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ESSAYS ON SLOT ALLOCATION AND EMISSION LICENSES IN AIRPORTS

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Contents

Abstract

Airport congestion is a growing problem in many airports worldwide. The movement towards reduction of government involvement at the micro level has led to the development of congestion-management approaches. This dissertation focuses on studying price and quantity based management solutions, in the presence of airport congestion. It comprises the following four chapters.

Airport Congestion and Inefficiency in Slot Allocation (joint with Pierre Picard, Alessandro Tampieri). This chapter analyses optimal slot allocation in the presence of airport congestion, and interests in the efficiency consequences of airport allocative strategies. We model peak and off-peak slots as vertically differentiated products, and congestion limits the number of peak slots that the airport can allocate. Inefficiency emerges when the airport does not exploit all its slots. We show that for a private airport, inefficiency may arise if the airport is not too congested and the per-passenger fee is small enough, while with a public airport it does not emerge. Furthermore the airport, irrespective of its ownership, tends to give different slots to flights with same destination if the underlying market is a duopoly, and a single slot if the underlying market is served by a monopoly.

Slot Allocation at Congested Airport with Endogenous Fee (joint with Alessandro Tampieri). This chapter extends the analysis of the previous chapter by considering endogenous airport charge. It contributes to the understanding of a peak period congested airport's optimal fee setting behaviour when it has power to allocate slots. By explicitly incorporating the endogenous fee setting behaviour into our previous framework with exogenous fee, we find that allocative inefficiency is precluded at a private airport if the fee can be determined by the airport.

Per-flight and per-passengers congestion pricing when airline quality differs. The third chapter investigates and compares congestion pricing scheme under Cournot competition and Bertrand competition, accounting for both per-passenger and per-flight charges. It studies a vertically differentiated airline market, where two airlines serve a same origin-destination route from a congested airport, and evaluates the mix of per-passenger and per-flight airport charge. We find out that under Bertrand competition, the low-quality airline partially internalizes congestion delay, whilst the high-quality airline does not internalize at all. For both, the magnitude of overprovision of frequency is greater under Bertrand competition than under Cournot competition. In addition, when the route is served by a monopoly airline, both passenger volume and flight frequency are undersupplied from a social viewpoint.

Uniform-price auction with endogenous supply: Should seller's reservation price be kept hidden? The last chapter examines and compares airlines' bidding behaviour in noise license auction where the monopolist seller has reservation price. In a uniform-price auction, the seller is empowered to decide how much to sell after receiving the bids. We find that when the reservation price is revealed to the bidders, they bid more aggressively when its realization is high. While when it is secret, there exists a unique equilibrium outcome for bidding behaviour in our model setting. Moreover, allotting licenses via auction results in fewer licenses traded than would have by a social planner.

Introduction

Global air traffic has grown substantially in the past decades. As a result, many airports around the world are working at or even beyond capacity at some operating periods. Notably, an airport is identified as being capacity constrained when the demand for movements exceeds the available airport handling capacity. According to the International Air Transport Association (IATA)[1], the number of capacity constrained airports (categorized as Level 3 airports in the IATA guideline) in the world increases markedly from 136 airports in the year 2000 to 179 in the year 2015. In addition, among the 119 airports that are currently subject to some level of congestion at certain periods of the day (categorized as Level 2 airports), many are forecasted to become fully congested in coming years. With traffic demand continues to rise, congestion is constantly a growing trend. In all instances, though the fundamental solution to a growing strain on capacity is by building new runways and other infrastructure, the long gestation and construction horizon make it hardly suitable as a short or medium term solution. Additionally, provision of extra capacity is often fraught with political opposition and public resistance, which typically revolve around negative environmental impacts such as noise and greenhouse gas emissions.

This raises the issue of finding interim solution to mitigate congestions through making better use of existing capacity, in particular through management of airport slots. A slot is an authorization granted to an airline to use the full range of airport infrustructure necessary for taking off and landing within a certain time window. In Europe, allocation of scarce runway capacity is implemented at 106 airports in accordance with the principles of IATA Worldwide Slot Guidelines (WSG). The guidelines set standard procedures and support a transparent, neutral and non-discriminatory allocation process. However, one important feature of slots has not been given due consideration, that demand is higher in peak periods than in off-peak periods at identical airport

[1] For more details see Worldwide Slot Guidelines, August 2015.

charge, because peak periods are more prefered than off-peak periods by all passengers. In the presence of the widespread practice of slot control and management, a study into slot allocation decision within the context of differentiated slots would provide useful implications.

Against this background particular attention should also be given to airport ownership. Indeed, historically most airports worldwide were public utilities and regulation on airport fees are held to non-profit levels. From the 1980s onwards, several waves of privatization have subsequently taken place in Europe, Asia, Australia and New Zealand and Latin America. Until 2013, 450 airports worldwide are entitled to some form of private-sector involvement. Among the 100 largest airports worldwide, 40 are either fully or partially owned or controlled by investors.[2] The reform process resulted in the emergence of varies ownership modes. In many instances the government maintains a controlling interest and plays a crucial role in establishing airport charges. Essentially, government entails price regulation to prevent airport imposing excessively high charge. The impact of different levels of economic regulation on airport's slot allocation decision remains to be analyzed. Relative to this point there is limited literature on the theory side.

This thesis is devoted to the investigation of two economic instruments to alleviate airport congestion, notably the slot allocation and congestion pricing. It also looks into one important type of airport externality, the noise pollution. Particular attention will be paid to the ownership and regulative intensity faced by an airport.

