Spatial Sparsity Based Direct Positioning for IR-UWB in IEEE 802.15.4a Channels

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Abstract—In this paper, we focus on the application of Compressive Sensing (CS) techniques to Impulse Radio (IR) Ultra-WideBand (UWB) positioning systems under indoor propagation environments. Direct Position Estimation (DPE) approaches can potentially improve the position estimation accuracy of conventional two-step techniques by directly estimating the position coordinates from the observed signal in a single step. Furthermore, DPE does not require a threshold selection upon which accuracy of two-step approaches depend on. Although in the presence of multipath the actual gains are not straightforward, recent evaluation of DPE positioning in IR-UWB systems proved accurate positioning estimate gains. However, it comes at a cost of higher computational complexity. This paper exploits the sparseness of the problem to reduce the computational load of the positioning estimation process and relax the requirements of the Analog to Digital Converter (ADC) when sampling UWB signals. Based on the fact that the number of unknown targets is small in the discrete spatial domain, this paper incorporates the multiple location hypotheses into an overcomplete basis, which highlights the sparseness of the spatial domain. This fact motivates the use of CS-based sampling and sparsity-based reconstruction techniques to jointly evaluate all possible hypotheses, thus avoiding the traditional position-by-position scanning where the multiple location hypotheses are evaluated independently. In so doing, we not only achieve a significant reduction in computational time but also we relax the sampling requirements.

Keywords—Compressive Sensing (CS), Direct Position Estimation (DPE), IR-UWB, localization.

I. INTRODUCTION

Impulse Radio Ultra-Wideband (IR-UWB) technology [1], [2] is the preferred candidate for the IEEE standardization group 802.15.4a, which specifies the physical layer for low data rate communications combined with positioning capabilities. The unique and unmatched characteristics of IR-UWB systems, namely its extremely short time domain transmitted pulses, have attracted wide interest for positioning passive nodes in wireless networks. To provide accurate position information of nodes, a number of reference nodes with known position (anchors) collect ranging information from radio signals emanating from the node with unknown position. Range accuracy depends on the narrowness of the pulse so that a pulse having a very short duration will result in the highest time resolution, which implies the highest range accuracy. The large signal bandwidth of IR-UWB not only offers outstanding ranging capabilities but also provide a means for resolving multipath indoor components. The latter is important since dense multipath environments, which are usually the propagation conditions in indoor positioning, make the shortest path delay (containing information about the distance) difficult to be accurately estimated. In fact, although Global Navigation Satellite System (GNSS) is the legacy solution for outdoor positioning, one of its major error sources is the multipath encountered in certain urban scenarios [3].

The conventional approach to solve the localization problem consists of a two-step procedure. First, some parameters of transmitted signal are measured such as Angle of Arrival (AoA), Signal Strength (SS) or Time of Arrival (ToA) [4], and second the estimated parameters are used to obtain the coordinates of unknown-location nodes by solving the geolocation problem. As demonstrated in [5], two-step procedures, which are common practice in most positioning systems, are suboptimal in general with respect to single-step schemes. The positioning approach omitting the intermediate step is known in the literature as Direct Position Estimation (DPE) [6], [7]. DPE has been applied to GNSS [8], narrowband emitters localization [6], passive geolocation [9], and UWB [10], [11] among others.

The improved performance of DPE over two-step procedures comes at the cost of higher receiver complexity. In order to overcome the computational complexity issue, sparsity-based DPE methods have been recently proposed [12], [13] which exploit the implicit sparse representation of the problem. Generally, potential targets cover a small part of the total discrete spatial domain, which allow the localization problem to be linearized over a set of possible locations by construction of an overcomplete dictionary [14] and solved using sparsity-based recovery techniques [15], [16].

In this paper, we investigate the feasibility of Compressive Sensing (CS) techniques applied to DPE for IR-UWB considering the localization paradigm where the actual processing is performed at the anchor nodes. This work extends previous authors publications [10], where a frequency domain receiver architecture was proposed for high resolution DPE based on the periodogram. The basic strategy in [10] was a grid-search evaluation of a power position profile defined as the signal energy distribution with respect to target location. Based on the fact that the number of unknown targets is small in the discrete spatial domain, this paper incorporates the multiple location hypotheses into an overcomplete basis, which highlights the sparseness of the spatial domain. This
fact motivates the use of CS-based sampling and sparsity-based reconstruction techniques to jointly evaluate all possible hypotheses. In essence, the use of an overcomplete basis avoids the position-by-position scanning where the multiple location hypotheses are evaluated independently, thus allowing not only a reduction in computational time but also to relax the strong requirements of Analog to Digital Converter (ADC) when sampling UWB signals and maintaining the location estimation accuracy.

