03]

**Philosophy:** Minimize the rk-construction efforts everywhere

Let $\Delta = \{ \varphi_i \Rightarrow \psi_i \models i \leq n \}$

We seek a “minimally constructed” $R^*_\Delta = \Sigma_i a_i[\varphi_i \land \neg \psi_i]$
**JZ-construction**

**Guiding principles** of the JZ-construction

- *Justifiable constructibility*: no superfluous shifting
- *No default redundancy*: equivalent defaults considered once
  \[ w.l.o.g.: ([\varphi_i \land \lnot \psi_i], [\varphi_i]) = ([\varphi_j \land \lnot \psi_j], [\varphi_j]) \]  
  implies \( i = j \)
- *Bottom-up plausibility maximization*: first construct the most plausible layers, ignoring the necessarily less plausible ones
- *Local shifting minimization*: when constructing a layer, realize the not-yet-settled defaults by lexicographically minimizing the longer shifts first

**Note:** Here the priorities are dynamic, not as for System Z, LEX
System JZ

**System JZ:** flagship proposal for rkm-based default inference

**JZ-idea:** Proceed rank by rank, trying to locally approximate ranking minimization (system Z) by local ranking constructions lexicographically minimizing the shifting efforts for each target rank

*Relative plausibility maximization:* $PM(R, \Delta)$

An auxiliary notion generalizing $R^{\Delta}_z = PM(R_0, \Delta)$

The most plausible rkm-model of $\Delta$ above $R$ for $R(\land \Delta \rightarrow) \neq \infty$

$$PM(R, \Delta)(A) = \inf_{\leq \{ R'(A) \mid R \leq R', R' \models_{rk} \Delta \}}$$
JZ-algorithm I

**Induction:** We jointly construct sequences \( R_i, R_i^*, \Delta_i, \Delta'_i, \)

**Start:** \( i = 0: R_0 = R, R_0^* = PM(R_0, \Delta) = R_\Delta^*, \Delta_0 = \emptyset, s_0 = 0 \)

**Step:** \( i \rightarrow i + 1: R_i \) preceding partial ranking construction, \( R_i^* = PM(R_i, \Delta), \Delta_i \) collection of settled defaults at stage \( i \)

\( s_{i+1} \) smallest \( s > s_i \) of the form \( s = R_i^*(\varphi_j \land \neg\psi_j) \) for \( \delta_j \in \Delta - \Delta_i \)

\( \Delta'_{i+1} = \{ \delta_j \in \Delta - \Delta_i \mid R_i^*(\varphi_j \land \neg\psi_j) = s_{i+1} \} \)
JZ-algorithm II

\[ R_{i+1} = R_i + \sum_{\delta_j \in \Delta_{i+1}} a_i[\varphi_j \land \neg \psi_j], \]

where \( \vec{a} \) is the lex-length-minimal tuple \( \vec{x} \) with, for all \( h \leq n, \)

\[
(R_i + \sum_{\delta_j \in \Delta_{i+1}} x_j[\varphi_j \land \neg \psi_j] + \\
\sum_{\delta_j \not\in \Delta_i \cup \Delta_{i+1}} \infty[\varphi_j \land \neg \psi_j])(\varphi_h \land \neg \psi_h) \
\geq s_{i+1}
\]

i.e. reaching \( s_{i+1} \) while ignoring all the shiftable propositions \( \varphi_h \land \neg \psi_h \) which verify \( R^*_i(\varphi_h \land \neg \psi_h) > s_{i+1} \)

\[ R^*_{i+1} = PM(R_{i+1}, \Delta) \text{ and } \Delta_{i+1} = \Delta_i \cup \Delta'_{i+1} \]

**Stop:** If \( s_{i+1} \) does not exist, then \( R^z_{\Delta} = R_{i+1} = R_i. \)
Examples I

Nested crossing:

\[ \Delta_{nc} = \{ T \leadsto a, T \leadsto b, T \leadsto r, T \leadsto s, T \rightarrow (a \land b \leftrightarrow r \land s) \} \]

\[ R_0^* = PM(R_0, \Delta) = 1[\neg a \lor \neg b] + \infty[\neg(a \land b \leftrightarrow r \land s)] \]

\[ R_0^*(\neg a) = ... = R_0^*(\neg s) = 1 < \infty = R_0^*(\neg(a \land b \leftrightarrow r \land s)) \]

hence \( s_1 = 1 \)