The first two chapters examine endogenous allocation of peak and off-peak slots, with the purpose of providing argument that could serve as a guidance for political proposals. The analysis in these two chapters suggests that privately managed airports that are subject to price regulation may induce allocative inefficiency, while their publicly owned counterparts would never engage in inefficient allocation. For such a private regulated airport, the emergence of allocative inefficiency is contingent on the level of airport charge, as well as peak slot scarcity. Peak slot scarcity can be associated to the classification of airport levels by IATA slot guidelines, that level 2 airports represent moderate peak slot scarcity, and level 3 airports represent severe peak slot scarcity.

Alternatively, other widely discussed congestion remedy is related to the pricing mechanism, namely congestion pricing. As is known, though the concept of congestion pricing stems from road traffic, a distinct characteristic of airline market is that

[2] See CAPA Centre for Aviation analysis report.

airlines are non-atomistic[3] agents. Hence they internalize self-imposed congestion (see e.g. Brueckner (2002, 2005), Pels and Verhoef (2004), Zhang and Zhang (2006), Basso (2008), Basso and Zhang (2008a), Morrison and Winston (2007)). As is typical in these literature, under congestion pricing scheme, externalities are corrected when each airline pays for the congestion toll. This toll should account for the portion of uninternalized congestion each airline imposes on all other airlines. Obviously, this toll is smaller than the toll charged for atomistic airlines. In light of this statement, it is to be expected that a smaller airline will be more susceptible to a higher toll than a larger airline, owning to a smaller fraction of self-imposed congestion. The application of differentiated tolls, especially higher tolls to the smaller airlines, is more often than not politically controversial, due to perceived unfairness (Brueckner (2002), Morrison and Winston (2007)).

Since airlines are non-atomistic agents and possess market power, the concern associated with market power effect from the part of the airlines emerges. It follows intuitively that the optimal toll would contain a component that essentially reduces the toll, and that airlines would restrict output. If the market power effect outweighs congestion effect, then there is too little traffic (Silva and Verhoef (2013)). When the airport is owned and operated by a public entity who pursuits socially optimal level of output, such as a municipality, state, or public authority, then rather than charging a toll, the airport may have to subsidize airlines. By and large, much of existing literature on airline competition in the presence of airport congestion has focused on homogenous airlines. Yet the prevailing of low-cost airlines leads one to question how optimal congestion pricing would work if competing airlines are heteogenous. What are airlines' internalization behaviors when they differ in quality, as proxied by flight frequency? Would the magnitudes of internalization vary substantively with competition type? These research questions are addressed in Chapter 3.

As mentioned previously, one significant factor impeding the capacity increment is local residents' resistance which is mostly due to noise disturbance. In effect, noise pollution has been a constant concern in many airports especially the major ones. In economic literature, in order to compensate residents in the vicinity of an airport who suffer from the noise nuisance of planes taking off and landing, some literature are in favor of setting up an emission license mechanism. Under such a regime, first of all noise is quantified according to some assessment criteria, then emissions licenses are assigned to the affected residents. Thereafter airports purchases licenses from the residents, in order to operate aircraft activities from this airport (Brechet and Picard (2010)).

[3] In the literature of congestion pricing, non-atomistic refers to airlines that have market power.

An airline could not emit more noise pollution than its holding of licenses. Chapter 4 of this dissertation examines the implementation of uniform-price format auction to the airport noise emission licenses market. Uniform-price auction has been discovered to lead to low-price equilibria (Wilson (1979), Ausubel and Cramton (2002)), due to the nature of bidders (airlines) strategic manipulation behavior, which is unfavorable to the seller (a single representative of all residents in this context). To minimize the extent of undesirable low-price equilibria, some literature introduces endogenous supply, i.e., the seller retains flexibility in determining supply quantity after collecting airlines' bid schedules (Back and Zender (1993), Lengwiler (1999)). Particularly, Chapter 4 compares airlines bidding behavior when the reservation price of emission license is either revealed or secret, and find that the choice of information relevation depends upon the magnitude of reservation price and policy maker's objectives.

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Chapter 1

Airport Congestion and Inefficiency in Slot Allocation

1.1 Introduction

Over the past decades, airline traffic growth has outpaced capacities at many of the world's major airports.[1] As a result, airport congestion has become a major issue faced by many airports worldwide. Airport congestion is likely to get worse in the coming decades, being generated by an expanding demand due to increase of income, and the growth of some developing countries.[2]

In the analysis of airport congestion, the economic literature focused mainly on "congestion pricing" (for which carriers pay a toll according to their contribution to congestion) as a regulatory tool to deal with congestion.[3] However, despite its theoretical feasibility, congestion pricing has not been practised in the real world. By contrast, slot allocation is the usual approach to management of congestion at airports. According to IATA World Scheduling Guidelines, a slot is "the permission given by a managing body for a planned operation to use airport infrastructure that is necessary to arrive or depart at an airport on a specific date and time".[4] Under a slot system, the airport

[1] For instance, over half of Europe's 50 largest airports have already reached or are close to their saturation points in terms of declared ground capacity (Madas and Zografos, 2008).

[2] The European Commision estimated that half of the world's new traffic will come from Asia Pacific region in the next 20 years. They expect that air traffic in Europe will roughly double by 2030, and that 19 key airports will be at saturation. See MEMO/11/857.

[3] For an early contribution on congestion pricing see Levine (1969). Recent representative studies include Brueckner (2002, 2005). Under congestion pricing, carriers could place as many flights as they wish provided they pay the toll, thus the overall level of congestion is determined by airline decisions.

