Universal scaling of quench-induced correlations in a one-dimensional channel at finite temperature

A. Calzona\textsuperscript{1,2,3,*}, F. M. Gambetta\textsuperscript{1,2}, M. Carrega\textsuperscript{4}, F. Cavaliere\textsuperscript{1,2}, T. L. Schmidt\textsuperscript{3}, M. Sassetti\textsuperscript{1,2}

1 Dipartimento di Fisica, Università di Genova, Via Dodecaneso 33, I-16146, Genova, Italy.
2 SPIN-CNR, Via Dodecaneso 33, I-16146, Genova, Italy.
3 Physics and Materials Science Research Unit, University of Luxembourg, 162a avenue de la Faîencerie, L-1511 Luxembourg.
4 NEST, Istituto Nanoscienze-CNR and Scuola Normale Superiore, Piazza San Silvestro 12, I-56127 Pisa, Italy.
* calzona@fisica.unige.it

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Abstract

We investigate the influence of thermal effects on the relaxation dynamics of a one-dimensional quantum system of interacting fermions subject to a sudden quench of the interaction strength. It has been shown that quantum quench in a one-dimensional interacting system induces entanglement between counter-propagating excitations, whose signature is reflected in finite two-point bosonic cross-correlators. At zero temperature, their relaxation dynamics is governed by a universal power law $\propto t^{-2}$, whose behavior can be detected in transport properties. Here, we consider the system initially prepared in a thermal state and we demonstrate that these quench-induced features are stable and robust against thermal effects. Remarkably, we argue that the long-time dynamics of the current injected from a biased probe still exhibits a universal power law relaxation $\propto t^{-2}$, even at finite temperature. This result is in sharp contrast with the non-quenched case, for which the current features a fast exponential relaxation towards its steady value, and thus represents a fingerprint of quench-induced dynamics.

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1 Introduction

Among all open problems in the field of quantum many-body systems, of special interest is the study of real-time dynamics far from equilibrium. In this context, recent state-of-the-art experiments performed on cold atoms [1, 2, 3, 4, 5, 6, 7] have shown the possibility to modulate in time various parameters and to detect transport properties as a useful tool to investigate the system response [8, 9, 10, 11]. Due to their high degree of tunability, cold atoms represent an ideal platform to study quantum quenches [12, 13, 14], which consist in the rapid variation over time of one of the system parameters in a controlled fashion. Such protocol naturally settles an out-of-equilibrium state whose subsequent time evolution will be highly non-trivial. Non-equilibrium physics of 1D systems and time-resolved dynamics has been also recently investigated in pioneering experiments in solid-state implementation, see for example [15, 16, 17, 18, 19].

When dealing with out-of-equilibrium systems, two important questions arise: do such systems eventually settle to a steady state in the long run? And if so, what characterizes their relaxation dynamics? Several theoretical studies [20, 21, 22] and experiments have addressed such topics, providing non-trivial results which strongly depend on the system considered. For example, some cold atom systems have shown a faster-than-expected relaxation towards a state well described by a grand-canonical ensemble [5]. Quantum quench experiments performed splitting a one-dimensional (1D) gas of cold atoms [4], initially prepared in a thermal state, have shown the system to settle towards a quasi-thermal state, which can be well-described in terms of an effective temperature lower than the initial one. On the contrary, other cold-atom implementations have reported relaxation towards a steady state which cannot be described as a thermal one [2, 23]. In this respect, integrable systems are particularly interesting, since it has been conjectured that, at least for local observables, they approach a steady state that can be characterized by the so-called generalized Gibbs ensemble [24, 25], whose associated density matrix is in general very different from the thermal one. However, a complete characterization of their relaxation dynamics is yet to be found.

Among all 1D integrable systems, a special role is played by Luttinger liquids [26, 27] which have been detected in several experiments [4, 6, 7, 28, 29]. Indeed, they are characterized by an infinite number of local conserved quantities which allow to promptly construct their generalized Gibbs ensemble. Moreover, the unavoidable presence of interactions give rise to several intriguing effects including charge and spin fractionalization [16, 17, 18, 27, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39], which have been also investigated in the presence of a quantum quench [40, 41]. Relaxation dynamics of Luttinger liquids have been the subject of recent theoretical activity [12, 14, 42, 43, 44, 45, 46, 47], even at finite temperature [48, 49, 50]. In a recent paper [41], the relaxation dynamics of a Luttinger liquid subjected to a sudden quench of the interaction strength has been studied at zero temperature. It has been shown that this generates entanglement between counter-propagating excitations [41, 51], resulting in finite values of bosonic cross-correlators of the 1D channel. This, in turn, affects the time evolution of local observables such as the charge and the energy currents locally injected from...
a biased probe. Indeed, in the long run they display a universal power-law relaxation dynamics $\propto t^{-2}$, which is remarkably insensitive to the details of the quench.

