Virtual-power-based Quasicontinuum Methods for Discrete Dissipative Materials

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Dissipative discreteness at small scales

Foams

Additive manufacturing

Collagen

Paper/cardboard

Textiles
Electronic textile
Elastoplastic spring lattice

![Elastoplastic spring lattice image]

[Graph showing engineering stress vs. engineering strain]

480 µm
Spring lattice with nodal sliding
Spring lattice with nodal sliding

\[ F_{sc} = k\ell_0 / 100 \]

\[ F_{sc} = k\ell_0 / 200 \]

\[ F_{sc} = k\ell_0 / 400 \]

\[ e_f = 8 \]

\[ e_f = 4 \]

\[ e_f = 2 \]
Quasicontinuum method (Tadmor et al, 1996)

\[ K \cdot u = f \]
Quasicontinuum method (Tadmor et al, 1996)

\[ K \cdot u = f \]

\[ \sum_{i=1}^{n} K_i \cdot u = \sum_{i=1}^{n} f_i \]
Quasicontinuum method (Tadmor et al, 1996)

\[ K \cdot u = f \]

\[ N^T \cdot K \cdot N \cdot u = N^T \cdot f \]

\[ \sum_{i=1}^{n} K_i \cdot u = \sum_{i=1}^{n} f_i \]
Quasicontinuum method (Tadmor et al, 1996)

\[
K \cdot u = f
\]

\[
N^T \cdot K \cdot N \cdot u = N^T \cdot f
\]

\[
\sum_{i=1}^{n} K_i \cdot u = \sum_{i=1}^{n} f_i
\]

\[
N^T \cdot \sum_{i=1}^{n} K_i \cdot N \cdot u = N^T \cdot \sum_{i=1}^{n} f_i
\]
Quasicontinuum method (Tadmor et al, 1996)

\[ K \cdot u = f \]

\[ \sum_{i=1}^{n} K_i \cdot u = \sum_{i=1}^{n} f_i \]

\[ N^T \cdot K \cdot N \cdot u = N^T \cdot f \]

\[ N^T \cdot \sum_{i=1}^{n} K_i \cdot N \cdot u = N^T \cdot \sum_{i=1}^{n} f_i \]

\[ N^T \cdot K_r \cdot N \cdot u = N^T \cdot f_r \]
Quasicontinuum method (Tadmor et al, 1996)

\[ K \cdot u = f \]

\[ \sum_{i=1}^{n} K_i \cdot u = \sum_{i=1}^{n} f_i \]

\[ N^T \cdot K \cdot N \cdot u = N^T \cdot f \]

\[ N^T \cdot \sum_{i=1}^{n} K_i \cdot N \cdot u = N^T \cdot \sum_{i=1}^{n} f_i \]

\[ N^T \cdot K_r \cdot N \cdot u = N^T \cdot f_r \]

\[ N^T \cdot \sum_{i=1}^{s} K_i \cdot N \cdot u = N^T \cdot \sum_{i=1}^{s} f_i \]
Quasicontinuum method (Tadmor et al, 1996)

- Ideal for local events in large-scale lattice computations
- Underlying lattice fully resolved where needed
- No continuum/constitutive assumptions
Virtual-power-based QC framework

Summation
Virtual-power-based QC framework

Summation 2
Virtual-power-based QC framework

Accuracy and efficiency

Plastic strain at 10% horizontal stretch

-0.04 0 0.12
Virtual-power-based QC framework

Bond failure and fiber sliding

99 unit cells
Results: bond failure and fiber sliding

Horizontal displacement, relative to uniform displacement
Conclusions

Virtual-power-based QC methodology

Summation: 1. exact rule
2. central rule

Dissipative effects included in QC via internal variables
- for elastoplasticity at sampling spring level
- for nodal sliding interpolated due to nonlocality
QC method for beams

QC method for irregular networks

Variational QC methods + adaptivity

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6943 sampling interactions

561 repeats

time step 0

damage 0
damage 1
Ongoing & future work

Applications: textiles, printed structures, foams

(goal-oriented) Adaptivity

Stochastics