[4] See Worldwide slot guidelines.

authority determines the total number of slots to make available, and slots are distributed among the airlines according to some allocation rule.[5] Given the prevalence of slot systems, a theoretical analysis investigating the interaction between slot allocation and congestion seems highly policy relevant.

In this chapter we analyse endogenous slot allocation in the presence of airport congestion, and we investigate the conditions under which the allocation choice is inefficient. We refer to a "slot" as the permission granted to a certain airline to use airport infrastructure for a planned operation at a specific time window of the day.[6] We examine a setting where an airport wants to maximize the number of passengers, and sorts slots according to different departure flights, while airlines compete in the flights market. As in Brueckner (2002), we model peak and off-peak slots as products of different qualities in a model of vertical differentiation.[7] Peak slots are congested, mirroring the situation of capacity shortages at peak hours faced by many airports. We analyze both a private and a public airport being restricted to levy a uniform per-passenger fee for flight activities, this being pre-determined by administrative bodies.[8] We consider separately the case where two flights towards a same destination (hereby denoted by "pairwise flights") are served by two airlines, and where they are served by a monopoly. In this complete information setting, airlines know the total provision of slots and the fact that each participating airline receives a single slot. Finally, we define as "allocative inefficiency" the situation in which not all the slots available are exploited.

Because peak slots are preferred by passengers to off-peak slots, the airport's slot

[5]For example, FAA (Federal Aviation Administration) capped peak hour flight movements at New York La Guardia, J.F.Kennedy, and Neward airports. As for Chicago's O'Hare airport, FAA persuaded two major airlines United and American Airlines to reduce peak flight activities while prohibiting smaller airlines from increasing flights to fill the gap.

[6]Though in practice congestion is not exclusively confined to runway congestion, and might embody other capacity dimensions such as environmental concerns, we nevertheless focus on runway congestion.

[7]Our approach differs from Brueckner (2002) as follows. In Brueckner (2002)'s framework, a monopoly airport chooses the critical points on the continuum that respectively define whether to fly or not and whether to fly in peak slots or off-peak slots. Focusing on finding the optimal congestion pricing, he implicity assumes that airport capacity is sufficient to meet peak hour demands. Unlike Brueckner (2002), our interest stems from the scarcity of peak hour slots. Thus we focus on the allocation instead of using the pricing tool.

[8]The analysis of a private airport also seems relevant. Although airports have long been owned by governments, there has been a significant worldwide trend towards government facilities privatization beginning from the middle of the 1980s. Following the United Kingdom, many major airports in Europe, Australia and Asia have followed suit and have undergone privatization or are in the process of being privatized. In principle, privatization is characterized by the transfer of ownership structure from state-owned to private enterprises.

assignment creates an exogenous quality differential between the carriers when the assigned slots are for different periods. The carriers compete conditional on this quality differential, and the resulting prices and passenger volumes therefore depend on the slot assignments. Depending on parameter values, the total passenger volume (and hence fee revenue for the airport) could be higher when the airport withholds a peak slot that it could allocate to the carriers, leaving the slot unused and the airport's peak capacity thus not fully exploited. This outcome is inefficient from society's point, but it is a possible feature of equilibrium in the model.

When the airport is private and each destination is served by a duopoly airline market, the results depend on whether the number of slots is lower than the number of destinations (the airport is "busy") or not (the airport is "not too busy"). A busy airport uses all the available peak slots to implement "peak/off-peak" configuration. The results are driven by differentiation which, on the one hand, increases the number of passengers, but on the other hand softens competition. The first effect more than outweighs the second effect, thus the airport prefers to adopt differentiation rather than to allocate both peak slots for the same destination.

A not-too-busy airport chooses its allocative strategy according to the amount of per-passenger fee. In particular, it implements a mix of "peak/off-peak" and "peak/peak" market configuration if the per-passenger fee is high, and "peak/off-peak" configuration in each market if the per-passenger fee is low. In the latter case, given that the number of destinations is lower than the number of slots, allocative inefficiency emerges. These results can be explained as follows. Low per-passenger fees imply cheap flight tickets, thus the airport can allocate slots in the peak/off-peak configuration without losing passengers, even with less competition. Allocative inefficiency is due to the fact that a not-too-busy airport does not need the extra slots to reach the optimal slot allocation. With high per-passenger fees, the airport allocates some peak/peak market configuration (undifferentiated slots) in order to induce more competition and to keep the price of flight tickets sufficiently low.

The emergence of allocative inefficiency in our results corresponds to the common practice in airport management of declaring a number of slots being lower than an airport's full capacity (Mac Donald, 2007, and De Wit and Burghouwt, 2008). Indeed, as De Wit and Burghouwt (2008) point out, "an efficient use of the slots at least requires a neutral and transparent determination of the declared capacity".

If each destination is served by a monopolist and one peak slot is assigned to them, the monopoly airline would choose to operate in the peak slot only, thus leaving unused the off-peak slot. Indeed, given the same (marginal) per-passenger fee for operating at a peak or off-peak hours, the airline prefers to put all seats in the peak flight. Moreover, if two peak slots are assigned to the monopoly airline, and assuming a preference for operating one flight at peak slot rather than two (for operating costs not modelled here), then the monopoly airline would also leave unused the extra peak slot. In turn, the airport would assign a single peak slot to each destination market, as long as peak slots are available. Naturally, in this case allocative inefficiency does not occur. Finally, the results are similar when the airport is public, with the exception than inefficiency does not emerge.