In this paper, we investigate whether such a universal power law survives when the system is prepared, before the quench, in a thermal state at finite temperature. We find that even in such a situation, a quantum quench induces non-vanishing cross-correlations between counter-propagating excitations at any finite time. We demonstrate that they show the same universal power law $\propto t^{-2}$ as in the $T=0$ case, with thermal corrections only resulting in a modification of the prefactor of such a power law. These properties determine the relaxation dynamics of the charge current injected from a biased probe, which displays a universal scaling law $\propto t^{-2}$ robust against thermal effects. Notably, this is in sharp contrast with a conventional non-quenched situation, in which a finite temperature induces a fast non-universal exponential decay towards the equilibrium value.

The outline of the paper is the following. In Sec. 2 the model is described. In Sec. 3 the fermionic Green functions, central to the evaluation of the transport properties, are introduced. Their basic building blocks, the bosonic two-point correlation functions are also defined and evaluated, and their time dynamics is analyzed in detail. In Sec. 4 we consider a possible transport setup, where a probe is assumed to be tunnel coupled to the system, and we evaluate the resulting charge current and its time-evolution after the quench. Section 5 contains our conclusions.

## 2 Model

We consider an interacting 1D channel of spinless fermions with short-range repulsive interactions. At times $t<0$, they are governed by the Hamiltonian (in all the paper, $\hbar = k_B = 1$)

\begin{equation}
H_i = v \sum_{r=R,L} \partial_r \int_{-\infty}^{+\infty} dx \, \psi_r(x)(-i\partial_x)\psi_r(x) + H_i^{(\text{int})},
\end{equation}

with $v$ the Fermi velocity, $\psi_r(x)$ the fermion field of the $r$-channel ($r = L/R$ for left-/right branches), and $\partial_{R/L} = +/-$. The interaction term is given by

\begin{equation}
H_i^{(\text{int})} = \frac{g_i^{(4)}}{2} \sum_r \int_{-\infty}^{+\infty} dx \left[ n_r(x) \right]^2 + g_i^{(2)} \int_{-\infty}^{+\infty} dx \, n_R(x)n_L(x),
\end{equation}

where $n_r(x) =: \psi_r(x)\psi_r(x)$ is the particle density on the $r$-channel and $g_i^{(2)}$, $g_i^{(4)}$ are the coupling strengths of the intra- and inter-channel interactions respectively \[26, 27\]. For simplicity, here and in the following we will consider the Galilean invariant case $g_i^{(2)} = g_i^{(4)} = g_i$. This fact has no relevant consequence, as the more general case $g_i^{(2)} \neq g_i^{(4)}$ leads to the same qualitative results discussed here.

The non-interacting part of $H_i$ can be diagonalized in terms of free bosonic fields $\phi_r(x)$, related to the fermionic field operator by means of the bosonization identity \[26, 27\]

\begin{equation}
\psi_r(x) = \frac{1}{\sqrt{2\pi a}} e^{-i\sqrt{2\pi} \phi_r(x)} e^{i\partial_r x},
\end{equation}

where...
where $a$ is a short-length cutoff, $q_F$ is the Fermi wave vector and we have omitted Klein factors. The Hamiltonian in Eq. (1) can then be fully diagonalized introducing new bosonic operators $\phi_i,\pm(x)$, connected to $\phi_r(x)$ by the canonical transformation

$$\phi_r(x) = \sum_{\eta=\pm} B_{\eta}\phi_i,\eta(x),$$

with $2B_{\pm} = K_i^{-1/2} \pm K_i^{-1/2}$ and

$$K_i = \sqrt{\frac{1}{1 + \frac{g_i}{\pi v}}} ,$$

a parameter encoding the interaction strength, with $0 < K_i \leq 1$ and $K_i = 1$ for non-interacting fermions. The Hamiltonian $H_i$ then reads

$$H_i = \frac{v_i}{2} \sum_{\eta=\pm} \int^{+\infty}_{-\infty} dx \ [\partial_x \phi_i,\eta(x)]^2 ,$$

where $v_i = v/K_i$ is the renormalized propagation velocity of the collective excitations described by the chiral fields $\phi_i,\eta(x, t) = \phi_i,\eta(x-\eta v_i t)$. At times $t < 0$ the system is prepared in a thermal state described by the equilibrium density matrix $\rho_{eq} = Z^{-1} e^{-H_i/T}$, where $Z = \text{Tr}\{e^{-H_i/T}\}$ is the system partition function.