The remainder of the paper is organized as follows. Section 2 describes the IR-UWB signal model. Section 3 reviews the conventional DPE proposed by the authors in [10] and introduces the novel CS-based DPE, which exploits the implicit spatial sparsity of the target scene. Results for simulated data are discussed in Section 4. Section 5 states the conclusions.

II. IR-UWB SYSTEM MODEL

We consider an IR-UWB system where transmission of an information symbol is typically implemented by the repetition of $N_f$ pulses of very short duration. The transmitted signal is modeled as,

$$s(t) = \sum_{k=-\infty}^{\infty} \sum_{j=0}^{N_f-1} a_j p(t - (kN_f + j)T_f - c_j T_e - b_k T_\delta)$$

where Pulse Position Modulation (PPM) is assumed with $\{b_k\}$ being the information symbols taking values $\{0, 1\}$ with equal probability. $p(t)$ refers to the single pulse waveform, being typically a Gaussian monocycle or one of its derivatives of duration $T_p$. $T_{sym} = N_f T_f$ is the symbol duration, where $T_f = N_c T_e$ and $T_\delta$ is the repetition pulse period also referred to as frame period, and $N_c$ is the number of frames per symbol, $T_e$ is the chip period, $T_\delta$ is the PPM interval, $N_c$ is the number of chips per frame and $\{c_j\}$ is the time hopping sequence which takes integer values in $\{0, 1, \ldots, N_c - 1\}$ and $a_j = \pm 1$ denotes a polarization sequence typically used for spectral shaping. Without loss of generality we assume in the sequel $a_j = 1 \forall j$.

Signal $s(t)$ propagates through an $M$-path fading channel whose response to $p(t)$ is $\sum_{m=0}^{M-1} h_m \tilde{p}(t - \tau_m)$, with $\tau_0 < \tau_1 < \ldots < \tau_{M-1}$, being $\tau_0$ the ToA that has to be estimated. The received signal is then the summation of multiple delayed and attenuated replicas of the received pulse waveform $\tilde{p}(t)$, which includes the antenna and filters distortion,

$$y(t) = \sum_{m=0}^{M-1} \sum_{j=0}^{N_f-1} h_m \tilde{p}(t - (j + T_{sym} + c_j T_e + b_k T_\delta) + w(t)$$

where $T_{sym} = j T_f + k T_{sym} + c_j T_e + b_k T_\delta$ and $w(t)$ is Additive White Gaussian Noise (AWGN). We assume that the received pulse from each $m$-th path exhibits the same waveform but experiences a different fading coefficient, $h_m$, and time delay, $\tau_m$.

The signal associated to the $j$-th transmitted pulse corresponding to the $k$-th symbol, in the frequency domain yields,

$$Y_{j,k}(\omega) = h_0 S_{j,k}(\omega) e^{-j\omega \tau_0} + \sum_{m=1}^{M-1} h_m S_{j,k}(\omega) e^{-j\omega \tau_m} + V_{j,k}(\omega)$$

where the Line-of-Sight (LOS) contribution is explicitly separated in the first term from the signal replicas associated to multipath as in [10]. The frequency component associated to the shifted pulse is given by,

$$S_{j,k}(\omega) = \tilde{P}(\omega) e^{-j\omega T_{jk}}$$

where $\tilde{P}(\omega)$ denotes the Fourier Transform of the pulse $\tilde{p}(t)$ and $V_{j,k}(\omega)$ is the noise in the frequency domain associated to the $j$-th frame interval corresponding to the $k$-th symbol. Sampling (3) at $\omega_n = \omega_0 n$ for $n = 0, 1, \ldots, N-1$ where $\omega_0 = \frac{2\pi}{N_T}$ and rearranging the frequency domain samples $Y_{j,k}[n]$ into the vector $Y_{j,k} \in \mathbb{C}^{N_T \times 1}$ yields

$$Y_{j,k} = h_0 S_{j,k} e_{\tau_0} + \tilde{V}_{j,k}$$

with $\tilde{V}_{j,k} = S_{j,k} e_h + V_{j,k}$ and $S_{j,k} \in \mathbb{C}^{N_T \times N}$ is a diagonal matrix whose components are the frequency samples of $S_{j,k}(\omega)$. The matrix $E \in \mathbb{C}^{N_M \times M-1}$ contains the delay-signature vectors associated to each arriving delayed signal (multipath).

$$E = [e_{\tau_1}, \ldots, e_{\tau_M}]$$

with $e_{\tau_m} = [1, e^{-j\omega \tau_m}, \ldots, e^{-j\omega (N-1)\tau_m}]^T$. The channel fading coefficients, except for $h_0$, are arranged in the vector $h = [h_1, \ldots, h_{M-1}]^T \in \mathbb{R}^{M-1 \times 1}$, and the noise samples in vector $V_{j} \in \mathbb{C}^{N_T \times 1}$.