To illustrate the empirical relevance of this mechanism, we have briefly investigated the slot allocation in the city-pair markets of the 5 most busiest US airports and of a random sample of 10 mid-range US airports (see Appendix A). Table 1 shows the numbers of origin-destination routes operated by monopoly, duopoly and oligopoly, and the pattern of slot occupancy. While a large bunch of city-pair service is supplied by monopoly airlines, there exists a significant portion of flight activity served by competing airlines.

	# Origin-destinations served by			# Slots occupation(%)	
	Mon. (%)	Duo.	Olig.	Peak	offpeak
Top 5	525 (62%)	161(19%)	157(19%)	29%	71%
Mid sized 10	200 (73%)	53 (20%)	19 (7%)	28%	72%

Table 1. Pattern of market structure and slot occupation.

Our theoretical analysis claims that slot allocation can be used by congested airports as a discrimination tool in markets where several firms compete. To highlight this, we focus on duopoly city-pair markets and construct an index $\mathcal{I}$ that measures whether the competing airlines are put in the same slot. Typically, along the discussion in Section 3, two competing airlines, each operating one flight to the same destination, are allocated in the same time slot if $\mathcal{I} = 0$ and are separated in a peak and an off-peak slot if $\mathcal{I} = 1$. A higher aggregated index $\mathcal{I}$ measures stronger use of a slot discrimination. Excluding inter-hub traffic and high frequency destinations, we then find evidence consistent with our model prediction that slot discrimination is stronger in the largest airports, see Table 2.

	Average Index $\mathcal{I}$	
	Largest 5 airports	Mid 10 airports
5-11am	0.38	0.23
11-16pm	0.32	0.2
16-23pm	0.44	0.44

Table 2. Average index $\mathcal{I}$

So far, slot allocation has drawn relatively little interest in the economic literature, with few but noteworthy contributions. Barbot (2004) models slots for airline activities as products of either high or low quality, and carriers choose the number of flights they operate. She shows that slot allocation improves efficiency according to the criteria for assessment, and welfare in fact decreases after re-allocation. Unlike the present paper, in Barbot (2004)'s model carriers could operate as many flights as they want. Our paper limits the number of peak slots, in order to address congestion. Verhoef (2008) and Brueckner (2009) compare the pricing and slot policy regimes. They show that the first best congestion pricing and slot trading/auctioning generate the same amount of passenger volume and total surplus. They investigate a single congested period. Their contributions do not distinguish between peak and off-peak hours, and allow the airport to allocate slots without charges. Although this seems a plausible description of some public airports, non-profit behavior does not seem likely for a private airport. Departing from Verhoef (2008) and Brueckner (2009), we assume that each airline operates a single flight. Our approach models certain time intervals that are most desired by all passengers as the peak period. In particular, the total number of slots that an airport could grant in the peak period does not meet the demand of passengers.

The remainder of the chapter is organized as follows. Section 1.2 introduces the model. Section 1.3 presents the baseline results, in which the airport is assumed to be private and an airline duopoly serves each destination. Section 1.4 and 1.5 show the changes in the results when either a monopoly airline serves for each destination or the airport is publicly owned, respectively. In section 1.6, we show the changes in the results when density is heterogenous among different flights. Section 1.7 concludes.

1.2 The model

In the baseline model, we consider a private airport that links N destinations $d \in D$.[9] Each destination is served by two flights f and $f' \in \mathcal{F}$ operated by independent air-

[9] Section 1.5 analyses the case with a public airport.

lines.[10] There are therefore $2N$ flights and $2N$ airlines ($\#(D) = N$ and $\#(\mathcal{F}) = 2N$). Formally, we define the mapping I from flights f to destinations d such that $I(f) = d$. The inverse mapping from flights to destinations is defined as $A(d) = \{f : I(f) = d\}$. We assume that airports at destinations are uncongested, so that the allocation decisions do not affect the flight scheduling of destination airports. Furthermore, we focus on the case of single trip departing flights. Return trip flights can be dealt with by either an identical analysis with two runways, or simply by adding a scale factor if there is a single runway. Destinations are independent in the sense that they are neither substitutes nor complements; therefore the demand for one destination is irrelevant to demands for other destinations. We assume that the quality differential is characterized only by the departing time. Although the quality of an airline depends on many factors, this approach allows us to concentrate on the congestion issue. There are two travel periods, denoted as peak and off-peak. A peak period represents the time window that consists of the most desirable travel times in a day, for instance early morning and late afternoon. The peak period may contain a collection of disjoint time intervals like 7:00-9:00 and 17:00-19:00. The off-peak period, by contrast, contains all other time intervals that do not belong to the peak period. In order to address the problem of peak slots congestion, the off-peak period is assumed to be uncongested, i.e., airport capacity can serve all flights in off-peak time intervals. Conversely, the peak period is congested in the sense that airport capacity cannot serve all flights within peak period. This assumption captures traffic patterns at many airports nowadays. All potential passengers agree that peak hours (denoted as subscripts h for higher quality) are more preferable than off-peak hours (denoted as subscripts l for lower quality) at an equal price. At peak hours the demand to use airport runways is much higher than off-peak hours, so that the perceived "qualities" of slots, s_l and s_h, satisfy exogenously $s_h > s_l > 0$. Finally, a slot allocation is defined as the mapping g from airline f to a slot type i, $g : \mathcal{F} \to \{l, h\}$, so that $g(f) = i$. For instance, $g(f) = h$ reads as airline f is allocated a peak time slot.

