A quantitative verification framework of SysML activity diagrams under time constraints

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\textbf{A B S T R A C T}

Time-constrained and probabilistic verification approaches gain a great importance in system behavior validation including avionic, transport risk assessment, automotive systems and industrial process controllers. They enable the evaluation of system behavior according to the design requirements and ensure their correctness before any implementation. Due to the difficulty of analyzing, modeling and verifying these large scale systems, we introduce a novel verification framework based on PRISM probabilistic model checker that takes the SysML activity diagram as input and produce their equivalent timed probabilistic automata that is expressed in PRISM language. To check the functional correctness of the system under test, the properties are expressed in PCTL temporal logic. To prove the soundness of our mapping approach, we capture the underlying semantics of both the SysML activity diagrams and their generated PRISM code. We found that the timed probabilistic equivalence relation between both semantics preserve the satisfaction of the system requirements. We present digital camera as case study to illustrate the applicability of the proposed approach and to demonstrate its efficiency by analyzing a performability properties.

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1. Introduction

Constraints on software development in terms of functionality, performance, reliability and time to market are becoming more stringent. Therefore, development reveals several challenges. Indeed, if in one side the applications are becoming increasingly complex, in the other side the market pressure for rapid development of these systems makes the task of their designs a challenge. One major challenge in software development process is to advance errors detection at early stages of life cycles. Researchers at the National Institute of Standards and Technology (NIST) attest: about 50 percent of software development budgets go to testing, yet flaws in software still cost the U.S. economy $59.5 billion annually (US Dept of Commerce, 2010). It has been shown in Fig. 1 (Rajlich, 2014) that the cost of system repairing during the maintenance in software development life cycle is approximately 67%. Therefore, accelerate the verification and maintenance process at preliminary design is extremely beneficial as compared to fixing them at the testing phase.

Recently, formal verification methods have become essential tools for developing safety–critical systems, where its behavioral correctness is a main concern. These methods require a mathematical expertise for specifying what the system ought to do and verifying it with respect to the requirements. There are mainly two ways for doing formal verification: \textit{Theorem proving} and \textit{model checking}.

Theorem proving needs to be operated by people with skills (Experts) in order to solve the problem. In addition, this approach need a precise description of the problem written in logical form and the user needs to think carefully about the problem with deeper understanding in order to produce an appropriate formulation. However, there is significant implementations of this approach such as HOL (Pientka, 2007), Coq (Bertot & Castérans, 2004).

Model checking is popular formal verification approach in software and hardware industry. For instance, SLAM (Ball, Bounimova, Kumar, & Levin, 2010) which is a Microsoft research project uses a model checking to verify device drivers are conform to their API specification. According to Clarke (2008), model checking is automatic and usually very fast. The user does not need to construct...
a correctness proof. If the specification is not satisfied, the model checker will produce a counterexample execution trace that shows why the specification does not hold. This verification method focuses on either qualitative or quantitative properties (Baier & Katoen, 2008). The qualitative properties assert that certain event will happen surely or not. The quantitative properties are based on the probability or expectation of a certain event (e.g. the probability of the system failure in the next $t$ time units is 0.85). Probabilistic model checking is an effective technique to verify probabilistically the satisfiability of a given property. In our paper we use PRISM language for probabilistic model checker (Kwiatkowska, Norman, & Parker, 2011) where the properties can be expressed in Probabilistic Computation Tree Logic (PCTL) or in Continuous-stochastic logic (CSL). Note that the properties prescribe what the system should do, and what it should not do, whereas the model description addresses how the system behaves.

In System Engineering, SysML (system modeling language) is a standard language (OMG, 2012) used for specification, analysis, design and verification of a broad range of complex systems (e.g. Hardware, software, information work flow). SysML reuses a subset of UML2 artifacts (Unified Modeling Language) (OMG, 2007) and provides additional extensions to specify requirements such as probability. Behavioral specification is achieved in SysML using three diagrams: state machine, communication and activity diagram. Activity diagram is particularly described in our paper.

This paper proposes a probabilistic verification of SysML activity diagram that are guarded with time constraints. The proposed framework is depicted in Fig. 2, it takes SysML activity diagrams (OMG, 2012) as inputs to produce the probability of the PCTL property. The kernel of the framework is based on transforming the activity diagram to its equivalent Probabilistic-Timed Automata (PTA) expressed in PRISM language. We extract the adequate semantics model related to SysML activity diagrams. Then, we present the underlying semantics related to the produced PRISM model. Furthermore, we show that the relation between both semantics preserves the satisfiability of PCTL properties.

The remainder of this paper is structured as follows: Section 2 discusses related work. The preliminaries needed for our work are presented in the next section. Section 4 describes and formalizes the SysML activity diagrams. The semantics PRISM model checker is presented in Section 5. Section 6 provides a mapping algorithm of SysML activity diagrams into PRISM code and its soundness is proved in Section 7. Section 8 illustrates the application of our framework on a case study. Section 9 draws conclusions and lays out future work.

2. Related works

In this section, we depict the recent works related to the verification of behavioral models then we compare them with our proposed approach.

Andrade, Maciel, Callou, and Nogueira (2009) and Carneiro, Maciel, Callou, Tavares, and Nogueira (2008) use SysML language and MARTE UML profile (Modeling and Analysis of Real-Time and Embedded systems) to specify ERTSs (Embedded Real-time Systems) constraints such as execution time and energy (Mallet & de Simone, 2008). They map only the states and the transitions into ETPN (Time Petri Net with Energy constraints). The approach is restricted on a sub set of artifacts with control flow (data flow is missed).

Doligalski and Adamski (2013) propose a verification and simulation of UML state machine. For this purpose, two mapping mechanisms are defined. The first consists on mapping the original model to Petri Network (PN) for verification according to the requirements. When the requirements are satisfied, the second mapping occurs to generate VHDL or Verilog description for simulation. The data on each transition is considered as a trigger for a new state. The formalization is restricted on Petri Networks.

Huang, Sun, Li, and Zhang (2013) propose a verification of SysML State Machine Diagram by extending the model with MARTE (Mallet & de Simone, 2008) to express the execution time. The tool has as input the state machine diagram and as output Timed Automata (TA) expressed in UPPAAL syntax (Behrmann, David, & Larsen, 2004). UPPAAL uses CTL (Computational Tree Logic) properties to check if the model is satisfied with liveness and safety properties.

Cafe, Hardebolle, Jacquet, and Boulanger (2013) develop a framework that maps the SysML diagrams describing the heterogeneous system (hardware and software) to SystemC language. The inputs are a set of SysML diagrams such as block diagram,
parametric diagram and behavioral diagram (e.g., Activity or state machine diagram). The result is simulated to check the satisfaction of timing and functional requirement. Karholm et al. (2007) define a mapping rules that takes as input a SaveCCM component model (Carlson, Hkansson, & Pettersson, 2006) and extracts a different tasks in XML format using SaveToTimes tool. The results of the mapping is encoded in UPPAAL syntax. Using CTL properties, the model can be checked (e.g., Minimum execution time) to schedule the tasks (i.e., Components) of general assembly.

Jarraya, Soeanu, Debbabi, and Hassaine (2007) propose a probabilistic verification of SysML activity diagram where the execution time of actions are represented as constraints (i.e., A note artifact in SysML activity diagram). The diagram is translated to its corresponding Discrete-Time Markov Chain (DTMC) and use PRISM model-checker for performance evaluation using PCTL. The approach is restricted on a sub set of SysML activity diagram constructs with control flow (data flow is missed).