The lex-length-minimal coeff. \( a_i \) s.t. for \( R_1 = a_1[\neg a] + ... + a_4[\neg s] \)
we have \( R_1 + \infty[...] (\neg a), ..., R_1 + \infty[...] (\neg s) \geq s_1 = 1 \) are \( a_i = 1/2 \)
Examples II

\[ \Delta_1 = \Delta'_1 = \{ \text{T} \sim a, \text{T} \sim b, \text{T} \sim r, \text{T} \sim s \} \]

\[ R_1^* = PM(R_1, \Delta) = R_1 + \infty[\neg(a \wedge b \leftrightarrow r \wedge s)], \text{ now } s_2 = \infty \]

The lex-length-minimal coefficient for \( \neg(a \wedge b \leftrightarrow r \wedge s) \) is \( \infty \)

\[ \Delta_2 = \Delta, \text{ hence induction stops here and} \]

\[ R^{jz}_{\Delta_{nc}} = 1/2[a] + ... + 1/2[s] + \infty[\neg(a \wedge b \leftrightarrow r \wedge s)] \]

is justifiably constructible.

**Observe:** The symmetries of \( \Delta_{nc} \), justifiable constructibility, and canonicity are enough to fix the result

**Note:** \( R^{jz}_{\Delta_{nc}}(\neg a \wedge \neg b) = 3/2 \) - thus we need rational rkm-values!
Examples III

**Big Birds Hammer light:** Birds are normally small, birds can normally fly, big birds are normally unable to fly

\[
\Delta_{bbl} = \{b \leadsto f, b \leadsto s, b \land \neg s \leadsto \neg f\}, \quad \Sigma_{bbl} = \{b, \neg s\}
\]

\[
R_0^* = R_{\Delta_{bbl}}^z = 1[b \land \neg f] + 2[b \land f \land \neg s] \quad \text{(just to describe it)}.
\]

Hence, in the first round we will only shift \(b \land \neg f\) and \(b \land \neg s\), ignoring the less plausible part \(b \land \neg s \land f\). We have \(s_1 = 1\) and \(\Delta_1 = \{b \leadsto f, b \leadsto s\}\).

Shifting \(b \land \neg s\) is then redundant and \(R_1 = 1[b \land \neg f] + 0[b \land \neg s]\).

\(R_1^*\) is now just \(R_1 + 2[b \land f \land \neg s]\), which puts \(b \land f \land \neg s\) to 2, the next target rank is thus \(s_2 = 2\). Hence \(R_2 = R_1 + 2[b \land f \land \neg s]\).

Because \(\Delta_2 = \Delta_{bbl}\), we stop and \(R_{\Delta_{bbl}}^{jz} = 1[b \land \neg f] + 2[b \land f \land \neg s] \)
While \( \{b, \neg f\} \cup \{b \rightsquigarrow f, b \rightsquigarrow s\} \models^j s, \)
we have \( \{b, \neg f\} \cup \{b \rightsquigarrow f, b \rightsquigarrow s, b \land \neg s \rightsquigarrow \neg f\} \not\models^j s, \neg s \)
Properties and principles

System JZ verifies:

• all KLM postulates
• Irrelevance principle (and exceptional inheritance) (IRR)
• Representation/Language Independence (RI)
• Local default equivalence

System JZ violates:

• LLE for defaults

But this is actually unavoidable if we insist on IRR and RI
Desirable inference

Let us call a default inference notion desirable iff:

- **Supraclassicality for** $\models w.r.t. \vdash: \vdash \subseteq \models$  
- **Basic nontriviality:** $\{\neg \varphi, T \models \varphi\} \not\models \varphi$ if $\varphi \not\models F$, $\neg \varphi \not\models F$
- **Representation invariance for** $\models$:
  for semantically invariant boolean isomorphisms $f : L \to L$
  $\Gamma(\varphi) \models \psi$ iff $\Gamma(f(\varphi)) \models f(\psi)$
- **LLE for defaults:** $\models_\Delta = \models_{\Delta'}$ if $\Delta \models_{rk} \Delta'$
- **Exceptional inheritance:**
  $\{\neg a\} \cup \{T \models a, T \models b\} \models b$ for logically independent $a, b$
  (follows from Irrelevance and and Representation invariance)
Exceptional inheritance paradox

Theorem: There are no desirable default inference notions!

What can we do? What to violate?

- Supraclassicality/nontriviality: untouchable cornerstones of default reasoning
- Representation invariance: conditio sine qua non for semantic-based approaches
- LLE for defaults: only very weak conditional logics for defaults
- Exceptional inheritance: only very weak inheritance patterns: e.g. System Z