Power and Rate Allocation in Cognitive Satellite Uplink Networks

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Abstract—In this paper, we consider the cognitive satellite uplink where satellite terminals reuse frequency bands of Fixed-Service (FS) terrestrial microwave links which are the incumbent users in the Ka 27.5-29.5 GHz band. In this scenario, the transmitted power of the cognitive satellite terminals has to be controlled so as to satisfy the interference constraints imposed by the incumbent FS receivers. We investigate and analyze a set of optimization frameworks for the power and rate allocation problem in the considered cognitive satellite scenario. The main objective is to shed some light on this rather unexplored scenario and demonstrate feasibility of the terrestrial-satellite co-existence.

In particular, we formulate a multi-objective optimization problem where the rates of the satellite terminals form the objective vector and derive a general iterative framework which provides a Pareto-optimal solution. Next, we transform the multi-objective optimization problem into different single-objective optimization problems, focusing on popular figures of merit such as the sum-rate or the rate fairness. Supporting results based on numerical simulations are provided which compare the different proposed approaches.

I. INTRODUCTION

The demand for broadband satellite services is growing at unprecedented rates and the licensed spectrum of 500 MHz for exclusive use, both for uplink and downlink, in the Ka band has been shown to be insufficient to meet the forthcoming demands [1], [2]. In this regard, the application of Cognitive Radio (CR) technology has emerged as a promising solution to enhance the satellite spectrum utilization [3]–[6]. In this paper, we focus on the cognitive Geostationary Orbit (GEO) satellite uplink where Return Channel Satellite Terminal (RCST) transmitters reuse the Ka-frequency bands of incumbent Fixed Service (FS) terrestrial microwave links. In this scenario, the primary goal is to provide good throughput to the RCSTs while protecting the FS links from harmful interference.

Very few works have investigated the cognitive satellite uplink scenario [7]–[10]. The applicability of CR in the aforementioned scenario was discussed in [7], concluding that both satellite and terrestrial systems could potentially operate in the same band without degrading each others’ performance. The same cognitive satellite uplink paradigm was considered in [8]. Specifically, [8] proposes a novel power control based on the inverse Signal-to-Interference plus Noise Ratio (iSINR). However, [8] neglects the aggregate interference caused by multiple RCSTs. It is important to note that, although a Multi-Frequency Time Division Multiple Access (MF-TDMA) scheme is employed in the DVB-RCS2 standard for the return link [11], it may happen that more than one RCST (while operating on different carrier frequencies) produce aggregated interference to the FS microwave network because the carrier bandwidth of the FS microwave links is usually higher than that of the RCSTs [12]. In our previous work [9], we took this into account and proposed a sub-optimal joint power and carrier allocation technique for the cognitive satellite uplink and terrestrial FS co-existence scenario. The technique presented in [9] was compared to different transmit power allocation strategies in [10], depending on the amount of side information available at each satellite terminal.

The cognitive satellite uplink can be cast as an underlay CR network, where cognitive users’ transmit power is limited by interference constraints imposed by the primary system [13]. Power control policies for cognitive underlay networks differ from conventional ones considering that they have severe transmit power limitations that arising from the spectrum sharing scenario. The use of game theory has proliferated as an important mathematical tool for distributed resource allocation and to model the primary-secondary users’ interaction [14], [15]. However, game-theoretic approaches usually lead to complex optimization problems. Simpler decentralized power control algorithms for wireless cellular systems can be found in [16], [17].

Herein, we investigate and analyze a set of optimization frameworks for power and rate allocation in cognitive satellite uplink networks over a specific set of carriers pre-assigned to the users. Our goal is to shed some light on this rather unexplored scenario and possibly pave the way for future research in this area. More precisely, we propose and solve different optimization problems depending on the objective function to be optimized. We start formulating a multi-objective optimization problem [18] where the rates of the RCSTs form the objective vector. For this, we derive a general iterative framework which provides a Pareto-optimal solution of the problem. However, there is no single optimal solution for multi-objective optimization problems. In practice, a single optimal alternative must be identified for engineering designs. To this end, we transform the multi-objective optimization problem into different single-objective optimization problems, whose main goal is to provide the best solution by finding the minimum or maximum value of a single objective function.
that lumps all different objectives into one. In this sense, we focus this paper in popular figures of merit for measuring the performance of a communication system such as the sum-rate or the rate fairness. These popular objective functions, which have been widely investigated for conventional communication networks, have never been examined for cognitive satellite uplink communications. It should be noted that the cognitive GEO satellite uplink differs from the conventional interference channel in that the interference from the FS terrestrial system to the satellite can be neglected due to large distance between them as well as the directivity and limited EIRP of the Ka band terrestrial communications [19]. Regarding fairness, we derive a simple and efficient algorithm which provides the optimal solution in terms of Max-Min fairness. Finally, we compare all the proposed transmit power allocation derived from the different optimization strategies through numerical simulation experiments.