Ouchani, Mohamed, and Debbabi (2014b) propose a verification framework of SysML activity diagram. The authors address a sub set of SysML activity diagram artifacts with control flow. The different artifacts have been formalized and a verification algorithm has been proposed for mapping these artifacts to PRISM language. The transformation result is a probabilistic automata to be checked by PRISM. The mapping approach is sound but the approach is limited to only the control flow artifacts without including object and time.

Debbabi, Hassane, Jarraya, Soeanu, and Alawneh (2010b) translate SysML activity diagrams into the input language of the probabilistic model-checker PRISM. The authors present the algorithm implementing the translation of SysML activity diagrams into Markov Decision Process (MDP) supported by PRISM model-checker. The mapping approach is limited to only the control flow artifacts without including object and time.

Ouchani, Jarraya, and Mohamed (2011) presents an approach based on probabilistic adversarial interactions between potential attackers and the system design models. These interactions result in a global model as composed SysML activity diagrams that support more additional artifacts than (Ouchani et al., 2014b) such as objects. The result diagram is translated into PRISM language and a set of security and functional properties are verified against the composed model.

George and Samuel (2012) propose a syntax verification of UML activity diagram to ensure the correctness of a design that may be causing incorrect results or/and incorrect workflow. The algorithm is used to analyze a sub set of activity diagram constructs such as action nodes, decision nodes, fork nodes, join nodes, edges, branches, initial state, final states and guard conditions. The main purpose of the algorithm is prevent the errors at design phase by decomposing activity diagram in its basic elements and verify its individually such as components interconnection or identical guard condition at decision node.

Karalappan, Koenig, and Kaliappan (2008) propose a verification approach for system workflow especially in communication protocol. The approach takes as input three UML diagrams: state machine diagram, activity diagram and sequence diagram. State machine diagram or activity diagram is converted into PROMELA code as a protocol model and its properties are derived from the sequence diagram as Linear Temporal Logic (LTL).

Lasnier, Zalila, Pautet, and Hugues (2009) develop a framework for automatic generation of embedded real time applications in C/C++/ADA. The input is AADL language (Architecture Analysis and Design Language) (Feiler, 2010) with typed software components (e.g., threads, data) and hardware components (e.g., processors, bus, memory). In addition, The design model is enriched with time execution properties (e.g., computation time, priority, deadline and scheduling algorithms) (Singhoff, Legrand, Nana, & Marcé, 2004). For timing estimation and optimization, a scheduling tool is used based on software components timing constraints. After time optimization, the framework generates C/C++/ADA code that can be simulated.

Knorreck, Apvrille, and de Saqui-Sannes (2011) develop a framework for timing constraints verification for customized profile applied for real time system specification incorporating a combined UML and SysML diagrams. Each block diagram and its state machine is converted to corresponding automata in UPPAAL syntax. The properties are derived from parametric diagram as Computational Tree Logic (CTL). Liu et al. (2013) propose a verification tool called UML State Machine Model Checking (USMMC). The tool verify UML state machine and processes verification using Linear Temporal Logic (LTL) properties. However, the mapping from UML state machine to USMMC input language is not given.

Grobelna, Grobelny, and Adamski (2014) present an approach to verify the correctness of UML activity diagrams for logic controller specification especially for embedded systems design. The authors formalize the activity diagram using rule-based logic model (Grobelna, 2013). The result model is mapped to NuSMV (Cimatti, Clarke, Giunchiglia, & Roveri, 1999) in order to verify system model against behavioral properties expressed in Linear Temporal Logic (LTL) formulas to detect some errors. When system is error free, the transformation of logical model into synthesizable model in VHDL language.

Noll (2015) presents the methodology that uses in COMPASS tool. The methodology targets aerospace systems and allows satisfactory for most safety–critical systems for aerospace on-board systems. The specification is based on the Architecture Analysis & Design Language (AADL). The analyses are mapped onto discrete, symbolic and probabilistic model checkers, but all of them are completely hidden away from the user by appropriate model-transformations. To model possible failures, the AADL model is enriched with behavioral error annex as state chart; when error occur, an error state is triggered. The toolset builds upon NuSMV (Cimatti et al., 1999), MRMC (Katoen, Zapreev, Hahn, Hermanns, & Jansen, 2011). Specifications are written in Probabilistic Computation Tree Logic (PCTL) and Continuous Stochastic Logic (CSL).

Abdulkhaleq and Wagner (2014) propose a method for safety analysis at the system level based on System-Theoretic Process Analysis approach (STPA). The process step for software development starts from specification level based on components interactions and identify the Unsafe Control Actions which are formalized into temporal logic. Symbolic Model Verifier (SMV) which was developed by McMillan (1992) is used to check the safety requirements. The limitation of the method is that the Component-based specification is not clearly described.

Marinescu et al. (2015) provides a framework for simulation and verification of EAST-ADL (Qureshi, Chen, Lönn, & Törngren, 2011). EAST-ADL is an architectural description language for automotive electronic systems. The specification level is a component-based architecture where behavior can be a set of xml-files and compiled
C-code or a Simulink model. The simulation is done by automatic transformation from the architectural model to a Simulink model. The verification is done by automatic transformation from the architectural model to a network of timed automata analysed by UPPAAL.

Jacobs and Simpson (2015) present a compositional approach to refinement and specification in the context of SysML. They consider the behavior of SysML blocks as a set of composed behavior of state machine and activity diagrams. The specification is mapped to a process-algebraic CSP where assertions about requirements can be proved or refuted (Jacobs & Simpson, 2013). The idea behind is to prove that the refinement is preserved and the automated analysis is computationally feasible.

Rodriguez, Fredlund, Herranz, and Marino (2014) present a tool for UML state machine diagram verification called UMerL. The tool is an interpreter of UML state machines implemented in Erlang (Fredlund & Svensson, 2007) that executes a system modelled as a collection of UML state machines. UMerL provides two functionalities: the system can be executed for simulation or verified. Using Linear Temporal Logic (LTL) the verification scenario can be checked and it provides a counterexample when an LTL property is violated.

Yan, Zhu, Yan, and Li (2014) use real-time model checking tool UPPAAL to find the possible best throughput and response time of MARTE models, and the best solution in the worst cases for both of them. The approach is based on adding the MARTE profile with its annotations (e.g. time processing) on UML diagrams such as a case diagram, a deployment diagram and a set of activity diagrams. The result of mapping these three diagrams is a network of timed automata. Using Computation Tree Logic (CTL) the verification scenario can be checked and it provides a counterexample when an LTL property is violated.

As a summary, in Table 1 we compare our framework to the existing works by taking consideration five criteria: SysML language, time constraints, data workflow, formalization, soundness and automation. The SysML criteria shows if an approach covers the probabilistic systems. The time constraint criteria shows if the approach includes the time specification at the design level. The data workflow criteria indicates if data objects exist at specification level. The formalization criteria confirms if the concept presents a semantics and formalizes the studied diagrams. The soundness feature shows if the soundness of the studied approach is proved. The automation criteria checks if the presented approach provides a tool. From the comparison, we observe that only few works formalize the behavioral diagrams, include data workflow and time constraint at specification level. Our works has for objective to support: data workflow, time constraints, and formalize the SysML activity diagram. In addition, we implement the tool that allows the automatic verification.