We assume that in each destination market the airlines engage in seat (quantity) competition.[11] We denote $p^f_{ii'}$ as the price charged for flight f flying to destination $d = I(f)$ that takes off at slot i while its competitor on the same destination takes off at slot i', $i, i' = \{l, h\}$. Similarly, $q^f_{ii'}$ denotes the number of passengers served by this flight. Following the general framework of vertical differentiation (Gabszewicz

[10] Section 1.4 extends the analysis to the case where a private airport interacts with airline monopolies.

[11] The assumption of quantity competition is common in the airline economics literature. See Brueckner (2002), Pels and Verhoef (2004), Basso (2008). Brander and Zhang (1990) find empirical evidence that the rivalry between duopoly airlines is consistent with Cournot behavior.

and Thisse, 1979), in each destination market the demand is generated from a unit mass of passengers indexed by a type parameter v. Passengers differ in tastes, the taste parameter is described by $v \in [0, 1]$, v being uniformly distributed with unit density. We assume each passenger flies at most once, and, if a passenger refrains from flying, her reservation utility is zero. Formally, a potential passenger in the destination market d with flights f and f', $d = I(f) = I(f')$, has the following preferences:

$$U^d = \begin{cases} vs_i - p^f_{ii'} & \text{if she takes flight } f \text{ in slot } s_i \text{ at price } p^f_{ii'} \\ vs_{i'} - p^{f'}_{i'i} & \text{if she takes flight } f' \text{ in slot } s_{i'} \text{ at price } p^{f'}_{i'i} \\ 0 & \text{if she does not fly.} \end{cases}$$

Without loss of generality, suppose $g(f') = h$ and $g(f) = l$. Pairwise flights obtain slots of different qualities. Under our assumption, flight f' obtains a peak slot, while flight f obtains an off-peak slot, so that $i' = h$ and $i = l$. It follows that the passenger with a taste parameter v flies with f' when $vs_h - p^{f'}_{hl} > vs_l - p^f_{lh} > 0$ and flies on flight f when $vs_l - p^f_{lh} > vs_h - p^{f'}_{hl} > 0$. The passenger indifferent between f' and f has taste

$$v_{hl} = \frac{p^{f'}_{hl} - p^f_{lh}}{s_h - s_l}. \tag{1.1}$$

Likewise, a passenger is indifferent between not flying and flying with airline f when $vs_l - p^f_{lh} = 0$, so that

$$v_{lh} = \frac{p^f_{lh}}{s_l}. \tag{1.2}$$

Hence with differentiated flights, the demand for flight f' is $1 - v_{hl}$, while the demand for flight f is $v_{hl} - v_{lh}$ (see Figure **??**). Moreover, there are $\underline{v}$ passengers that do not fly.[12]

For $g(f') = g(f) = i \in \{h, l\}$, i.e., pairwise flights obtain slots of same quality, then a passenger is indifferent between flights. In this case, the pairwise flights are homogeneous and hence evenly share the destination market. There are two possible configurations: both flights obtain either peak or off-peak slots. In the peak/peak configuration, a passenger v is willing to fly when

$$v \geq v_{hh} \equiv \frac{p^f_{hh}}{s_h} \left(= \frac{p^{f'}_{hh}}{s_h} \right), \tag{1.3}$$

while in the off-peak/off-peak configuration, she is willing to fly when

$$v \geq v_{ll} \equiv \frac{p^f_{ll}}{s_l} \left(= \frac{p^{f'}_{ll}}{s_l} \right). \tag{1.4}$$

[12]Motta (1993) shows that Cournot competition can be studied only with partial market coverage, since the demand function can not be inverted with full market coverage.

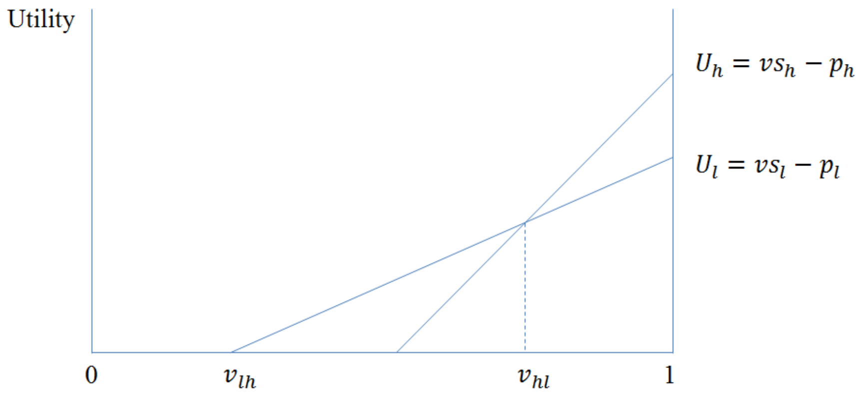

Figure 1: Valuation of slot quality

The demand for each flight is thus $\frac{1}{2}\left(1-\frac{p_i}{s_i}\right)$. Hence the three configurations are characterized by the following demand functions, respectively:

$$(i)\ \begin{cases} q^{f}_{lh}(p^{f}_{lh}, p^{f'}_{hl}) & = \dfrac{p^{f'}_{hl}-p^{f}_{lh}}{s_h-s_l} - \dfrac{p^{f}_{lh}}{s_l}, \\ q^{f'}_{hl}(p^{f}_{lh}, p^{f'}_{hl}) & = 1-\dfrac{p^{f'}_{hl}-p^{f}_{lh}}{s_h-s_l}, \end{cases} \tag{1.5}$$

$$(ii)\ q^{f}_{ll}(p_{ll}) + q^{f'}_{ll}(p_{ll}) = 1-\frac{p_{ll}}{s_l}, \tag{1.6}$$

$$(iii)\ q^{f}_{hh}(p_{hh}) + q^{f'}_{hh}(p_{hh}) = 1-\frac{p_{hh}}{s_h}. \tag{1.7}$$