At time $t = 0$, the bath is disconnected and the interaction strength is suddenly quenched $g_i \to g_f (K_i \to K_f)$, resulting in a sudden switch of the Hamiltonian $H_i \to H_f$ with

$$H_f = \frac{v_f}{2} \sum_{\eta=\pm} \int^{+\infty}_{-\infty} dx \ [\partial_x \phi_f,\eta(x)]^2 .$$

Here, $v_f = v/K_f$ and $K_f = [1 + (g_f/\pi v)]^{-1/2}$ is the new Luttinger parameter describing the interaction strength in the post-quench state. The fields $\phi_i,\eta(x)$ and $\phi_f,\eta(x)$ are connected by the canonical transformation

$$\phi_f,\eta(x) = \sum_{\ell=\pm} \theta_{\ell}\phi_i,\ell(x), \quad \text{with} \quad \theta_{\pm} = \frac{1}{2} \left[ \sqrt{\frac{K_i}{K_f}} \pm \sqrt{\frac{K_f}{K_i}} \right] .$$

Note that the non-quenched case ($K_i = K_f$) is represented by $\theta_- = 0$ and $\theta_+ = 1$. The fermionic field after the quench can be expressed as

$$\psi_r(x, t) = \frac{1}{\sqrt{2\pi a}} \exp \left\{ -i\sqrt{2} \sum_{\eta} A_{\eta}\phi_r,\eta(x, t) + i\vartheta_r q_F x \right\} ,$$

with

$$A_{\pm} = \frac{1}{2} \left[ \frac{1}{\sqrt{K_f}} \pm \sqrt{K_f} \right] .$$
3 Fermionic and bosonic correlation functions

In the following we will discuss the lesser Green functions of the 1D channel. These are defined as

\[ G_r(t, t - \tau) = i \langle \psi_r^\dagger(t - \tau) \psi_r(\tau) \rangle_{eq}, \quad (r = L, R) \]  \hspace{1em} (11)

where the fermionic fields are evaluated at the same point in space (not written here for notational convenience). The brackets \( \langle \ldots \rangle_{eq} \) denotes a quantum average performed on the initial thermal density matrix \( \rho_{eq} \). Note that the quantum quench breaks the time-reversal symmetry and, therefore, all two-point correlation functions depend in principle on two time coordinates \( t \) and \( \tau \), the absolute and relative one respectively.

Using the bosonization identity of Eq. (3) the Green functions can be written as

\[ G_r(t, t - \tau) = i 2\pi a \exp \left\{ \pi \left[ A^2_{\bar{\theta}_r} D_{+,+}(t, \tau) + A^2_{\bar{\theta}_r} D_{-,+}(t, \tau) + 2A_+A_- D_{+,+}(t, \tau) \right] \right\}, \]  \hspace{1em} (12)

where the local, finite-temperature two-point bosonic correlation functions have been introduced

\[ D_{\alpha,\beta}(t, \tau) = 2\langle \phi_{f,\alpha}(0, t - \tau) \phi_{f,\beta}(0, t) \rangle_{eq} - \langle \phi_{f,\alpha}(0, t - \tau) \phi_{f,\beta}(0, t) \rangle_{eq} - \langle \phi_{f,\alpha}(0, t) \phi_{f,\beta}(0, t) \rangle_{eq}, \]  \hspace{1em} (13)

with \( \alpha, \beta = \pm \) and the bosonic fields evaluated at same point in space. Since these bosonic correlation functions lie at the heart of transport properties, we will devote the rest of this Section to their analysis.

We will concentrate on the regime \( \tau < t \), which is the relevant one for the study of transport properties. The time evolution of all the bosonic fields appearing in Eq. (13) is thus dictated by \( H_f \) and, exploiting their chirality, this allows to rewrite Eq. (13) in terms of equal-time correlators at \( t = 0 \). Using Eq. (8), it is possible to express all the correlators in terms of the initial bosonic fields \( \phi_{i,\eta}(x, t) \) which satisfies

\[ \langle \phi_{i,\pm}(x, 0) \phi_{i,\pm}(0, 0) \rangle_{eq} - \langle \phi_{i,\pm}^2(0) \rangle_{eq} = \frac{1}{2\pi} \ln \left[ \frac{\Gamma(1 + T\omega_f^{-1} - iT\bar{v}_f^{-1})^2}{\Gamma(1 + T\omega_f^{-1})^2} \right] + \frac{1}{2\pi} \ln \left( 1 + iT\omega_f^{-1} \right), \]  \hspace{1em} (14)

where \( \Gamma(z) \) is the Euler Gamma function and \( \omega_f = v_f q_F \) is a cutoff frequency. Here, we have made the identification \( a = q_F^{-1} \). The whole correlation functions can be thus decomposed as

\[ D_{\alpha,\beta}(t, \tau) = D_{\alpha,\beta}^{(0)}(t, \tau) + \Delta D_{\alpha,\beta}(t, \tau) \]  \hspace{1em} (15)

with \( D_{\alpha,\beta}^{(0)}(t, \tau) \) a zero-temperature contribution and \( \Delta D_{\alpha,\beta}(t, \tau) \) corrections due to the finite-
temperature. We obtain

\[
D^{(0)}_{\alpha,\alpha}(t,\tau) = \sum_{\eta=\pm} \frac{\theta_{\eta}^2}{\pi} \log \frac{1}{1-i\eta \omega_f \tau},
\]

(16)

\[
\Delta D^{(0)}_{\alpha,\alpha}(t,\tau) = \frac{\theta_+^2 + \theta_-^2}{\pi} \log \frac{|\Gamma(1+T\omega_f^{-1}+iT\tau)|^2}{\Gamma^2(1+T\omega_f^{-1})},
\]