3. Preliminaries

3.1. SysML activity diagram

SysML activity diagram is a graph-based diagram where activity nodes are connected by activity edges. The activity diagrams is a primary representation for modeling flow based behavior in OMG (2012). Fig. 3 shows the set of interesting artifacts used for verification in our framework. The artifacts consist of activity nodes, activity edges, activity control and constraints. Activity nodes have three type: Activity invocation, objects and control nodes. Activity invocation includes action, call behavior, send/receive signals (objects). activity control includes: flow initial, flow final, activity final, fork, merge and join node. Activity edge includes: control flow and object flow. Object flow can connect the output pin of one action to the input pin of next action to enable the passage of tokens. Control flow provides additional constraints on when and in which order the action within an activity will be executed. A control token on an incoming control flow enables the execution of an action and offers a control token on outgoing control flow when action completes its execution. Control nodes such as join, fork, merge, decision, initial and final are used to control the routing of control token over edges and specify the sequence of actions (concurrency, synchronization). In addition, the constraints can be used as SysML notes stereotyped with “Local Post Condition” and “Local Pre Condition”. Next, we define the rules for actions to begin and end execution during the activity workflow and we show the expression manner of time in activity diagram.

3.1.1. Actions execution

The following rules (Friedenthal, Moore, & Steiner, 2008) describe the requirements for actions to begin and end execution during the activity workflow:

(a) The action owned by the activity must be executed.
(b) The action is executed if the number of tokens at each required input pins is equal or greater than its lower multiplicity bound.
(c) The action is executed if a token is available in each of the action’s incoming control flow.
(d) Once these perquisites are met, the action will start executing and tokens at its input pins are available for consumption.

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<tr>
<th>Approach</th>
<th>SysML</th>
<th>Time constraint</th>
<th>Data workflow</th>
<th>Formalization</th>
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3.1.2. Time expression using SysML/MARTE

Friedenthal et al. (2008), Jarraya et al. (2007) and Debbabi, Hassane, Jarraya, Soeanu, and Alawneh (2010a) provide a manner to add the time constraints to actions with specialized form of constraint that can be used to specify the time duration of an action’s execution. The constraint is shown using standard constraint notation: a note attached to the action which is constrained (see Fig.2). However, annotation of time constraints on top of SysML activity diagrams is not clearly defined in the standard (OMG, 2012).

According to OMG SysML (OMG, 2012) the actions can be triggered using wait time action artifact (Time Event). Wait time action artifact becomes enabled when control token arrives on its incoming control flow and the precedent action has finished. If the time event specified by execution time elapses then it completes immediately and offer token to its outgoing control flow, else waits for that time event to occur. If the Time event has not an incoming edge then it becomes enabled as soon as the activity begins executing.

MARTE is a new UML profile standardized by the OMG (Mallet & de Simone, 2008) aims to replace the UML profile SPT (Profile for Schedulability, Performance and Time) MARTE was defined to make quantitative predictions regarding real-time and embedded features of systems taking into account both hardware and software characteristics. The core concepts are design and analysis parts. The design parts are used to model real time embedded systems. On the other hand, the analysis parts provide facilities to support the resources analysis such as execution time, energy and memory usage. In our paper, we use execution time for quantitative verification.

Fig. 4 illustrates how the probability value is specified on the outgoing edges of the decision nodes testing their corresponding guards. In addition, the time is specified using MARTE profile with the stereotype $\text{resourceUsage}$.

The action TurnOn requires exactly 2 units of time to terminate; Action AutoFocus terminates within the interval $[1,2]$. The action ‘TakePicture’ execution time is negligible. To model probabilistic systems, the probabilities are assigned to edges emanating from decision nodes where the assigned values should sum up to 1. For instance, the decision node testing the guard memFull has the following semantic interpretation: the probability is equal to 0.2 that the outcome of the decision node will be (memFull = true) and the corresponding edge traversed.

3.2. Probabilistic model checking

Model checking is a formal verification technique that given a finite-state model of a system and a formal property, systematically checks whether this property holds for that model. If not a counter example is generated. Probabilistic model checking is based on the analysis of systems that exhibit a probabilistic behavior. The model checked can be expressed as Discrete-Time Markov Chains (DTMC), Continuous-Time Markov chains (CTMC), Probabilistic Automata (PA) based on Markov Decision Process (MDP) or recently as Probabilistic Timed Automata (PTA) to describe a probabilistic model under time constraints. In this paper, we focus on Probabilistic Timed Automata (PTA).

Probabilistic Timed Automata (PTA) (Norman, Parker, & Sproston, 2013) are modeling formalism for systems that exhibit probabilistic, non-deterministic and real-time characteristic. Verification of PTAs permits analysis of a wide range of quantitative properties such as reliability and performance, e.g:

- The minimum probability to take a picture within 2 s;
- The minimum probability to take a picture within 2 s when flash is on.

PTAs can also be augmented with additional quantitative informations in the form of costs or rewards, e.g:

- The minimum expected time for completion of all tasks;
- The minimum expected energy to take a picture when flash is on.

This paper provides a formal definition of PTAs to specify and verify properties such as those listed above. Probabilistic Timed Automata (PTA) is a Probabilistic Automata equipped with a finite set of real-valued clock variables called clocks. Conditions on the values of the clocks are used as enabling conditions (i.e., guards) of actions: only if the condition is fulfilled the action is enabled.
and capable of being taken; otherwise, the action is disabled. Conditions which depend on clock values are called clock constraints which is used to limit the amount of time that may be spent in a location. The following definition prescribes how constraints over clocks are to be defined.

**Definition 1 (Clock Constraint).** A clock constraint over set $X$ of clocks is formed according to the grammar
$$g ::= x < c \mid x \leq c \mid x > c \mid x \geq c \mid \neg g$$

where $c \in \mathbb{N}$ and $x \in X$. Let $CC(X)$ denotes the set of clock constraints over $X$.

A Probabilistic Timed Automata (PTA) is an Probabilistic Automata with clocks variable. The clocks are used to formulate the real-time assumptions on system behavior. An edge in a PTA is labeled with a guard (when is it allowed to take an edge?), an action (what is performed when taking the edge?), and a set of clocks (which clocks are to be reset?). A state is equipped with an invariant that constrains the amount of time that may be spent in that location. The formal definition is:

**Definition 2.** Probabilistic Timed Automata (PTA). PTA is a tuple $M = (s, S, \mathbf{X}, \mathbf{Act}, \mathbf{Inv}, \mathbf{Prob}, \mathbf{L})$, where:

- $s$ is an initial state, such that $s \in S$,
- $S$ is a finite set of states,
- $\mathbf{X}$ is a set of clocks,
- $\mathbf{Act}$ is a set of actions,
- $\mathbf{Inv} : S \rightarrow CC(\mathbf{X})$ is an invariant condition, imposes restrictions on the allowable values of clock variable,
- $\mathbf{Prob} : S \times \mathbf{Act} \rightarrow \text{Dist}(2^\mathbf{X} \times S)$ is a probabilistic transition function assigning for each $s \in S$ and $a \in \mathbf{Act}$ a probabilistic distribution $\mu \in \text{Dist}(2^\mathbf{X} \times S)$,
- $\mathbf{L} : S \rightarrow 2^\mathbf{AP}$ is a labeling function mapping for each state a set of atomic propositions.
Transitions (Edges) in PTA are labeled with tuple (g, ϕ) where g is a clock constraint of the PTA, ϕ is an action. The intuitive interpretation of $s \xrightarrow{g, ϕ} s'$ is that the PTA can move from state s to state $s'$ when clock constraint g holds. Besides, when moving from state s to $s'$, any clock will be reset to zero and action ϕ is performed according to the distribution $μ ∈ Dist(2^X × S)$. Function $Inv$ assigns to each state a state invariant that specifies how long the PTA may stay there (maximum elapsed time). For state s, $Inv(s)$ constrains the amount of time that may be spent in s.