Solving (1.5)-(1.7) for prices, the inverse demand functions corresponding to the three possible destination market structures are given respectively as follows:

$$(i)\ \begin{cases} p^{f}_{lh} & = s_l\left(1-q^{f}_{lh}-q^{f'}_{hl}\right), \\ p^{f'}_{hl} & = s_h\left(1-\frac{s_l}{s_h}q^{f}_{lh}-q^{f'}_{hl}\right), \end{cases} \tag{1.8}$$

$$(ii)\ p_{ll} = s_l\left(1-q^{f}_{ll}-q^{f'}_{ll}\right), \tag{1.9}$$

$$(iii)\ p_{hh} = s_h\left(1-q^{f}_{hh}-q^{f'}_{hh}\right). \tag{1.10}$$

Airlines choose the number of seats in order to maximize profits. It should be stressed that the one-stage quantity competition can be interpreted as the result of a two-stage sequential game in the spirit of Kreps and Scheinkman (1983). In the two-stage game, airlines first simultaneously choose aircraft sizes as well as frequencies, then compete on flight fares. Aircraft size, once set, is hard to adjust, thus airlines face capacity constraints at later stage. Airline costs include airport per-passenger charge ϕ, while marginal operating costs are normalized to zero. We do not consider the entry of airlines in the airport and assume that fixed costs are sunk. Thus the profit of a flight f competing in the destination market d with another flight f', $d = I(f) = I(f')$, is given by:

$$\begin{aligned} \pi^f_{lh} &= \left(p^f_{lh}(q^f_{lh}, q^{f'}_{hl}) - \phi\right) q^f_{lh} \text{ if } g(f) = l \text{ and } g(f') = h, \\ \pi^{f'}_{hl} &= \left(p^{f'}_{hl}(q^f_{lh}, q^{f'}_{hl}) - \phi\right) q^{f'}_{hl} \text{ if } g(f) = h \text{ and } g(f') = l, \end{aligned} \tag{1.11}$$

for an peak/off-peak slot configuration and

$$\begin{aligned} \pi^f_{ii} &= \left(p^f_{ii}(q^f_{ii}, q^{f'}_{ii}) - \phi\right) q^f_{ii} \text{ if } g(f) = g(f') = i \in \{l, h\}, \\ \pi^{f'}_{ii} &= \left(p^{f'}_{ii}(q^f_{ii}, q^{f'}_{ii}) - \phi\right) q^{f'}_{ii} \text{ if } g(f) = g(f') = i \in \{l, h\}, \end{aligned} \tag{1.12}$$

for the peak/peak and off-peak/off-peak slot configuration.

The airport earns the charge ϕ for each passenger. It chooses the slot allocation mapping $g(\cdot)$ that maximizes its profits. We get the following program:

$$\max_{g(\cdot)} \Pi = \Sigma^N_{d=1} \phi \left(q^f_{g(f),g(f')} + q^{f'}_{g(f'),g(f)}\right) \text{ where } I(f) = I(f') = d, \tag{1.13}$$

subject to the peak slot capacity constraint

$$\#\{f : g(f) = h\} \leq M, \tag{1.14}$$

where M is the total number of peak slots. To avoid a cumbersome discussion of ties, we assume that M is even. Constraint (1.14) implies that the overall allocated peak slots cannot exceed the total number of available peak slots. Peak capacity cannot accommodate all flights ($M < 2N$), while off-peak capacity can accommodate all flights (there is no constraint for the off-peak slots).

We then define allocative inefficiency as follows.

Definition 1 *Allocative inefficiency emerges when at least one peak slot is not used.*

This definition seems natural. In the presence of airport congestion, leaving some peak slots unused represents a degree of inefficiency.

Figure 2 shows the timing of the game. In the first stage the airport allocates peak and off-peak slots for a given fee ϕ. In the second stage airlines choose number of seats to supply $q^f_{ii'}$ based on the slot allocation. In the third stage passengers in each destination decide whether to fly with a peak period airline, an off-peak period airline, or not to fly at all. The equilibrium concept is the subgame perfect equilibrium.

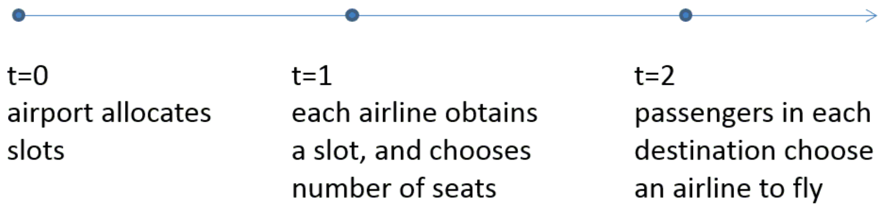

Figure 2: Timing

1.3 Results

In this section we show the baseline results of the model. As mentioned earlier, we first focus on a private airport that links destinations operated by duopoly airlines with symmetric demand and cost structures. We first discuss the competition between the airlines and then the optimal slot allocation the airport.