The remainder of this paper is organized as follows. In Section II we present the problem description and formulation. Section III focuses on the multi-objective optimization problem. In Section IV we transform the multi-objective optimization problem into different single-objective optimization problems. In Section V we provide supporting results based on numerical data. Finally, Section VI concludes the paper.

II. PROBLEM DESCRIPTION

Let us consider a cognitive satellite network consisting of $K$ RCSTs and $L$ FS microwave stations, as shown in Fig. 1. As mentioned earlier, although MF-TDMA scheme is employed in DVB-RCS2 standard, the aggregate interference may occur at the FS microwave network. We assume that the return link between each RCST transmitter and the corresponding satellite is not affected by the terrestrial transmissions due to the limited EIRP of the terrestrial networks, the large distance between them and the over-the-horizon directivity of FS terrestrial communications [19].

Regarding resource management, each satellite network is coordinated by a Network Control Center (NCC), which, in current systems, collects Signal-to-Noise Ratio (SNR) values from the RCSTs and manages the network resources and rate demands accordingly. Assume that the NCC has a perfect knowledge of the interference channel link gains, namely $\{a_{k,l}\}$, between the $k$-th RCST and the $l$-th FS station. The interference channel links of a simplified satellite-terrestrial co-existence network with $K = 2$ RCSTs and $L = 3$ FS stations are shown in Fig. 2.

Let $p_k$ denote the transmit power of the $k$-th RCST. Let $\mathbf{p} = [p_1 \ p_2 \ \ldots \ p_K]^T$ represent the power allocation vector, and $p_{\text{max}}$ denote the maximum power budget that a RCST can afford, i.e., $0 \leq p_k \leq p_{\text{max}}$. The achievable rate by the $k$-th RCST is a function of the corresponding transmitted power $p_k$ and is given by

$$r_k = \log_2 \left( 1 + \frac{d_k p_k}{\sigma_k^2} \right) \text{ [bits/sec/Hz]},$$

where $d_k$ denotes the channel power gain of the link from the $k$-th RCST to the satellite (including transmit and receive antenna gains and propagation losses) and $\sigma_k^2$ denotes the noise power level of the $k$-th satellite link.

To protect the terrestrial stations against excessive interference, the aggregated interference at the $l$-th station $I_a(l)$ has to satisfy the $I_a(l) \leq I_{\text{thr}}$, where $I_{\text{thr}}$ denotes the maximum tolerable interference level which is defined by the regulatory authorities. Typical reference limitations are given by ITU recommendations such as ITU-R F.758, where the interference level is recommended to be $-10$ dB below the receiver noise.

Our goal is to maximize the performance of the cognitive satellite uplink while protecting the FS links from harmful interference. Therefore, the user rates $r_k$, $k = 1, \ldots, K$, are our objective functions, which cannot be treated separately because they may be conflicting due to the interference constraint $\mathbf{A} \mathbf{p} \leq \mathbf{I}_{\text{thr}} \mathbf{1}$, where $\mathbf{1}$ is the all-one vector and the channel gains $\{a_{k,l}\}$ between the $k$-th RCST and the $l$-th FS station have been rearranged in a matrix format as

$$\mathbf{A} = \begin{bmatrix} a_{1,1} & \cdots & a_{K,1} \\ \vdots & \ddots & \vdots \\ a_{1,L} & \cdots & a_{K,L} \end{bmatrix}.$$ 

We resort to multi-objective optimization [20]–[22] to consider an optimization of rates for various RCSTs in the network. In
particular, the associated multi-objective optimization problem can be formulated as follows:

$$\begin{align*}
\max_p & \quad r \\
\text{s.t.} & \quad Ap \leq I_{k+1}1 \\
& \quad 0 \leq p_k \leq p_k^{\max}, \; k = 1, \ldots, K
\end{align*}$$

(3)

where $r = [r_1 \; \ldots \; r_K]^T$, and $\{r_k\}$ are as defined in (1).

In this paper, we propose different alternatives to tackle the optimization in (3).