A location in PTA is a pair $(s, ϕ)$ in $R_0$ such that for a set X of clocks, $ϕ$ is a clock valuation with $ϕ : X → R_0$, assigning to each clock $x ∈ X$ its current value $ϕ(x)$. In any location $(s, ϕ)$, either a certain amount of time $t_ε ∈ R_0$ elapses, or an action $x ∈ Act$ is performed. If time elapses, the choice of $t_ε$ requires that the invariant $Inv$ remains continuously satisfied while time passes i.e. $ϕ(t_ε) = Inv(s)$. The resulting location after this transition is $(s, ϕ + t_ε)$. In the case where an action is performed, an action $x$ can only be chosen if it is enabled i.e. when clock constraint $g$ holds, $ϕ(t_ε) = g$. Once an enabled action $x$ is chosen, a set of clocks will be reset to zero and a successor location are selected at random, according to the distribution $μ_{s, Act} = Prob(s, x)$.

There are two possible ways in which a PTA can proceed by taking a transition in the PTA (transition action) or by letting time progress while remains in a state (delay transition):

- **Action transition**: $(s, ϕ) \xrightarrow{(g, ϕ)} (s', ϕ')$ if the following conditions hold:
  - (a) there is a transition $s \xrightarrow{g} s'$ such that $t_ε ∈ R$
  - (b) $ϕ + t_ε = g$
  - (c) $ϕ' = (ϕ + t_ε)[X := 0]$
  - (d) $ϕ + t_ε = Inv(s)$

- **Delay transition**: $(s, ϕ) \xrightarrow{[t]} (s, ϕ + d), d ∈ R$
  - (e) if $ϕ + d = Inv(s)$

For a transition that corresponds to (a) traversing a transition $s \xrightarrow{g} s'$ in PTA it must hold that (b) $ϕ$ satisfies the clock constraint $g$ (ensuring the transition is enabled), and (c) the new clock valuation in $s'$ is reset, should (d) satisfy the state invariant of s ($ϕ + t_ε = Inv(s)$). To remain in the same state (e) if the location invariant remains true while time progresses.

A reward structure are defined at the level of PTA equivalently referred to as costs or prices using a pair $r = (r_ε, r_{Act})$, where $r_ε : S → R^{0,5}$ is a function assigning to each state the rate at which rewards are accumulated as time passes in that location and $r_{Act} : S × Act → R^{0,5}$ is a function assigning the reward of executing each action in each state.

**Definition 3** (Parallel composition of PTAs). The parallel composition of PTAs $M_1 = (S_1, Σ_1, X_1, Act_1, Inv_1, Prob_1, L_1)$, $M_2 = (S_2, S_2, X_2, Act_2, Inv_2, Prob_2, L_2)$ is the PTA $M = M_1||M_2 = (S × S_1 × S_2, X_1 × X_2, Act_1 × Act_2, Inv_1 × Inv_2, Prob(L_1 × L_2))$: where $Prob(L_1 × L_2) = Act_1 × Act_2$ is the set of transitions, such that one of the following requirements is met.

1. $s_1 \xrightarrow{r_ε} S_1 × S_2, S_1 × S_2 \xrightarrow{r_ε} S_1 × S_2, S_1 × S_2$ if $s_1 \in Act_1 ⊆ Act_2$, $s_1, t_1, t_2 ∈ S_1, t_2, t_3 ∈ S_2$.
2. $s_1 \xrightarrow{r_{Act}} S_1 × S_2, S_2 \xrightarrow{r_{Act}} S_2 × S_1$ if $s_1, s_2 ∈ S_1 × S_2$.
3. $s_1 \xrightarrow{r_{Act}} S_1 × S_2, S_2 \xrightarrow{r_{Act}} S_2 × S_1$ if $s_1, s_2 ∈ S_1 × S_2, s_2, s_3 ∈ S_2 × S_1$.

If PTA $M_1$ has associated with rewards structure $(r_ε, r_{Act})$, then the reward structure of $r = (r_ε, r_{Act})$ for $M_1||M_2$.

1. $r_1^ε(s_1, s_2) = r_1^ε(s_1) + r_2^ε(s_2), r_{Act}((s_1, s_2, x)) = r_{Act}^ε(s_1, x) + r_{Act}^ε(s_2, x)$, and $x ∈ Act_1 ⊆ Act_2$
2. $r_{Act}((s_1, s_2, x)) = r_{Act}^ε(s_1, x)$, and $x ∈ Act_1 \Act_2$
3. $r_{Act}((s_1, s_2, x)) = r_{Act}^ε(s_2, x)$, and $x ∈ Act_2 \Act_1$

The PRISM tool supports three PTA model checking techniques: Digital Clocks (Kwiatkowska, Norman, Parker, & Sproston, 2004) and abstraction refinement using stochastic games (Kwiatkowska, Norman, & Parker, 2009) where stochastic games supports the reward operator and the Digital clocks supports the probabilistic operator. The structure of our temporal logic is expressed by the following BNF grammar:

**3.3. Property specification for PTAs**

The syntax of our logic is given by the following grammar:

$ϕ ::= \text{true} \mid \text{ap} \mid ϕ \land ϕ \mid ¬ϕ \mid P_{ϕ}[ϕ] \mid R_{ω}^{[ϕ]}(p)$

$ψ ::= ϕ \land ϕ \mid ϕ \lor ϕ$

$ρ ::= ψ \mid C \lor F \mid ϕ$

Where “ap” is an atomic proposition, $P$ is a probabilistic operator and R is a reward. Operator $P_{ϕ}[ϕ]$ means that the probability of path formula $ψ$ being true always satisfies the bound $\bowtie p$, $p ∈ [0, 1]$. $R_{ω}^{[ϕ]}[p]$ means that the expected value of reward function $ρ$ on reward structure $r$ meets the bound $\bowtie q$, $q ∈ Q$, “$_ffl” is in $<, =, >$, “$ω” represents the conjunction operator and “$-$” is the negation operator. Two paths formulas are included bound until $ϕ \lor ϕ$ and time-bound until $ϕ \lor ϕ$. Bound until means that a state satisfying $ϕ$ is eventually reached and that, at every time-instant prior to that, $ψ$ is satisfied. The time-bound variable has the same meaning, where the occurrence of $ϕ$ occur within time k. The reward operator $I_k$ refers to the reward of the current state at time instant k, $C_ω$ refers to the total reward accumulated up until time point k and $F ϕ$ to the total reward accumulated until a state satisfying $ϕ$ is reached, e.g:

$R_{ω}^{[ϕ]} = [F Success]:$ what is the expected reward accumulated before the system successfully terminates?

$P_{ω} = [true \land ω^{100} Complete]:$ the minimum probability of the system eventually completing its execution successfully after 100 time units?

Let P is PTA and $[[P]] = (S, Σ, T, Act, lab, Step_0)$ its semantics where $S$ is a set of states, $I$ is the initial state, $T$ is a set of clocks, lab is a labeling function and $Step_0 : S × Act → Dist(2^X × S)$ is a probabilistic transition function and let $r$ denotes the reward structure over $[[P]]$ corresponding to the reward structure over $P$. The satisfaction relation of a PCTL formula $ϕ$ denoted by “$|=ϕ$” and defined as follows where $(s, ϕ)$ is a location and Path is a sequence of states:

- $s, ϕ |= true$ is always satisfied,
- $s, ϕ |= ap ⇐⇒ ap ∈ I(s)$ and I is a labeling function,
- $s, ϕ |= ϕ_1 \land ϕ_2 ⇐⇒ s, ϕ_1 \land ϕ_2,
- s, ϕ |= ¬ϕ ⇐⇒ s, ¬ϕ,
- s, ϕ |= P_{ϕ}[ϕ] ⇐⇒ Prob_{ϕ}[ϕ](π ∈ Path_{ϕ})π |= ϕ ⇐⇒ p, for all $σ ∈ Ad_{ϕ}[ϕ]$
- $s, ϕ |= R_{ω}^{[ϕ]}[p] ⇐⇒ Exp_{ϕ}[ϕ](rew(r, ρ)) ⇐⇒ q$ for all $σ ∈ Ad_{ϕ}[ϕ]$

Where for any finite path $π$ of $[[P]]$:
In this section, we formalize SysML activity diagram with extended artifacts by providing an adequate calculus.