III. DIRECT POSITION ESTIMATION IN IR-UWB

This section reviews the DPE approach for IR-UWB proposed by the authors in [10] and introduces the proposed CS-based DPE technique.

A. Conventional DPE

DPE attempts to solve the positioning problem on a single step allowing to perform a multidimensional search directly over the spatial coordinates. Let us define the observation frequency sample vector as the concatenation of signals received from all anchors, $Y = (Y^{(1)}, \ldots, Y^{(L)}, \ldots, Y^{(N_A)})^T$, with $Y^{(l)} = h_0 S_{l} e_{\tau_0} + \tilde{V}^{(l)}$ according to (5) (symbols and pulse indexes are dropped in order to ease notation). The system model can then be written in terms of LOS contribution as,

$$Y = S_0 e_p + \tilde{V}$$

with $S_0 = \text{diag}(h_0 S_1, \ldots, h_0 S_N)$ being a block diagonal matrix with pulse spectral components weighted by the LOS channel fading coefficients, $e_p = (e_{\tau_1}(p), \ldots, e_{\tau_N}(p))^T$ the steering vector as a function of the target spatial coordinates $p = [x, y]^T$, and $\tilde{V} = (\tilde{V}^{(1)}, \ldots, \tilde{V}^{(N_A)})^T$. The steering vectors $e_{\tau}(p)$ are defined as in (6) but in this case the delay is related to the position vector $p$ by the geometrical relation $f_l(p) = \|p - p_l\|/c_l$ with $p_l$ being the two-dimensional coordinates of the $l$-th anchor. The DPE position estimate is given by the position vector that maximizes the following cost function,

$$\hat{p} = \arg\max_p e_p^H R e_p$$

where $R = \frac{1}{N_s} \sum_{n=1}^{N_s} Y Y^H \in \mathbb{C}^{N_M \times N_M}$, being $N_s$ the total number of observed data blocks. The optimization in (8) can be performed by a grid search or other stochastic optimization methods.
B. CS-based DPE

CS theory states that a sparsely representable signal can be reconstructed using very few number of measurements compared to the signal dimension [15],[16]. Based on the sparsity of the target area, i.e., the number of unknown targets is small in the discrete spatial domain, let us consider a subset of the complete observation \( \mathbf{Y}^{(f)} \),

\[
\tilde{\mathbf{Y}}^{(f)} = \Phi \mathbf{Y}^{(f)}
\]

where \( \Phi \) is a matrix that randomly selects \( \kappa \) samples of \( \mathbf{Y}^{(f)} \) \((\kappa < N)\). This matrix \( \Phi \) is known as sampling matrix and is given by randomly selecting \( \kappa \) rows of the identity matrix \( \mathbf{I}_N \).

Assuming the periodogram defined in (8) is partitioned into a finite number of positions to be evaluated, \( N \times N \), in the X-axis and the Y-axis, the scene can be represented by the indicator function \( \sigma(k,l), k = 0, \ldots, N_x-1, l = 0, \ldots, N_y-1 \). Let \( \sigma \) be the concatenated \( N_xN_y \times 1 \) scene indicator vector corresponding to the spatial sampling grid. For the positioning problem at hand, the scene vector, \( \sigma \), and the observations are related by,

\[
\tilde{\mathbf{R}} = \sum_{i=0}^{N_xN_y-1} \sigma(p_i) \mathbf{e}_p \mathbf{e}_p^H + \tilde{\mathbf{R}}_e
\]

where \( \tilde{\mathbf{R}} \in \mathbb{C}^{\kappa N_A \times \kappa N_A} \) is the compressed sample covariance matrix computed as \( \mathbf{R} = \frac{1}{N_y} \sum_{n=1}^{N_y} \mathbf{Y} Y^H \), with \( \mathbf{Y} = \begin{bmatrix} \mathbf{Y}^{(1)} \cdots \mathbf{Y}^{(f-N)} \end{bmatrix}^T \) denoting the concatenation of compressive signals from all anchors. \( \mathbf{R} \), represents the concatenation of compressive signals from all anchors. \( \mathbf{R} \) is given by randomly selecting \( \kappa \) rows of the identity matrix \( \mathbf{I}_N \).