(17)

\[
D^{(0)}_{\alpha,-\alpha}(t,\tau) = \frac{\theta_+\theta_-}{2\pi} \log \frac{|1+4\omega_f^2(t-\tau)^2|(1+4\omega_f^2t^2)}{[1+\omega_f^2(2t-\tau)^2]^2},
\]

(18)

\[
\Delta D^{(0)}_{\alpha,-\alpha}(t,\tau) = \frac{2\theta_+\theta_-}{\pi} \log \frac{|\Gamma(1+T\omega_f^{-1}-iT(2t-\tau))|^2}{|\Gamma(1+T\omega_f^{-1}+2iT(t-\tau))||\Gamma(1+T\omega_f^{-1}+2iTt)|}.
\]

(19)

The above expressions are general and hold for all \( t \) and \( \tau \), provided that \( \tau < t \). As a general feature, we note that auto-correlators \( (\alpha = \beta) \) only depend on \( \tau \) and not on \( t \). By contrast, cross-correlators \( (\alpha = -\beta) \) feature a full dependence on both \( t \) and \( \tau \).

A direct comparison between the quenched and the non-quenched cases turns out to be particularly useful. In the latter one has \( \theta_- = 0 \) and cross-correlators vanish

\[
D^{(0)}_{\alpha,-\alpha}(t,\tau) = \Delta D^{(0)}_{\alpha,-\alpha}(t,\tau) = 0.
\]

(20)

This is a consequence of the fact that chiral fields \( \phi_{f,\eta}(x,t) \) are completely decoupled from each other in the Hamiltonian \( H_f \). Conversely, we observe that a quantum quench always leads to finite cross-correlations, see Eq. (18) and (19). At zero temperature this fact has been understood as a consequence of quench-induced entanglement between counter-propagating bosonic excitations [41, 52, 53]. Here, Eq. (19) demonstrates that these finite cross-correlations, and thus a certain amount of entanglement, survive even if the system is initially prepared in a thermal state. This important result has remarkable and peculiar consequences on the whole relaxation dynamics, as we are going to show. As for the auto-correlators, they are also affected by the quantum quench, although in a less relevant way: they get renormalized by an interaction dependent coefficient and an additional term appears in \( \Delta D^{(0)}_{\alpha,\alpha} \).

To study the relaxation dynamics of the quenched system, it is useful to derive asymptotic expressions for \( D_{\alpha,-\alpha}(t,\tau) \). In the case of \( D^{(0)}_{\alpha,-\alpha}(t,\tau) \) the relevant time scale is \( \tau \) and for \( t \gg \tau \) one finds

\[
D^{(0)}_{\alpha,-\alpha}(t,\tau) = -\frac{\theta_+\theta_-}{\pi} \left( \frac{\tau}{2t} \right)^2 + O\left( \frac{\tau^3}{t} \right).
\]

(21)

Concerning \( \Delta D_{\alpha,-\alpha}(t,\tau) \), an additional time scale \( T^{-1} \) emerges. For \( \tau \ll t \ll T^{-1} \) one has

\[
\Delta D_{\alpha,-\alpha}(t,\tau) = \frac{\theta_+\theta_-}{\pi} \left[ 2(T\tau)^2 \Gamma(1,1+T\omega_f^{-1}) - 4(T\tau)^2 \Gamma(3,1+T\omega_f^{-1})(Tt)^2 \right] + O\left( \frac{\tau^3}{t} \right) + O(Tt^3),
\]

(22)

where \( \Gamma(n,z) \) is the \( n \)-th order polygamma function, while for \( \tau \ll T^{-1} \ll t \)

\[
\Delta D_{\alpha,-\alpha}(t,\tau) = \frac{\theta_+\theta_-}{\pi} (1+2T\omega_f^{-1}) \left( \frac{\tau}{2t} \right)^2 + O\left( \frac{\tau^3}{t} \right) + O\left( \frac{T^{-1}}{t} \right)^3.
\]

(23)
Combining the above expressions and considering the reasonable temperature regime $T \ll \omega_f$, the scaling laws simplify to

$$\Delta D_{\alpha,-\alpha}(t,\tau) \approx \frac{\theta_+\theta_-}{\pi} \left\{ \frac{(2T\tau)^2\Gamma(1,1+T\omega_f^{-1}) - \left(\frac{\tau}{T}\right)^2}{2T\omega_f^{-1}\left(\frac{\tau}{T}\right)^2} \right\} \quad (\tau \ll t \ll T^{-1}),$$

$$\left(\frac{\tau}{t} \ll T^{-1} \ll t\right).$$

(24)

The above expression shows that the evolution of cross-correlation follows a universal power-law decay $\propto t^{-2}$ even in the presence of a thermal initial state, in perfect analogy with what already discussed for the $T = 0$ case [41]. The preparation of the system at finite temperatures only leads to a modification of the prefactor of the scaling law, visible at times $t \gg T^{-1}$. Thus, we can conclude that the universal decay of cross-correlators, whose presence is a fingerprint of quench-induced entanglement, is robust against thermal effects. As we shall see in the following Section, this fact reflects also on the transport properties of the system.

