Several classes of operations $f : A^n \to A$ admit a decomposition scheme

$$f(x) = \Pi(x_k, f(x^1_k), f(x^0_k)), \quad k \leq n, \quad (1)$$

where $\Pi : A^3 \to A$ and $\{0, 1\} \subseteq A$, and where $x^c_k$ denotes the $n$-tuple obtained from $x$ by substituting its $k$-th component by $c \in A$. Shannon decomposition (or Shannon expansion) [2,5] is an instance of (1) for Boolean functions, where $\Pi(x, y, z) = xy + (1 - x)z$. More recent examples include the class of polynomial operations over a bounded distributive lattice. They are decomposable [3] as in (1) for $\Pi(x, y, z) = (x \land y) \lor (x \land z) \lor (y \land z)$.

More generally, if $\Pi : A^3 \to A$, we say that a class of operations on $A$ is $\Pi$-decomposable [4] if it is the class of those operations that satisfy (1). In this talk, we investigate those $\Pi$-decomposable classes of operations that constitute clones. The results presented in this talk are published in [1].

References