1.3.1 Duopoly airlines

In the second stage, airlines set their optimal supply of seats. We analyze each possible configuration (peak/off-peak; peak/peak; off-peak/off-peak) separately. Consider first a destination market d where pairwise flights $(f, f') = A(d)$ obtain different slots. Then according to (1.11), airlines' profits are expressed by:

$$\pi^f_{lh} = \left[s_l(1 - q^f_{lh} - q^{f'}_{hl}) - \phi \right] q_{lh} \tag{1.15}$$

$$\pi^{f'}_{hl} = \left(s_h - s_l q^f_{lh} - s_h q^{f'}_{hl} - \phi \right) q_{hl}. \tag{1.16}$$

Airlines choose the number of seats to maximise profits, for any given ϕ. The first-order conditions are:

$$\frac{\partial \pi^f_{lh}}{\partial q^f_{lh}} = -\phi + (1 - q^f_{lh} - q^{f'}_{hl})s_l - q^f_{lh} s_l = 0, \tag{1.17}$$

$$\frac{\partial \pi^{f'}_{hl}}{\partial q^{f'}_{hl}} = -\phi + s_h - 2q^{f'}_{hl} s_h - q^f_{lh} s_l = 0. \tag{1.18}$$

Solving (1.17) and (1.18) simultaneously with respect to q^f_{lh} and $q^{f'}_{hl}$ yields:

$$q^f_{lh} = q_{lh} \equiv \frac{s_h s_l - \phi\,(2s_h - s_l)}{(4s_h - s_l)s_l}, \tag{1.19}$$

$$q_{hl}^{f'} = q_{hl} \equiv \frac{2s_h - \phi - s_l}{4s_h - s_l}. \tag{1.20}$$

To ensure interior solutions, we assume the condition $0 < \phi < \phi_1 \equiv \frac{s_h s_l}{2s_h - s_l}$. Note that $q_{lh} < q_{hl}$ for all $0 < \phi < \phi_1$. If a destination market obtains different slots, then the airline with the peak slot serve more passengers in equilibrium than its off-peak competitor. Since quantities are symmetric across destinations, prices and profits are also symmetric and we can dispense the variables with the superscripts f and f' in the sequel without loss of clarity. Plugging (1.19) and (1.20) into (1.8) yields

$$p_{lh} = \frac{s_h(2\phi + s_l)}{4s_h - s_l}, \tag{1.21}$$

$$p_{hl} = \frac{2s_h^2 + (3s_h - s_l)\,\phi - s_h s_l}{4s_h - s_l}, \tag{1.22}$$

both of which are positive, and where

$$p_{hl} - p_{lh} = \frac{(s_h - s_l)\,(s_h + 2\phi)}{4s_h - s_l} > 0.$$

Naturally, prices are functions of ϕ, with $\frac{\partial p_{lh}}{\partial \phi}, \frac{\partial p_{hl}}{\partial \phi} > 0$.

Given $q_{lh} < q_{hl}$, $p_{lh} < p_{hl}$ and (1.11), the airline flying during the peak slot earns higher profit than its off-peak slot counterpart.

Consider next the optimal number of seats provided in the same destination d market where both airlines $(f, f') = A(d)$ obtain the same slots. The airlines face the demand

$$q_{ii}^{f} + q_{ii}^{f'} = 1 - \frac{p_i}{s_i}. \tag{1.23}$$

Plugging (1.23) and (1.25) into airline profits (1.12) yields:

$$\begin{cases} \pi_{ii}^{f} = \left[(1 - q_{ii}^{f} - q_{ii}^{f'})s_i - \phi\right] q_{ii}^{f}, \\ \pi_{ii}^{f'} = \left[(1 - q_{ii}^{f} - q_{ii}^{f'})s_i - \phi\right] q_{ii}^{f'}. \end{cases} \tag{1.24}$$

The first order conditions of π_{ii}^{f} and $\pi_{ii}^{f'}$ with respect to q_{ii}^{f} and $q_{ii}^{f'}$, respectively, are:

$$\begin{cases} -\phi - q_{ii}^{f} s_i + s_i(1 - q_{ii}^{f} - q_{ii}^{f'}) = 0, \\ -\phi - q_{ii}^{f'} s_i + s_i(1 - q_{ii}^{f} - q_{ii}^{f'}) = 0. \end{cases}$$

By solving the above two equations for q_{ii}^{f} and $q_{ii}^{f'}$ we obtain the optimal number of seats served by two airlines, which are identical due to symmetry:

$$q_{ii}^{f} = q_{ii}^{f'} = q_{ii} \equiv \frac{s_i - \phi}{3s_i}. \tag{1.25}$$

To ensure interior solutions, we make the assumptions $0 < \phi < s_i$ and $s_i > \phi_1$, $i \in \{h, l\}$. Again quantities are symmetric across destinations so that prices and profits are symmetric and can be dispensed with the superscript f and f'. Hence $0 < \phi < \phi_1$ is a sufficient condition for q_{lh}, q_{hl}, q_{ii} to be positive. For all $0 < \phi < \phi_1$, we have $q_{hl} > q_{hh} > q_{ll} > q_{lh}$. Plugging (1.25) into (1.24) and (1.10) yields:

$$p_{ll} = \frac{s_l + 2\phi}{3},\ p_{hh} = \frac{s_h + 2\phi}{3}. \tag{1.26}$$

Again, prices are functions of ϕ, with $\frac{\partial p_{ii}}{\partial \phi} > 0$.