To confirm the validity of the asymptotic expansions outlined above, we have numerically evaluated the full cross-correlation functions in Eq. (18) and (19). Figure 1(a) shows the behavior of $\Delta D_{\alpha,-\alpha}(t,\tau)$ as a function of time for different temperatures. For short times $t \ll T^{-1}$ the thermal component of the cross-correlator saturates to a constant value while a power law decay $\propto t^{-2}$ is evident for $t \gg T^{-1}$, in accordance with Eq. (24). The full cross-correlator $D_{\alpha,-\alpha}(t,\tau)$ is instead shown in Fig. 1(b) for the same temperatures shown in the previous Figure. For the case of the two lowest temperatures considered, the power law $\propto t^{-2}$ can be observed in both regimes of short (but still $\tau < t$) and long ($t \gg T^{-1}$) times. This fully agree with the expansions in Eq. (24). Deviations only occur in a narrow window of times roughly centered around $t = T^{-1}$.

Therefore, we can conclude that for a quenched system the bosonic cross-correlators, signaling
the entanglement of counter-propagating excitations induced by the quench, are non-zero even at finite temperature and exhibit a universal power-law decay $\propto t^{-2}$, similarly to the $T = 0$ case.

4 Transport properties

The decay properties of the cross-correlators $D_{\alpha,-\alpha}(t,\tau)$ directly affect the transport properties of the system. The latter thus represent a promising tool to experimentally verify the results presented here. To this end, we study the injection of fermions into the channel from a tunnel-coupled non-interacting 1D probe. Fermions in the probe are described by the Hamiltonian

$$ H_p = -iv \int_{-\infty}^{+\infty} dx \left( \chi^\dagger(x) \partial_x \chi(x) \right), \tag{25} $$

with $\chi(x)$ the fermionic field of the probe, which is kept at temperature $T$ and is subject to a bias voltage $V$ with respect to the Fermi level of the channel. A localized injection at $x = x_0$ is switched on at $t = 0^+$ – immediately after the quench – and is described by the tunneling Hamiltonian [40, 41, 54, 55, 56]

$$ H_t(t) = \vartheta(t) \lambda \sum_{r=L,R} \psi^\dagger_r(x_0) \chi(x_0) + \text{h.c.}, \tag{26} $$

with $\lambda$ the tunneling amplitude and $\vartheta(t)$ a step function. We are interested into the total injected current $I(V,t) = I_+(V,t) + I_-(V,t)$, where

$$ I_\eta(V,t) = q \partial_t \int_{-\infty}^{+\infty} dx \langle \delta n_\eta(x,t) \rangle \tag{27} $$

is the chiral current of the channel, with $q$ the fermion charge. Here,

$$ \langle \delta n_\eta(x,t) \rangle = \text{Tr} \{ n_\eta(x,t) [\rho(t) - \rho_{eq}(0)] \} \tag{28} $$

is computed in the interaction picture with respect to $H_t$, with the chiral particle density defined as

$$ n_\eta(x,t) = -n \sqrt{\frac{K_f}{2\pi}} \partial_x \phi_{f,\eta}(x - \eta v_f t). \tag{29} $$

At the lowest order in the tunneling, the total current can be written as [41]

$$ I(V,t) = 2q|\lambda|^2 \text{Re} \left\{ i \int_0^t d\tau \left\{ \sum_{r=L,R} G_r^<(t,t-\tau) \right\} G_p^>(\tau) \sin(qV\tau) \right\}, \tag{30} $$

where $G_r^<(t,t-\tau)$ is the lesser Green function of the channel defined in Eq. [41] and

$$ G_p^>(\tau) = -\frac{i}{2\pi a} \left| \frac{\Gamma(1 + T \omega_f^{-1} K_f^{-1} + iT\tau)}{\Gamma^2(1 + T \omega_f^{-1} K_f^{-1})} \right|^2 \frac{1}{1 - i \omega_f K_f \tau}. \tag{31} $$
is the greater Green function of the probe. Plugging Eq. (12), with $D_{\alpha,\beta}(t, \tau)$ given in Eqs. (16-19), and Eq. (31) into Eq. (30) one can eventually write the total current as

$$I(V, t) = \frac{4g|\lambda|^2}{(2\pi a)^2} \text{Re} \left\{ \int_0^t d\tau \frac{1}{\Gamma^2(1 + T\omega_f^{-1}K_f^{-1} + iT\tau)} \frac{1}{1 - i\omega_f K_f\tau} \right. \times \left\{ \frac{\Gamma(1 + T\omega_f^{-1} + iT\tau)}{\Gamma^2(1 + T\omega_f^{-1})} \right\}^{\nu_+ + \nu_-} \left[ 1 - i\omega_f\tau \right]^{\nu_+} \left[ 1 + i\omega_f\tau \right]^{\nu_-} \times \left[ (1 + 4\omega_f^2(1 + 4\omega_f^2t^2)) \right]^{-\gamma} \times \left[ \frac{\Gamma(1 + T\omega_f^{-1} - iT(2t - \tau))}{\Gamma(1 + T\omega_f^{-1} + 2iT(2t - \tau))} \right]^{-4\gamma} i\sin(qV\tau) \right\}, \tag{32}$$