### 4.1. SysML activity diagrams syntax

Based on textual specification of SysML (OMG, 2012) we formalize the SysML activity diagram by developing its Calculus: Timing Activity Calculus (TAC). In Table 2, each SysML artifact is represented and described formally by its related TAC term. The TAC terms in Fig. 5 extends NuAC defined by Debbabi et al. (2010a) and enhanced by Ouchani et al. (2014b) adding the time and data. In this calculus we distinguish between two syntactic concepts: Marked and Unmarked terms. A marked terms are activity diagrams with tokens. The unmarked terms corresponds to the static structure of the activity diagram. A marked term denotes the configuration of diagram in the workflow process.

To support tokens we augment the “over bar” operator with integer value n such that the $\overline{N^n}$ denotes the term $N$ marked with n tokens with the convention that $\overline{N^0} = N$ and $\overline{N^0} = N$. Multiple tokens can be observed when the loop encompass a fork in its body. Furthermore, we use a prefix label $l$: for each node to uniquely reference it in the case of a backward flow connection. Particularly, labels are useful for connecting multiple incoming flows towards merge and join nodes. Let $\mathcal{L}$ be a collection of labels ranged over by $l, b, h, ...$ and $N$ be any node (except initial) in the activity diagram. We write $l: N$ to denote a l-labelled activity node $N$. It is important to note that nodes with multiple incoming edges (e.g. join and merge) are visited as many times as they have incoming edges. Thus, as a syntactic convention we use only a label (i.e. $l$) to express a TAC term if its related node is encountered already. We denote by $D(A, g, N, N')$ and $D(A, p, g, N, N')$ to express a non-probabilistic and a probabilistic decisions, respectively. For the workflow observation on SysML activity diagrams, we use structural operational semantics (Milner, 1999 & Norman et al., Norman, Palamidessi, Parker, & Wu, 2007) to formally describe how the computation steps of TAC atomic terms take place. We define $\mathcal{A}$ the set of actions labeling the transitions (i.e. the alphabet of TAC, to be distinguished from action nodes in activity diagrams). An element $a$ is the label of the executing action node or $x(y)$ inputs an object name on $x$ and stores it in $y$. An element $t$ is the time for action transition and $p$ be probability values such that $p \in [0, 1]$. The general form of a transition is $A^{\overline{a\overline{p}}}. A'$. The probability value specifies the likelihood of a given transition to occur and it is denoted by $P(A, a, t, x, A')$. The case of $p = 1$ presents a non-probabilistic transition and it is denoted simply by $A^{\overline{a}} A'$. For simplicity, we define $A|N$ to specify $N$ as a sub-term of $A$ and by $|A|$ to denote a term $A$ without tokens. For the call behavior case of $a: N$, we denote by $A[a: N]$ by $A_{a=N}$. In the sequel, we describe the operational semantic rules of the TAC calculus in Table 3.

The semantics of SysML activity diagrams expressed using $\mathcal{A}$ is the result of the defined inference rules. The semantics can be described in terms of PTA as stipulated by Definition 4.

![Fig. 5. Syntax of Timing Activity Calculus (TAC).](image-url)
Definition 4. (TAC-PTA). A Probabilistic Timed Automata of TAC term \( A \) is the tuple \( M_A = \langle S, X, Act, Inv, Prob, L \rangle \), where:

- \( S \) is an initial state, such that \( I(S) = \{ l : I \rightarrow N \} \).
- \( S \) is a finite set of states reachable from \( S \), such that, \( S = \{ s_{0, x, c} | s(I(s)) = \{ N \} \} \).
- \( X \) is a set of clocks,
- \( Act \) is a set of actions including three types: label of the executing action, \( x(y) \) inputs a name on \( x \) and stores it in \( y \),
- \( Inv : S \rightarrow CC(X) \) is an invariant condition (i.e. constraints over clocks \( X \)),
- \( Prob : S \times Act \rightarrow Dist(2^X \times S) \) is a probabilistic transition function assigning for each \( s \in S \) and \( a \in Act \) a probabilistic distribution \( \mu \) where:
  - For each \( S \subseteq S \) and \( t \in X \) such that \( S = \{ s_{i : 0 \leq i \leq n} | s_I(s) = \{ N \} \} \), each transition \( s^{a}_{i} \rightarrow s \) satisfies one TAC semantic rule and \( \mu(S, \theta) = \sum_i p_a = \sum_{s_{i : 0 \leq i \leq n}} \mu(s, \theta + t) = 1 \).
  - For each transition \( s^{a}_{i} \rightarrow s \) satisfies one TAC semantic rule and \( \mu(S, \theta + t) = 1 \).

- \( L : S \rightarrow \{ true, false \} \) is a labeling function where: \( [[L]] = \{ true, false \} \).

5. PRISM formalization

In this section, our formalization focus on Probabilistic Timed Automata (PTA) that extends the standard probabilistic automata (PA) considered as appropriate semantic model for SysML activity diagram (Ouchani, Mohamed, & Debbabi, 2013). The PRISM model checker supports the PA with ability to model real-time behavior by adding real-valued clocks (i.e. clocks variable) which increases with time and can be reset (i.e. updated).

A Timed Probabilistic System (TPS) that represents a PRISM program \( P \) is composed of a set of “m” modules \( m > 0 \). The state of each module is defined by the evaluation of its local variables \( V_i \). The global state of the system is defined as the union of the evaluation of local and global variables; \( V = V_i \cup V_g \). The behavior of each module is described as a set of statements in the form of:

\[
act \text{guard} \rightarrow p_1 : u_1 + p_2 : u_2
\]

Where act is an (optional) action labeling the command, the guard is a predicate consists of boolean, integer and clock variables and propositional logic operators, \( p \) is a probability. The update \( u \) is a set of evaluated variables expressed as conjunction of
assignments \( V'_i = eval(V'_i) \) and \( V''_i = eval(V''_i) \) where \( V'_i \in V_i \cup V_c \) and \( \text{eval} \) are values evaluated via expressions denoted by “eval” \( \text{eval} : V \rightarrow \mathbb{R} \cup \{\text{True}, \text{False}\} \). The formal definition of a command is given in Definition 5.

**Definition 5.** A PRISM command is a tuple \( c = (a, g, u) \),

- “act” is an action label.
- “guard” is a predicate over \( V \).
- “\( u = \{ (p_i, u_i) \} \) m \( i \in \) \( 1 \leq m \), \( 0 \geq p_i \geq 1 \) and \( \sum_{i=1}^{m} p_i = 1 \) and \( u = \{ (v, \text{eval}(v)) \} : \) is initial values of global variables.
- “\( \text{exp} \) is a set of global logic operators.
- “\( M \) is a set of modules composing a System.
- “\( \text{CSPexp} \) is CSP algebraic expression.