### III. PARETO-OPTIMAL USER RATE MAXIMIZATION

Inspired by the literature on economics, Pareto optimality describes a state in optimization problems in which resources are distributed such that it is not possible to improve a single objective without causing at least one other objective to become worse than before the change [23]. The set of Pareto optimal points is typically referred to as the Pareto boundary. All feasible combinations form the Pareto feasible set which is enclosed by the Pareto boundary.

We note that as $\{r_k\}$ in (3) are monotonically increasing functions of the corresponding $\{p_k\}$, the multi-objective problem in (3) is equivalent to

$$\begin{align*}
\max_p & \quad p \\
\text{s.t.} & \quad p \in \Omega
\end{align*}$$

(4)

where $\Omega$ denotes the set of feasible vectors $p$ satisfying the two constraints of (3). Following the notation in [18], we define the Pareto feasible set as $\mathcal{P} = \{p : p \in \Omega\}$, which contains all the combinations of possible values $p_k$ that are simultaneously attainable with the available resources.

To find a Pareto-optimal solution to (4), meaning a solution that lies on the Pareto-boundary of the problem, we first propose a general iterative framework and then discuss its application to (4).

#### A. General iterative framework for pareto-optimization

Consider the following optimization problem:

$$\begin{align*}
\max_x & \quad \text{all } f(x, y) \\
\text{s.t.} & \quad x \in \Gamma
\end{align*}$$

(5)

**Proposition 1:** Assuming that all Pareto-optimal solutions of (5) are finite, a Pareto-optimal solution of (5) can be obtained using the following iterative approach. Given $x^{(t)} \in \Gamma$, ($t \geq 0$), obtain $x^{(t+1)}$ as the solution to the following (single-objective) optimization problem,

$$\begin{align*}
\max_{x^{(t+1)}} & \quad \min_y \left\{ \frac{f(x^{(t+1)}, y)}{f(x^{(t)}, y)} \right\} \\
\text{s.t.} & \quad x^{(t+1)} \in \Gamma
\end{align*}$$

(6)

Proof: At $x^{(t+1)} = x^{(t)}$, we have $f(x^{(t+1)}, y)/f(x^{(t)}, y) = 1$, and consequently, $\min_y \left\{ f(x^{(t+1)}, y)/f(x^{(t)}, y) \right\} = 1$. As a result, for the optimal $x^{(t+1)}$ of (6),

$$\begin{align*}
\min_y \left\{ \frac{f(x^{(t+1)}, y)}{f(x^{(t)}, y)} \right\} & \geq 1
\end{align*}$$

(7)

which implies that $f(x^{(t+1)}, y) \geq f(x^{(t)}, y), \forall y$. □

#### B. Application to the cognitive satellite uplink

Returning to the specific problem of power allocation in (4), the previous general framework can be employed simply as follows. Given $p_k^{(t)} \in \mathcal{P}$ ($t \geq 0$), the power allocation for the time instant $(t + 1)$, $p_k^{(t+1)}$, is given by the solution of

$$\begin{align*}
\max_{p_k^{(t+1)}} & \quad \min_k \left\{ \frac{p_k^{(t+1)}}{p_k^{(t)}} \right\} \\
\text{s.t.} & \quad p_k^{(t+1)} \in \Omega
\end{align*}$$

(8)

which requires solving a simple Linear Program (LP) at each iteration. Note that the value of $p_k^{(t+1)}/p_k^{(t)}$ should approach one as $t \to \infty$.

The optimization problem in (8) can be reformulated in a simpler form observing that $\alpha = \min_k \left\{ p_k^{(t+1)}/p_k^{(t)} \right\}$ is equivalent to $\alpha p_k^{(t)} \leq p_k^{(t+1)}$, $k = 1, \ldots, K$, for $\alpha$ being the largest possible real number. In particular, we can rewrite (8) as

$$\begin{align*}
\max_{p_k^{(t+1)}, \alpha} & \quad \alpha \\
\text{s.t.} & \quad p_k^{(t+1)} \in \Omega \\
& \quad \alpha p_k^{(t)} \leq p_k^{(t+1)}, \; k = 1, \ldots, K
\end{align*}$$

(9)

in which the optimal $\alpha$ would be always larger than or equal to one. The solution to (9) is always a rate tuple on the Pareto rate boundary of (3), regardless of the initial point $p^{(t=0)} \in \mathcal{P}$.