where $\nu_\pm = \theta_\pm^2 (A_+^2 + A_-^2)$ and $\gamma = -A_+ A_- \theta_+ \theta_-$. Equation (32) is exact for all temperature regimes and it is thus suitable for numerical investigation. Useful analytical expansions in the long-time limit can be obtained by observing that the integrand actually contributes to the integral only in two well separated regions near the boundary of the integration domain, i.e. for $\tau \approx 0$ and $\tau \approx t$. Indeed, in the central region $0 \ll \tau \ll t$ both $G_r^0(t, t - \tau)$ and $G_p^0(\tau)$ are slowly varying and the presence of the oscillating factor $\sin(qV\tau)$ makes the contribution of this region to the integral essentially negligible as long as $qVt \gg 1$. As a consequence, in the long-time limit the current decomposes into two terms

$$I(V, t) \approx I_A(V, t) + I_B(V, t), \tag{33}$$

the former stemming from an expansion of the integrand for $\tau \to 0$ and the latter from an expansion for $\tau \to t$. In particular, one finds

$$I_A(V, t) = I^\infty(V) + \frac{\kappa(t)}{q^2} \frac{\gamma}{\kappa^2} q^2 \partial_V I^\infty(V), \tag{34}$$

where

$$\kappa(t) = \begin{cases} \frac{1}{T\omega_f^{-1}} & (t \ll T^{-1}) \\ T\omega_f^{-1} & (t \gg T^{-1}) \end{cases} \tag{35}$$

and the steady-state current $I^\infty(V)$ reads

$$I^\infty(V) = \frac{4g|\lambda|^2}{(2\pi a)^2} \text{Re} \left\{ \int_0^t d\tau \frac{1}{\Gamma^2(1 + T\omega_f^{-1}K_f^{-1} + iT\tau)} \frac{1}{1 - i\omega_f K_f\tau} \right. \times \left\{ \frac{\Gamma(1 + T\omega_f^{-1} + iT\tau)}{\Gamma^2(1 + T\omega_f^{-1})} \right\}^{\nu_+ + \nu_-} \left[ 1 - i\omega_f\tau \right]^{\nu_+} \left[ 1 + i\omega_f\tau \right]^{\nu_-} \times \left[ \frac{\Gamma(1 + T\omega_f^{-1} - iT(2t - \tau))}{\Gamma(1 + T\omega_f^{-1} + 2iT(2t - \tau))} \right]^{-4\gamma} i\sin(qV\tau) \right\}. \tag{36}$$

As for the term $I_B(V, t)$, it is useful to introduce $z = t - \tau$ and expand the integrand of Eq. (32), retaining only the leading term in $z/t$. One eventually obtains

$$I_B(V, t) = \Phi_1(V, t)\Phi_2(t) \tag{36}$$
where
\[ \Phi_1(V, t) = \frac{-4q|\lambda|^2}{(2\pi a)^2 K_f} \cos \left[ \frac{x}{2} (\nu_+ - \nu_-) \right] \int_0^{+\infty} dz \frac{\Gamma(1 + T \omega_f^{-1} + 2iTz)}{[1 + (2\omega_f^2 z^2)]^{\gamma}} \sin[qV(t - z)], \]  
(37)
is a bounded oscillating function of time. The decay in time of \( I_B(V, t) \) is therefore entirely governed by
\[ \Phi_2(t) = \frac{2^{-2\gamma}}{(\omega_f t)^{1+\nu_+ + \nu_- - 2\gamma}} \left[ \frac{\Gamma(1 + T \omega_f^{-1} K_f^{-1} + iTt)}{\Gamma^2(1 + T \omega_f^{-1} K_f^{-1})} \right]^{\gamma + \nu_-} \times \left[ \frac{\Gamma(1 + T \omega_f^{-1} - iTt)}{\Gamma(1 + T \omega_f^{-1} + 2iTt)} \right]^{-2\gamma} \]
(38)
which, exploiting the expansion
\[ \left[ \frac{\Gamma(1 + T \omega_f^{-1} + iTt)}{\Gamma^2(1 + T \omega_f^{-1})} \right]^{\gamma + \nu_-} \approx 2\pi(Tt)^{1+2T\omega_f^{-1}e^{-\pi Tt}} e^{-\pi Tt} \quad \text{for} \quad t \gg T^{-1}, \]
(39)
can be approximated as
\[ \Phi_2(t) \approx \left\{ \begin{array}{ll}
2^{2\gamma T \omega_f^{-1}} (2\pi T \omega_f^{-1})^\rho \Gamma(1 + T \omega_f^{-1} K_f^{-1})^{-2} \left[ \Gamma(1 + T \omega_f^{-1}) \right]^{-\nu_+ - \nu_-} \\
\times (Tt)^{2T \omega_f^{-1} K_f^{-1}} e^{-\pi T(1+\nu_+ + \nu_-) t} 
\end{array} \right\} \]
(40)
for \( t \ll T^{-1} \), and
\[ \Phi_2(t) \approx \left\{ \begin{array}{ll}
2^{2\gamma T \omega_f^{-1}} (2\pi T \omega_f^{-1})^\rho \Gamma(1 + T \omega_f^{-1} K_f^{-1})^{-2} \left[ \Gamma(1 + T \omega_f^{-1}) \right]^{-\nu_+ - \nu_-} \\
\times (Tt)^{2T \omega_f^{-1} K_f^{-1}} e^{-\pi T(1+\nu_+ + \nu_-) t} 
\end{array} \right\} \]
(41)
for \( t \gg T^{-1} \).