**Definition 6.** A PRISM module is tuple \( M = (V_i, I, \text{Inv}, C) \), where:

- \( V_i \) is a set of local variable associated with a module.
- \( \text{Inv} \) is a time constraint of the form \( t \geq d / t \in \{ <, \geq \} \) and \( d \in \mathbb{N} \).
- \( I \) is the initial value of \( V_i \).
- \( C = \{ i, 0 < i < k \} \) is a set of commands that define the module behavior.

To describe the composition between different modules, PRISM use CSP communication sequential process operators (Hoare, 1985) such as Synchronization, Interleaving, Parallel Interface, Hiding and Renaming. **Definition 7** provides a formal definition of PRISM system.

**Definition 7.** A PRISM system is tuple \( P = (V, \text{exp}, M, \text{CSPexp}) \), where:

- \( V = V_1 \cup \cdots \cup V_m \) \( V_1 \) is the union of a set local and global variables.
- \( I_p \) is initial values of global variables.
- \( \text{exp} \) is a set of global logic operators.
- \( M \) is a set of modules composing a System.
- \( \text{CSPexp} \) is CSP algebraic expression.

5.1. **PRISM syntax**

The syntax of PRISM PTA is defined by the BNF grammar presented in **Fig. 6**. To clarify the syntax we define the following:

- \( \{\text{min} \ldots \text{max}\} \) is a set of ranges such that \( \text{min} \leq \text{max} \). \( \text{min} \in \mathbb{N} \) and \( \text{max} \in \mathbb{N} \).
- \( p \in \{0, 1\} \) is a probability value.
- \( \text{eval} \) is an evaluation expression that can be composed of the following operators: \(+, -, \times, /, <, <=, >, >=, !=, ==, g, a : b\).
- \( \text{eval} \) is a string describing a variable \( (v \in V) \) and \( \text{init} (v) \) is its initial value.
- \( \text{name} \) is a string describing the module name. For the module \( i \), name is denoted by \( M_i \).
- Invariant impose restrictions on the admissible values of clock variable.
- \( \text{CSPexp} \) is a CSP expression composed of the following operators \( |, ||, ||a, b, \ldots||, /|a, b, \ldots|, \) and \( |a \rightarrow b, c \rightarrow d, \ldots| \).

5.2. **PRISM semantics**

The Probabilistic Timed Automata of a PRISM program \( P \) is based on the atomic semantics of a command \( c \) denoted by \( ||c|| \). The latter is a set of transitions defined as follows: \( ||c|| = \{ (s, a, \mu) | s \geq g \} \) where \( \mu \) is a distribution over \( S \) such that \( \mu(s, a + \nu v) = \{ 0 < p_i < 1, v \in V, s' (v) = \text{eval}(V) \} \).

**Table 4**

| PRISM-PTA operational semantic rules | \( \text{INIT} \) \( (V, \text{init}(V)) \rightarrow (V, ||(\text{init}(V))||) \) \( \text{- INIT} \) initializes variables. For a module \( M_i \), \( \text{init} \) returns the initial value of the local variable \( V_i \in V_i \) | \( \text{CNJ-UPD} \) \( (V, \nu, \text{eval}(V) \land \nu' = \text{eval}(V') \rightarrow (V, ||V||, ||V'||) \) \( \text{CNJ-UPD} \) implements the conjunction of a set of assignments |
| \( \text{ENB-CMD1} ||V, e'|| \rightarrow ||V'|| \) \( \text{ENB-CMD1} \) enables the execution of a probabilistic command | \( \text{SYNC} \) \( (V, e, V_1, e_1, V_2, e_2) \) \( \text{SYNC} \) derivation rule permits the synchronization between modules on a given action |
| **Fig. 6.** The syntax of PRISM Probabilistic Timed Automata. | **LOOP** \( (V_i, \nu) \rightarrow (V) \) This axiom presents a loop in a state without changing variables evaluations. It can be applied to avoid a deadlock | **UPDATES** \( (V, \nu' = \text{eval}(V)) \rightarrow (V, ||(\text{eval}(V))||) \) **UPDATES** axiom describes the execution of a simple assignment for a given variable \( v \). Its evaluation is updated in \( V_i \) of \( M_i \). |
| **PRB-UPD1** \( (V, p : (v_i = \text{eval}(V)) \rightarrow (V_{(i)}[\nu_i]) \) \( 0 < p' < 1 \) \( \text{PRB-UPD1} \) describes the execution of a command in a module |
| **PRB-UPD2** \( (V, p : (v_i = \text{eval}(V) \land v_i' = \text{eval}(V')) \rightarrow (V_{(i)}[\nu_i]) \) \( 0 < p < 1 \) \( \text{PRB-UPD1} \) and \( \text{PRB-UPD2} \) describe probabilistic updates | |
Definition 1 stipulates the formal definition of PRISM Probabilistic Timed Automata denoted by $MP$. The states of $MP$ take the form $V_1; ...; V_n; e_{val}$. The stepwise behavior of $MP$ is described by the operational semantic rules provided in Table 4.

Definition 8. (PRISM-PTA). A Probabilistic Timed Automata of PRISM program $p$ is the tuple $MP = (si, S, X, Act, Inv, Prob, L)$, where:

- $si$ is an initial state, such that $L(si) = [\text{init}(V_1)]$.
- $S$ is a finite set of states reachable from $si$, such that, $S = \{S_{0,0,0,0} | L(S_i) \in \{AP\}\}$.
- $X$ is a set of PRISM clocks variables.
- $Act$ is a finite set of actions,
- $Inv$: imposes restrictions on the allowable values of clock variable,
- $Prob : S \times Act \to \text{Dist}(2^X \times S)$ is a probabilistic transition function assigning for each $s \in S$ and $a \in Act$ a probabilistic distribution $\mu$ where: For each $s \in S$, $\nu \in V$ is a PRISM variable, $a \in Act$ and $v_t \in V$ is a clock variable such that $s(\nu, v_t) \sim \mu_{[v_t + v_t]}$, $\nu + v_t \models \text{Int}(t)$ and $s(\nu, v_t) \models g$.
- $L : S \to 2^{\text{AP}}$ is a labeling function that assigns for each state a set of valuated propositions.

6. The verification approach

This section describes the transformation of SysML activity diagrams $A$ into a PTA written in PRISM input language. Algorithm 1

\begin{verbatim}
\begin{verbatim}
\end{verbatim}
\end{verbatim}

illustrates the transformation algorithm T that takes A as input and returns its PRISM code by PrismCode. The diagram A is visited using a depth-first search procedure (DFS) and the algorithms output produces PRISM synchronized modules.

```
Algorithm 1 Transformation Algorithm T of SysML activity diagrams into PRISM Code

Input: SysML activity diagram A
Output: PRISM Code
1 Nodes: Stack;
2 cNode, nNode as Node;
3 nNodes, vNodes List_Of_Node;
4 maxDuration, minDuration: Integer;
5 action as Action;
6 ProcedureT(A)
7 Nodes.push(init); // Push the initial node ∈ A
8 While Nodes.notEmpty()
9    cNode:= Nodes.pop(); // Pull out the first node
10   if (cNode not in vNode)
11      vNodes.add(cNode); // The current is considered as visited
12      nNodes:=cNode.Successors(); // Store the successors in buffer list
13      if (cNode.hasDuration()) // set Invariant
14         maxDuration:= cNode.maxDuration();
15         PrismCode.add(\( \phi(cNode,maxDuration) \));
16      End if
17      minDuration:= cNode.minDuration();
18      PrismCode.add(\( T(cNode,nNodes, minDuration, action) \));
19   End if
20   For all n in nNodes
21      Then Nodes.add(n); // Add the successors to the stack
22   End For
23 nNodes.clear(); // Empty the buffer list and search ends
24 End While
25 End Procedure T
```

First, the initial node is pushed into the stack of nodes denoted by Nodes (line 7). While the stack is not empty (line 8–29), the algorithm pops a node from the stack into the current node denoted by cNode (line 9). The current node is added to the list vNode of visited nodes (line 12) if it is not already visited (line 10). PrismCode is constructed by calling the function T and \( \phi \). The first has four arguments which are the current node, its successors, the minimum duration and action type (line 22). The second has two arguments which are the current node and the maximum duration to impose the maximal clocks supported by the state (line 18). The explored successors are pushed into the stack nodes (line 25–27). The algorithm terminates when all nodes are visited.