**Remark** Due to the convexity of the constraint set in (8) and (9), the proposed iterative method should converge in exactly one iteration, as finding the global optimum of (9) implies that some of the $\{p_k\}$ cannot be increased any further while $p \in \Omega$.

### IV. MULTI-OBJECTIVE TO SINGLE-OBJECTIVE TRANSFORMATION

In the previous section, we have seen that the solution of a multi-objective optimization problem consists of a set, namely the Pareto boundary. However, from a practical point of view, a communication system ultimately requires a single solution for operation. Picking a desirable point out of the set of the Pareto boundary requires the incorporation of preferences or priorities into the problem [24].

We divide this section into two parts. In the first part, we focus on the weighted sum approach which transforms the multi-objective optimization problem into a single-objective optimization problem. The weighted sum is the simplest multi-criteria decision making method but it is a compensatory method in the sense that “poor” user rates can be compensated...
by “good” ones. Moreover, the relation between weights and user rate requirements remains unsolved [25].

The second part focuses on the user rate fairness. Rate fairness essentially tries to avoid undesirable situations in which a user maximizes its rate at the expense of some other users. In this case, the rate of all users will be degraded to match the rate of the user with the lowest quality channel.

### A. Maximization of weighted sum-rate

The multi-objective optimization problem introduced in (3) can be reformulated as maximization of a weighted sum of user rates, which is one of the most popular figures of merit for measuring the performance of a communication system. The maximization of the weighted sum of user rates with adaptive power control can be expressed using the weighted sum approach as follows:

\[
\max_{\mathbf{p}} \sum_{k=1}^{K} w_k \log_2 \left(1 + \frac{d_k p_k}{\sigma^2} \right)
\]

subject to:

\[
\mathbf{p} \in \Omega
\]

where non-negative \( \{w_k\}, k = 1, \ldots, K \) are the given weights assigned to different RCSTs, with \( \sum_{k=1}^{K} w_k = 1 \). Note that the objective function in (10) is concave with respect to the power values, so it can be solved numerically using interior-point methods. To solve (10) we used the CVX package [26], which makes use of a primal/dual solver to deal with log-based objective.

### B. User rate fairness

With the emergence of heterogeneous networks, fairness becomes crucially important along with concerns for excellent throughput. There are many definitions of “fairness” in the optimization literature [27], hence no consensus about a unique definition is yet obtained.

Here, we consider the two most used definitions of fairness, namely Max-Min fairness and proportional fairness.

1) **Max-Min fairness**: Max-Min fairness can be achieved if and only if the allocation of available resources is feasible and an attempt to increase the rate of any participants necessarily results in the decrease in the rate of some other participants with the smallest rate. In other words, it maximizes the user with the minimum rate:

\[
\max_{\mathbf{p} \in \Omega} \min_k \{r_k\}
\]

On the downside, this definition of fairness does not perform well in the presence of bottleneck users: if one user imposes strong interference constraints it may prevent the others from improving.

There are several algorithms for computing the Max-Min fair allocation depending on the area of application. In general, the most widely used algorithm for obtaining max-min fairness is the water-filling algorithm (WF) [28]. Intuitively, WF satisfies users with a poor conditions first, and distributes evenly the remaining resource to the remaining users enjoying a good condition. In our case, we will focus first on assigning the power of the RCST transmitters (the bottleneck RCSTs) affecting the worst FS station, i.e., the FS station which receives the highest level of aggregate interference. The proposed max-min algorithm for the cognitive satellite uplink is presented as Algorithm 1.

### Algorithm 1 Max-Min Fairness

**Require**: Interference link gains \( \mathbf{A} \), number of FSS terminals \( L \), interference thresholds \( l_{\text{thr}} \), maximum transmission power \( P_{\text{max}} \) and step size \( \alpha \).