Here, \( \rho = 1 + \nu_+ + \nu_- - 2\gamma \geq 2 \) for any \( 0 < K_{i,f} \leq 1 \).

Summarizing what we have obtained so far, the long-time evolution of the current \( I(V, t) \) towards its steady value \( I^\infty(V) \) consists of two regimes. For \( t \ll T^{-1} \), the transient of the injected current reads
\[ I(V, t) - I^\infty(V) \approx \gamma \frac{\omega_f^2}{2q^2} \frac{\partial^2 I^\infty(V)}{\partial t^2} + \frac{1}{(t\omega_f)^2} \Phi_1(V, t) \left[ 2\pi(Tt)^{1+2T\omega_f^{-1}} e^{-\pi Tt} \right]^{-2\gamma} \frac{1}{(t\omega_f)^\rho}. \]
(41)

The leading term is the universal power-law decay \( \propto t^{-2} \), whose origin traces back to the quench-induced entanglement. The sub-leading term is a power law with a non-universal exponent \( \rho > 2 \), multiplied by an oscillating factor \( \Phi_1(V, t) \) whose period is proportional to \( (qV)^{-1} \). For reasonable quenches, \( \rho \) is still close to 2 and therefore the competition between the two decays is strong. The behavior described in Eq. (41) is qualitatively similar to the \( T = 0 \) case discussed in Ref. [31][37]. On the other hand, in the regime \( T^{-1} \ll t \) the transient of the current reads
\[ I(V, t) - I^\infty(V) \approx T \gamma \frac{\omega_f^2}{q^2} \frac{\partial^2 I^\infty(V)}{\partial t^2} \left[ \frac{1}{(t\omega_f)^2} \right] \Phi_1(V, t) \left( Tt \right)^{2T \omega_f^{-1} (1+K_f^{-1})} e^{-\pi T(1+\nu_+ + \nu_-) t}. \]
(42)
Figure 2: Solid lines show $|\delta I(V,t)|$ (units $4q^2|\lambda|^2 \omega_f v_f^{-2}$) as a function of time in a quenched system with $K_i = 0.9$ and $K_f = 0.6$. Crosses show $|\delta I(V,t)|$ in a non-quenched system with $K_i = K_f = 0.6$. Different colors refer to different temperatures: $T = 10^{-1}$ (blue), $T = 10^{-2}$ (orange), $T = 10^{-3}$ (green). The inset displays $|\delta I(V,t)|$ in the absence of quench with $K_i = K_f = 0.6$ and $T = 10^{-3}$ on a logarithmic plot. The black dashed line shows an exponential decay $\propto \exp[-\pi T t (1 + \nu_+ + \nu_-)]$. Here $qV = 0.1 \omega_f$, time is in units $\omega_f^{-1}$, and temperature in units $\omega_f$.

Remarkably, the leading term is still represented by the universal power law $\propto t^{-2}$, a fingerprint of quench-induced entanglement, while the sub-leading one features a temperature-dependent exponential decay

$$\sim e^{-\pi T t (1 + \nu_+ + \nu_-)}$$

and it is therefore negligible with respect to the leading one.

All these features clearly emerge in Fig. 2 where we plotted on a bi-logarithmic scale the numerically evaluated $|\delta I(V,t)| = |I(V,t) - I^\infty(V)|$ for different temperatures, a bias $qV = 0.1 \omega_f$ and a quench with $K_i = 0.9$ and $K_f = 0.6$ (solid lines). It is easy to identify the existence of the two regimes $t \ll T^{-1}$ and $T^{-1} \ll t$. The former features lots of oscillations as a result of the strong competition between the universal power-law decay and the non-universal one. Note that the quench considered here leads to a non-universal exponent $\rho \simeq 2.3$. By contrast, the latter regime displays a very clean decay $\propto t^{-2}$, resulting from the exponential suppression of the non-universal oscillating terms.