The function T presented in Listing 1 and Listing 2 produces the appropriate PRISM command for each TAC term. The action label of a command is the label of its related term n. The guard of this command depends on how the term is activated and minimal clock valuation. The flag related to the term is its label l that is initialized to false except for the initial node it is true which conforms to the premise of the TAC rule INIT-1. The updates of the command deactivate the propositions of the term, activate that ones related to its successors, reset the clock variable of its successors and assign an object values to its successors inputs pins. For a term \( n \in A \), we define three useful functions are: \( L(n); S(A), \) and \( E(A) \) that return the label of a term \( n \), the initial and the final terms of the diagram \( A \), respectively. For example, the call behavior action \( i : a \in A \) (line 26) produces two commands (line 30), and it calls the function \( \Gamma \) (line 57). The first command in (line 30) synchronizes with the first command in (line 61) produced by the function \( \Gamma \) in the action 1 from the diagram \( A \). Similarly, the second command in (line 30) synchronizes with the command of line 65 in the action \( L(E(A)) \) from the diagram \( A \). The first synchronization represents the TAC rule BH-1 where the second represents the rule BH-2. The function \( \Gamma \) is similar to the function \( \Gamma \) except for the initial and the final nodes as shown in (line 52) and (line 56), respectively. The region behavior calls the sub actions within the region in the iterative manner Listing 2 (line 3). After each execution an object is produced, the number of execution has the size of the collection in the output. Due to the unsupported collection type in PRISM we use the simple object type integer, double. Thanks for prism renaming ability, we can rename the sub module name end its variables to produce multiple objects. At the end of Region execution, multiple parallel commands are synchronized and objects are assigned Listing 2 (line 7). The region is interrupted Listing 2 (Line 10), including accept event actions in the region, when a token traverses an interrupting edge. At this point the interrupting token has left the region and is not terminated Listing 2 (Line 38). The \( \phi \) function Listing 2 (line 49–54) associate with each action the maximum integer number supported by the clock variable (i.e. clock invariant). The generated PRISM fragment of each diagram \( A \) is bounded by two PRISM primitives: the module head Module\( A \), and the module termination endmodule.

7. The transformation soundness

Our aim is to prove the soundness of the transformation algorithm \( \Gamma \) by showing that the proposed algorithm preserves the satisfiability of PCTL properties. Let \( A \) be a TAC term and \( M_\alpha \) is its corresponding PTA constructed by the TAC operational semantics denoted by \( S \) such that \( S(A) = M_\alpha \). For the program \( P \) resulting after transformation rules, Let \( M_\beta \) its corresponding PTA constructed by PRISM operational semantics denoted \( S' \) such that \( S'(P) = M_\beta \). As illustrated in Fig. 7, proving the soundness of \( \Gamma \) algorithm is to find the adequate relation \( \mathcal{R} \) between \( M_\beta \) and \( M_\alpha \).

To define the relation \( M_\beta \mathcal{R} M_\alpha \), we have to establish a step by step correspondence between \( M_\beta \) and \( M_\alpha \). First, we introduce the notion of the timed probabilistic bisimulation relation (Ben-Menachem, 2010; Segala, 1995) in Definitions 9 and 10. This relation is based on the probabilistic equivalence relation \( \mathcal{R} \) defined in Definition 8 where \( \delta \mathcal{R} \) denotes the quotient space of \( \delta \) with respect to \( \mathcal{R} \) and \( \equiv \mathcal{R} \) is the lifting of \( \mathcal{R} \) to a probabilistic space.

**Definition 8.** (The equivalence \( \equiv \mathcal{R} \)). If \( R \) is an equivalence on \( \delta \), then the induced equivalence \( \equiv \mathcal{R} \) on Dist(\( \delta \times 2^A \)) is given by: \( \mu \mathcal{R} \mu' \) iff \( \mu(\delta, \vartheta + d) \equiv \mathcal{R} \mu(\delta, \vartheta + d') \).

**Definition 9 (Timed Probabilistic Bisimulation Relation).** A binary relation \( \mathcal{R} \) over the set of states of PTAs is timed bisimulation iff whenever \( s_1 \mathcal{R} s_2, \alpha \) is an action and \( d \) is a delay:

- if \( s_1 \xrightarrow{\alpha} \mu(s_1, \vartheta + d) \) there is a transition \( s_2 \xrightarrow{\alpha} \mu(s_2, \vartheta + d') \), such that \( s_1 \mathcal{R} s_2 \). The delay \( d \) can be different from \( d' \);
- two states \( s, s' \) are time probabilistic bisimilar, written \( s \sim s' \), iff there is a timed probabilistic bisimulation related to them.
Definition 10 (Timed Probabilistic Bisimulation of PTAs). Probabilistic Timed Automata $A_1$ and $A_2$ are timed probabilistic bisimilar denoted $(A_1 \sim A_2)$ iff their initial states in the union of the probabilistic timed transition systems $T(A_1)$ and $T(A_2)$ generated by $A_1$ and $A_2$ are timed probabilistic bisimilar.

For our proof, we stipulate herein the mapping relation $\mathcal{R}$ denoted by $M_{\mathcal{R}}$ between a TAC term $A$ and its corresponding PRISM term $P$.

Definition 11 (Mapping relation). The relation $M_{\mathcal{R}}$ between a TAC term $A$ and a PRISM term $P$ such that $C(A) = P$ is a timed probabilistic bisimulation relation.

Finally, proving that $\Gamma$ is sound means showing the existence of a timed probabilistic bisimulation between $M_A$ and $M_P$.

Lemma 1 (Soundness). The mapping algorithm $\Gamma$ is sound, i.e. $M_A \sim M_P$.

Proof. We prove $M_A \sim M_P$ by following a structural induction on TAC terms and their related PRISM terms. For that, let $s_1, s_0^1 \in S_A$ and $s_2, s_0^2 \in S_P$. We distinguish the following cases where $L(s)$ takes different values:

1. $L(s_1) = I : R \rightarrow N$ such as $x = (i, a) \Rightarrow \exists s_1 x \frac{d_x}{1} s_1$, $L(s_1') = I : x \rightarrow \neg N$. For $L(s_2) = \Gamma(L(s_1))$, we have $L(s_2) = (L(x), \neg L(N))$ then $\exists s_2 x \frac{d_x}{1} s_2$ where $L(s_2') = (\neg L(x), L(N))$.


\[ \Gamma : A \rightarrow P \]

1. $R : A \rightarrow N$, case(a) of

\[ b(y), \text{ for } \xi, y \in \mathcal{J}_{\mathcal{A}} \]

\[ (\mathcal{I}) \rightarrow (false) \cup \{ (L(E(A_1))) \cup (L(N')) \rightarrow (false) \} \cup (L(N)) \]

2. $\mathcal{E} : (A_1, A_2) \rightarrow N$

\[ (\mathcal{I}) \rightarrow (false) \cup \{ (L(E(A_1))) \cup (L(N')) \rightarrow (false) \} \cup (L(N)) \]

Fig. 7. The transformation soundness.
3. If \( L(s_1) = I : D(g_1, N_1, N_2) \) then \( \exists s_2 \in 1 : N_1 \), \( L(s_2) = I : D(g_1, N_1, N_2) \). For \( L(s_2) = \Gamma(L(s_1)) \), we have \( L(s_2) = (l, \neg l_{i_1}, \neg l_{i_2}) \), then \( \exists s_3 \in 1 : s_2 \) where \( L(s_3) = (\neg l, l_{i_1}, l_{i_2}) \).