The model in (10) can be conveniently rewritten into a sparse notation as follows,

\[
\tilde{\mathbf{r}} = \mathbf{A} \sigma + \tilde{\mathbf{r}}_e
\]

where \( \tilde{\mathbf{r}} \) is a \((\kappa N_A)^2 \times 1\) vector formed by the concatenation of the columns of \( \mathbf{R} \). From now on, to clarify notation, the concatenation of columns will be denoted with the operator \( \mathbf{vec}(\cdot) \). Therefore, \( \tilde{\mathbf{r}} = \mathbf{vec}(\mathbf{R}) \). The columns of \( \mathbf{A} \) contain the candidate covariance matrices corresponding to the different tentative target locations \( p_i \), that conform the spatial scanning grid and is defined as,

\[
\mathbf{A} = \begin{bmatrix} \mathbf{r}(p_0) & \mathbf{r}(p_1) & \cdots & \mathbf{r}(p_{N_xN_y-1}) \end{bmatrix}
\]

where \( \mathbf{r}(p_i) = \mathbf{vec}(\mathbf{e}_p \mathbf{e}_p^H) \). The variable \( \tilde{\mathbf{r}}_e \) encompasses undesired contributions and imperfections of the model. Note that the elements different from zero vector \( \sigma = \begin{bmatrix} \sigma(p_0) & \sigma(p_1) & \cdots & \sigma(p_{N_xN_y-1}) \end{bmatrix} \) correspond to the grid point where we find periodogram values different from zero.

The sparse vector \( \sigma \) can be recovered from the compressive measurements by solving the following optimization problem,

\[
\min_{\sigma} \| \sigma \|_1 \quad \text{subject to} \quad \tilde{\mathbf{r}} \approx \mathbf{A} \sigma
\]

where \( \| \sigma \|_1 = \sum_i |\sigma(p_i)| \). Several methods are available in the literature to solve the optimization problem in (13). In this paper, we choose Orthogonal Matching Pursuit (OMP) [17] to solve (13), which is an iterative algorithm known to provide a fast and easy to implement solution. OMP iteratively computes the local optimum solutions expecting that these will lead to the global optimum solution. In iterative-based reconstruction algorithm, the stopping rule applied to halt the iterative reconstruction process plays an important role in the final estimation performance. Finding robust stopping criteria in CS-based iterative algorithms is a long standing problem [18]. Here, since we are particularly interested in the maximum (see (8)), we only need to run a single iteration which, by definition of the OMP, determines the column of \( \mathbf{A} \) that is most correlated with \( \tilde{\mathbf{r}} \), or which contributes to \( \tilde{\mathbf{r}} \) most.

IV. SIMULATION RESULTS

The algorithm has been analyzed by means of Monte-Carlo MATLAB-based simulations with the aim of evaluating the potential gains of CS-based DPE over conventional DPE approach in IR-UWB. For numerical evaluation we consider the channel models developed within the framework of the IEEE 802.15.4a [19]. In particular we evaluated the localization performance for the CM3 Office LOS and CM7 Industrial LOS channel models, whose propagation channel is characterized to exhibit dense multipath components, specially the CM3. Without loss of generality, we present preliminary results for a simplified signal model where time-hopping is not explicitly considered and \( \{b_k\} = \{0\} \forall k \). The transmitted pulse is a Gaussian monochromatic pulse with duration \( T_p = 1 \) ns. The observation window coincides with the frame duration \( T_f \) which is equal to 56 ns. The non-compressive sampling rate is given by \( 2 \) GHz, which translates into an observation window of \( N = 128 \) samples. The sampling rates of \( \tilde{\mathbf{Y}}^{(f)} \) and \( \mathbf{Y}^{(f)} \) are related through the compression rate \( \rho = \frac{\kappa}{N} \). To strictly focus on the performance behavior due to compression and remove the effect of insufficient data records, the size of the compressed observations is forced to be the same for any compression rate. Thus, for a high compression rate, the estimator takes samples for a larger period of time. The simulation parameters are summarized in Table I, where \( N_s \) denotes the number of observed data frames. The position algorithm is evaluated in a 2D setting with the target randomly placed within a square room of \( 6 \times 6 \) m\(^2\), with 4 anchor nodes placed at the corners of the room. The area of interest is divided into a \( 200 \times 200 \) grid and the resolution is 0.03m.