1: **Initialize**: 
   - The powers \( \mathbf{p}^{(t=0)} \leftarrow \mathbf{0} \)
   - The interference constraints \( \mathbf{I}^{(t=0)} \leftarrow l_{\text{thr}} \mathbf{1} \)
   - The RCSTs with updated power \( \mathbf{A}^{(t=0)} = \emptyset \)
   - The iteration counter \( t = 1 \)
2: **repeat**
   3: Identify the worst FS station in terms of received interference (the bottleneck) assuming that all RCSTs transmit with the same power:
      \[
      l_w = \min_l \left[ \frac{\mathbf{I}^{(t-1)}(l)}{\sum_{\mathbf{A}^{(t)}(t)} a_{l,k}} \right].
      \]
4: Assuming all RCSTs transmit with the same power, find the RCST that contributes the most to the interference of \( l_w \):
   \[
   k_w = \max_{k \in \mathbf{A}^{(t)}} a_{l_w,k}.
   \]
5: Derive what is the maximum transmit power that the RCSTs can transmit without exceeding the interference constraint of \( l_w \):
   \[
   p_w = \frac{\mathbf{I}^{(t-1)}(l_w)}{\sum_{k \in \mathbf{A}^{(t)}} a_{l_w,k}}.
   \]
6: Assign \( \mathbf{p}^{(t)} \leftarrow \mathbf{p}^{(t-1)} \) and update the the \( k_w \)-th RCST, with value \( p_{k_w} = p_w \), and update \( \mathbf{A}^{(t)} = \mathbf{A}^{(t-1)} \cup \{k_w\} \)
7: Find new interference constraints:
   \[
   \mathbf{I}^{(t)} = \mathbf{I}^{(t-1)} - [\mathbf{A}]_{k \in \mathbf{A}^{(t)}} \mathbf{p}^{(t)}_{k \in \mathbf{A}^{(t)}}.
   \]
   where \( [\mathbf{A}]_{k \in \mathbf{A}^{(t)}} \) denotes the matrix formed with the columns of \( \mathbf{A} \) indicated by the index set \( \mathbf{A}^{(t)} \).
8: \( t \leftarrow t + 1 \) and return to step 1 if the stopping criterion is not met.
9: **until** \( t = K + 1 \) (all the RCSTs have been updated).
problem can be formulated as,

$$\max_{\mathbf{p}} \sum_{k=1}^{K} \log_{10}(p_k)$$

subject to $\mathbf{p} \in \Omega$ (16)

which can be solved using CVX package [26].

V. SIMULATION RESULTS

In order to evaluate the proposed optimization solutions, we consider a simple scenario with $K = 2$ RCSTs like the one depicted in Fig. 2. The parameters are defined as follows,

$$\mathbf{A} = \begin{bmatrix} 0.4 & 0.25 \\ 0.1 & 0.3 \\ 0.2 & 0.1 \end{bmatrix}, \quad I_{thr} = 2, \quad p_{\text{max}} = 10, \quad d_k = 1, \sigma_k^2 = 1, \forall k$$

(17)

Figures 3(a) and 3(b) illustrate the power and rate Pareto regions and the corresponding Pareto power and Pareto rate boundary for the cognitive satellite uplink sharing frequency resources with the terrestrial network.

Figures 3(c) and 3(d) show the results of the proposed techniques and compare the result with the gradient-based power control proposed in [10], [17] with $\alpha_k = 0.1, \forall k$, and 200 iterations. For comparison, we solved and plotted the result of $\max \sum_{k=1}^{K} w_k p_k$, subject to $\mathbf{p} \in \Omega$, which corresponds to the maximization of the sum-power optimization. We considered $w_k = 1, \forall k$, both for the sum-rate and sum-power optimization. For (9), we selected the initial point randomly over the power Pareto region. Both power and rate results shown in Fig. 3 have been summarized in Table I and II, respectively.

In both Figures 3(c) and 3(d), we observe the technique presented in [10] perfectly matches with the solution of the maximization of the sum-powers, which goes in line with the conclusion in [17]. However, this is not the optimal in terms of sum-rate neither in terms of fairness. As expected, the max-min fairness gives the same rate to both users. The PF, on the other hand, allows a small difference between individual rates to achieve higher sum-rate compared to the max-min. The Pareto optimal solution lies in the Pareto boundary, but its value strongly depends on the initial power assignment.

According to the achieved results, PF seems to be the best solution since it provides a good trade-off between fairness and overall satellite throughput. Even so, the choice of appropriate algorithm depends on the design criteria we want to follow.

VI. CONCLUSION

In this paper, we investigated and analyzed a set of optimization approaches to solve the power and rate allocation in cognitive satellite uplink where RCSTs reuse frequency bands of Ka-band terrestrial microwave links. We solve and compare the proposed methods concluding that proportional fairness is the best solution in terms of the good trade-off it offers regarding fairness and rate. Results have demonstrated the potential of power allocation optimization to satisfy the interference requirements imposed by the incumbent terrestrial network.

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Fig. 3: Cognitive satellite uplink with $K = 2$ RCSTs and $L = 3$ FS stations: (a) Power Pareto and feasible set, (b) Rate Pareto and feasible set, (c) Power performance evaluation, and (d) Rate performance evaluation.