4. If \( L(s_1) = I : \emptyset \) then \( \exists s_2 = s_1 \), \( L(s_2) = I : \emptyset \). For \( L(s_2) = \Gamma(L(s_1)) \), we have \( L(s_2) = (l) \) then \( \exists s_3 = s_1 \) where \( L(s_3) = (l) \).

5. If \( L(s_1) = I : \emptyset \) then \( \exists s_2 = s_1 \), \( L(s_2) = I : \emptyset \). For \( L(s_2) = \Gamma(L(s_1)) \), we have \( L(s_2) = (l) \) then \( \exists s_3 = s_1 \) where \( L(s_3) = (l) \).

Proof. The preservation of PCTL properties is proved by induction on the PCTL structure and its semantics. Since \( M_A \sim M_P \) and by relying to the semantics of each PCTL operator \( \xi \in \{ U, \mathcal{U}, \Gamma, \mathcal{F}, \mathcal{P}, \mathcal{R}_{m*} \} \), we find that \( (M_A \models \xi) \Longleftrightarrow (M_P \models \xi) \) which means: \( (M_A \models \phi) \Longleftrightarrow (M_P \models \phi) \).

8. Implementation and experimental results

In this section, we apply our verification framework on Digital camera case study (Debbabi et al., 2010a). The related SysML activity diagrams are modeled on Topcased2 then mapped into Prism code via our Java implementation. Listing 3 shows a simplified code for the Digital Camera module. In the purpose of providing experimental results demonstrating the efficiency and the validity of our approach, we verify four system functional requirements. They are expressed in PCTL as follows:

1. The maximum probability value that the TakePicture action should not be activated if either the memory is full memFull = true or the AutoFocus action is still ongoing. T is a constant value referring to the time bound

\[
P_{\text{max}} = \min \{E^{<T}(\text{memFull} \land \text{AutoFocus}) \wedge \text{TakePicture}\}
\]

2. The maximum probability to complete all tasks after turning on the camera. T is a constant value referring to the time bound

\[
P_{\text{max}} = \min \{E^{<T}(\text{WriteMem}) \wedge \text{WriteMem} \}
\]

Listing 3. Digital camera PRISM code fragment.
\( P_{\text{max}} = ?[F^{\geq T}\text{Final}] \)

3. The minimum expected time that the \text{TurnOff} action should be activated after turning on the camera.

\[ R(\text{"time"})_{\text{min}} = ?[F(\text{TurnOff})] \]

The verification results of the above two properties are done using an i7 CPU 2.67 GHz with 8.0 GB of RAM and shown in Fig. 8. For different values of time \( T \), Fig. 8(a) shows that the verification result for Property 1 converges to 0.6 after 4 time units. Fig. 8(b) shows that the verification result for Property 2 converges to 0.916 after 6 time units. For the third property, the minimum reward or minimum expected time that the \text{TurnOff} action should be activated after turning on the camera is equal to 3.448 time units.

Now, we want to verify the abstraction effects over SysML activity diagram described above Fig. 4 to cope with state explosion problem when we check the property

\( P_{\text{max}} = ?[F^{\geq T}\text{TakePicture}] \)

![Fig. 8. The verification of PCTL properties on the digital camera.](image)

![Fig. 9. The abstract SysML activity diagram for Property 4.](image)
For this purpose we apply the algorithm and rules defined by Ouchani, Mohamed, and Debbabi (2014a). This abstraction approach depends on the activity diagram and the PCTL properties as input and produces an abstracted model. However, the abstraction rules do not focus on time constraints. To take the advantages of the algorithm we hide the state (Activity Action) that does not appear in PCTL property proposition set such as: Flash, TurnOff, FinalFlow and Fork2 except those are constrained. The result is shown in Fig. 9 and Table 5 presents its different verification results in function of time interval T. To evaluate the verification cost, we measure the time required for verifying a given property, denoted by \( T_v \) and time required to construct the model denoted by \( T_c \). The results are depicted in Fig. 10. The number of states and transitions for the concrete model are 9364 states and 23,653 transitions, respectively and 4580 states, 12,221 transitions for the abstracted model. As conclusion, we notice that the abstraction rules preserves the results while verification and construction time are optimized.

9. Conclusion

In this paper, we presented a formal verification approach of systems modeled by using SysML activity diagram. The objective is to alleviate errors and failures that can emerge in system design-flow in order to reduce the cost of maintenance and repairing as soon as possible from the implementation. The proposed approach use SysML activity diagram with time annotations using MARTE profile to evaluate the system behavior at different periods of time. Compared to Grobelna et al. (2014), which use NuSMV (Cimatti et al., 1999) for state reachability in activity diagram, our approach leverages probabilistic and timed modeling, allowing the assessment of properties expressed in probabilistic temporal logic. With respect to Ouchani et al. (2014b) which use a probabilistic model to check the SysML activity diagram with difficulty to predict the system behavior due to the static description, we employ a probabilistic and timed model capturing system governed by time constraints. Moreover, in contrast to Marinescu et al. (2015), which use C programs as component behavior but only components (without extracting C behavior) are mapped to UPPAAI, that result in difficulties to evaluate accurately the system (the core behavior is hidden), our approach make decisions at specification level using SysML activity diagram.

The research contribution of this work consists on capturing the activity diagram with time features as Probabilistic Timed Automata supported by PRISM language. We proposed a calculus dedicated to SysML activity diagrams that captures precisely the underlying semantics. In addition, we formalized PRISM language and showing its semantics. Moreover, we proved the soundness of our proposed approach by defining the relation between the semantics of the mapped diagrams and the resulting PRISM models. By this relation, we proved the preservation of the satisfiability of PCTL properties. We have shown the effectiveness of our approach in realistic case study: digital Camera where time and probability are evaluated. In addition, we evaluated the time construction and time checking when the size of diagram is reduced according to PCTL properties.

The practical advantages of the proposed approach in the context of predicting the behavior of systems, consists on providing key decision for errors minimization and product cost reducing via probabilistic model checking. The limitations of the proposed approach relate to the problem of state explosion, we explain in our work how to reduce the number of state by removing just the negligible states that are not time-constrained.

The presented work can be extended in the following four directions. First, we want to reduce the size of the model generated after mapping the SysML activity diagrams into PRISM code that affects memory and time especially for the timed states. Second, we intend to extend our approach to support more constraints that affect the entire system behavior such as energy. Third, we want to study the problem of partitioning using model checking in case of HW/SW CoDesign starting from the component based

<table>
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<tr>
<th>Time interval</th>
<th>Concrete model ( T_v )</th>
<th>Abstract model ( T_v )</th>
<th>Concrete model ( T_c )</th>
<th>Abstract model ( T_c )</th>
<th>Results</th>
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<tr>
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</tr>
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</table>

(a) Verification Time (b) Construction Time

Fig. 10. The abstraction effects on digital camera activity diagram.
specifically. Finally, we intend to translate the system modeled using Component based specification with activity diagram as behavior model to specific language such as VHDL/C/C+/NesC.

References