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>1</th>
<th>0.50</th>
<th>0.25</th>
<th>0.13</th>
</tr>
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<tbody>
<tr>
<td>( \kappa )</td>
<td>128</td>
<td>64</td>
<td>32</td>
<td>16</td>
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<tr>
<td>( N_s )</td>
<td>128</td>
<td>256</td>
<td>1024</td>
<td>2048</td>
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Fig. 1(a) and Fig. 1(b) depict the results obtained in a particular realization for a CM3 Office LOS scenario of the conventional DPE and the CS-based DPE where only 25% of the Nyquist samples have been considered for solving the positioning problem, i.e., \( \rho = 0.25 \). We considered an average SNR at each anchor node of \( 4 \) dB. Clearly, the conventional DPE cost function (8) exhibits a large number of local maxima of similar amplitude in close vicinity of the true target position, as shown in Fig. 1(a), thereby making the detection of the global maximum more difficult. The proposed CS-based DPE was applied to the reduced set of
measurements and the recovered sparse vector $\sigma$ is depicted in Fig. 1(b), where one single peak associated with the true target location (indicated by an arrow in the figure) can be easily identified. It is evident that conventional DPE approach generates many artifacts in close vicinity of the global maximum (indicated by an arrow in the figure) corresponding to the true target location. The less cluttered representation obtained with CS-based DPE underscores the importance of imposing sparsity on the solution via $l_1$ minimization.

For the evaluation of the position estimation accuracy, we consider 100 channel realizations for each scenario. The position error of conventional DPE and CS-based DPE with $\rho = 0.5$ is depicted in Fig. 2 in terms of Cumulative Density Function (CDF) for Industrial LOS (CM7) scenarios. Again, we considered an average SNR at each anchor node of 4dB. It is worth mentioning, however, that similar results were obtained for other SNRs (from -5dB to 10dB). From Fig. 2 it can be conclude that, in a large percentage (about 80%), the proposed CS-based DPE with half of the original samples provide the same localization capabilities as conventional DPE, i.e., the position of the global maximum is not affected by the reduction of data. However, as we will see later, an excessive reduction in the number of measurements translates into a degradation of the localization performance. Finding the value of $\kappa$ that ensures perfect recovery of $\sigma$ is a problem of longstanding interest in the CS community [15], [16]. Existing results in this regard showed that the minimum $\kappa$ depends on the number of non-zero components of the sparse vector $\sigma$, which in our case is a function dependent on the multipath components present in the propagation channel under consideration. In future work, we will study the relationship between the multipath and the minimum number of measurements.

To test the effect of reducing acquired samples, Fig. 3 shows the CDF of the proposed detector under CM7 Industrial LOS channel, SNR=4dB, for different compression rates. From Fig. 3 we observe that, as expected, the positioning error degrades as the compression rate decreases.

Generally, Industrial LOS (CM7) is a more friendly environment compared to Office LOS (CM3), where the much denser multipath poses additional challenges for the positioning problem. To illustrate the performance difference of these two propagation scenarios, Fig. 4 shows the periodogram obtained with DPE and the CS-based reconstructed target space with $\rho = 0.5$. This single realization is used to motivate the discussion. An exhaustive analysis would include averaging over several Monte Carlo trials as done in Fig. 2 for instance. It is in CM3 where both the conventional DPE and the proposed CS-based DPE experience larger difficulties to accurately estimate the target position due to the strong and dense multipath components. This statement is confirmed by the comparison of Fig. 4(a) and Fig. 4(b), where conventional DPE is evaluated in CM7 and CM3, respectively. The periodogram obtained with conventional DPE in CM7 have less clutter residuals compared to the corresponding CM3 periodogram, leading to
Fig. 4. Periodogram realization with 4 anchors and SNR=4dB for: (a) Conventional DPE with $\rho = 1$ evaluated in CM7 Industrial LOS, (b) Conventional DPE with $\rho = 1$ evaluated in CM3 Office LOS, (c) CS-based DPE with $\rho = 0.5$ evaluated in CM7 Industrial LOS and (d) CS-based DPE with $\rho = 0.5$ evaluated in CM3 Office LOS.

an enhanced localization performance. The ambiguity caused by the CM3 strong multipath components also affects the localization performance of CS-based DPE, as evident from the comparison of Fig. 4(c) and Fig. 4(d), where CS-based DPE is evaluated in CM7 and CM3, respectively. The estimated position obtained with CS-based DPE is located further away from the true target position when considering IEEE 802.15.4a CM3 due to the dense multipath.

V. Conclusion

A CS-based Direct Position Estimation (DPE) for IR-UWB localization based on frequency domain signal has been introduced and assessed under realistic channel models developed by the IEEE 802.15.4a standardization group. The proposed method transforms the target location estimation problem into a spatial sparse target representation optimization problem. Specifically, we show with numerical results that the proposed scheme can provide similar position estimate accuracy with less samples than a grid-based solution for DPE. Further work on the multipath effect in the CS-based strategy will be considered in the future as well as more detailed comparison with other localization approaches such as two-step procedures.

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