The presence of the universal power-law decay in the regime $t \gg T^{-1}$ is rather peculiar of the quench-induced dynamic and therefore represents an important fingerprint of it. To stress this point, it is interesting to discuss how Eq. (41) and (42) look like in the non-quench case. Since $\theta_- = 0$ implies $\gamma = 0$, the universal terms disappear while the non-universal ones are not qualitatively modified. Therefore, the regime $t \gg T^{-1}$ in a non-quenched system features a fast exponential decay, which can be easily distinguished from the universal power law induced by a quench. This fact distinctly emerges in Fig. 2 where we superimposed crosses showing $|\delta I(V,t)|$ in the non-quenched case with $K_i = K_f = 0.6$ (note that, to improve the readability of the Figure, the sampling rate is kept very low). A more detailed plot of the
Figure 3: (a) Density plot of $|\mathcal{F}(V,T)|$ (units $4q\gamma|\lambda|^2\omega_f v_f^{-2}$) as a function of the bias voltage $qV$ (units $\omega_f$) and the temperature $T$ (units $\omega_f$). (b) Plot of the transient current $|\delta I(V,t)|$ (units $4q|\lambda|^2\omega_f v_f^{-2}$) as a function of time, for temperature $T = 10^{-3}\omega_f$ and for two different bias voltage: $qV = 0.1\omega_f$ (solid line) and $qV = 0.004\omega_f$ (dotted line). In both Panels, the quench is from $K_i = 0.9$ to $K_f = 0.6$.

non-quenched current is presented in the inset for $T = 10^{-3}\omega_f$, using a logarithmic scale. Here the oscillations of this non-universal term are clearly visible as well as its exponential decay, which is in good agreement with Eq. (43) (dashed black line).

Finally, the $t \gg T^{-1}$ regime is the most interesting one in order to assess the different time evolution of quenched and non-quenched systems at finite temperature. It is therefore worth to discuss how the prefactor

$$\mathcal{F}(V,T) = T\gamma\frac{\omega_f^2 I^\infty(V)}{q^2}$$

of the universal power law depends on the temperature $T$ and on the bias $V$ in this regime. In Fig. 3(a), we plot $|\mathcal{F}(V,T)|$ focusing on a reasonable range of the parameters. Interestingly, this function turns out to be non-monotonic with respect to both $T$ and $V$. This fact leads to an unexpected feature: considering temperatures lower than $10^{-2}\omega_f$, a decrease of the voltage bias can actually determine an increase by orders of magnitude of the prefactor $|\mathcal{F}(V,T)|$. As an example of this behavior, in Fig. 3(b) we plotted the current $|\delta I(V,t)|$ as a function of time for two different voltage biases but with the same temperature $T = 10^{-3}\omega_f$. In the $t \gg T^{-1}$ regime, the dotted line ($qV = 0.004\omega_f$) lies more than one order of magnitude above the solid one ($qV = 0.1\omega_f$). Therefore, a proper tuning of the voltage bias can be useful to magnify and to detect the peculiar features induced by the presence of the quantum quench.

5 Conclusions

In this paper we have investigated the relaxation dynamics of a 1D channel of spinless interacting fermions, initially prepared in a thermal state at $T > 0$, subject to a sudden quench
of the interaction strength. The system is modeled as a Luttinger liquid. The two-point correlation functions between the chiral bosonic fields, a key quantity to evaluate transport properties, are analytically evaluated and studied in details. One of the main results is that, even in the presence of a finite temperature, entanglement between counter-propagating excitations occurs, leading to non-zero cross-correlators between the chiral bosonic fields. They decay in time following a universal power law $\propto t^{-2}$ which has proven to be stable against thermal effects. The latter only leads to a renormalization of its prefactor in the long-time limit $t \gg T^{-1}$. The universal scaling of correlation functions implies an analogous behavior in the fermionic Green functions of the channel, which determine observable quantities such as transport properties. In particular, we have analyzed in details the charge current flowing to the system from a locally tunnel-coupled biased probe. If the tunneling is switched on immediately after an interaction quench, the current decays towards its steady value with the same universal power law $\propto t^{-2}$ which characterizes the two-point cross-correlators. Remarkably, thermal effects only lead to a renormalization of the prefactor of the power law for $t \gg T^{-1}$. The persistence of such a universal power law confirms the robustness of the results previously obtained in the $T \to 0$ limit \cite{41}. Moreover, such a behavior is in sharp contrast with what happens in the non-quenched case, for which the current features a fast exponential relaxation. This remarkable difference represents a strong signature of the peculiarity of quenched-induced out-of-equilibrium states with respect to conventional thermal ones.

For a typical system of cold atoms \cite{4}, one can estimate temperatures of about $T \sim 100 \text{ nK}$, for which the thermal time scale is about $T^{-1} \sim 500 \mu s$. Typical experiments in such a system can explore time domains well in excess of the hundred of ms, so it seems feasible, at least in principle, to observe the striking differences between quenched and non-quenched systems in the presence of thermal effects.

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References