1.3.2 Airport

In the first stage, the airport maximizes its profit by allocating peak slots subject to congestion. In this set-up, destination markets can have only three types of slot allocations: n_1 destination markets have peak/off-peak allocations, n_2 get peak/peak allocations and n_3 receive off-peak/off-peak allocations. The airport allocation problem (1.13) simplifies to the following linear program:

$$\max_{n_1, n_2, n_3} \left[n_1 \left(q_{lh} + q_{hl}\right) + 2n_2 q_{hh} + 2n_3 q_{ll}\right] \phi \tag{1.27}$$

subject to

$$n_1 + n_2 + n_3 = N \tag{1.28}$$

$$n_1 + 2n_2 \leq M \tag{1.29}$$

$$0 \leq n_1, n_2, n_3 \leq N. \tag{1.30}$$

where N denotes the number of destinations, $2N$ the number of flights, and M the number of available peak slots. The first constraint checks the count of destination markets while the second one expresses the airport peak slot capacity. The optimal slot allocation depends on how the number of passengers in each type of slot allocation ($q_{lh} + q_{hl}$, $2q_{hh}$ and $2q_{ll}$) compare with each other.

According to (1.19) and (1.20), the number of passengers in a destination market is given by

$$q_{lh} + q_{hl} = \frac{s_h(3s_l - 2\phi) - s_l^2}{s_l(4s_h - s_l)}, \tag{1.31}$$

whereas with configuration peak/peak, or off-peak/off-peak, the number of passenger in a destination market is

$$2q_{ii} = \frac{2(s_i - \phi)}{3s_i},\ i \in \{h, l\}. \tag{1.32}$$

To get intuition, consider that the capacity constraint (1.29) is not binding. Reallocation of flights must satisfy only the constraint (1.28). The airport can reorganize the slot in three ways. First it can add a peak/off-peak configuration at the expense of an off-peak/off-peak configuration (i.e. $\Delta n_1 = 1, \Delta n_3 = -1$). Since $s_h > s_l > 0$, the additional number of passengers is given by

$$q_{lh} + q_{hl} - 2q_{ll} = \frac{(s_h - s_l)(2\phi + s_l)}{3s_l(4s_h - s_l)} > 0.$$

Second, it can add a peak/peak configuration at the expense of the same off-peak/off-peak configuration (i.e. $\Delta n_2 = 1, \Delta n_3 = -1$) and gain

$$2q_{hh} - 2q_{ll} = \frac{2\phi(s_h - s_l)}{3s_h s_l} > 0$$

passenger. As a result, the off-peak/off-peak configuration should never be chosen by the airport. Finally, the airport may substitute a peak/off-peak configuration for an peak/peak configuration (i.e. $\Delta n_1 = 1, \Delta n_2 = -1$) and gain $(q_{lh} + q_{hl}) - 2q_{hh}$ passengers. Using (1.31) and (1.32), this gain is shown to be positive if and only if

$$\phi \leq \phi_2 \equiv \frac{s_l s_h}{6s_h - 2s_l}$$

where

$$\phi_1 - \phi_2 = \frac{s_l(2s_h - s_l)}{2(3s_h - s_l)} > 0.$$

At low airport fees ϕ, the number of passengers is larger when the flights to the same destination are differentiated in their departure times. The opposite holds for high fees. There are two forces in this setting. On the one hand, competition is softer under peak/off-peak configuration as each airline targets either the high or low valuation passengers. In equilibrium they offer seats to "low cost" passengers. On the other hand, airlines intensively compete for the same high valuation passengers under a peak/peak configuration. While they attract more consumers with high valuation they do not reach the "low cost" ones. As a consequence, when the airport fee ϕ is small enough ($\phi < \phi_2$), flight fares remain sufficiently low to attract many low valuation passengers. The first effect outweighs the second effect, so that $q_{lh} + q_{hl}$ is greater than $2q_{hh}$. The airport then has an incentive to separate departing schedules. By contrast, if the airport fee ϕ is large ($\phi > \phi_2$), the flight fares are too high to attract many low valuation passengers. The airport favors high valuation passengers and offers the most valued departure slot to all of them.

Figure 3 plots number of passengers $q_{lh} + q_{hl}$, $q_{hh} + q_{hh}$ and $q_{ll} + q_{ll}$ as the fee $\phi \in (0, \phi_1)$ varies. The off-peak/offpeak configuration is always dominated. When

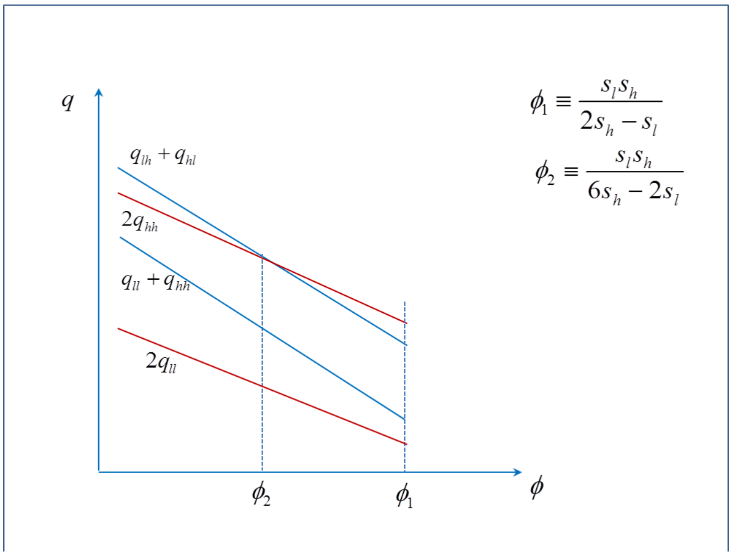

Figure 3: Passenger seats to a destination in each slot configuration

ϕ is small ($\phi < \phi_2$) the airport gets a larger number of passenger in a peak/off-peak configuration $q_{lh} + q_{hl}$ than with a peak/peak one $q_{hh} + q_{hh}$.

Consider now that the airport hits its capacity constraint (1.29). Feasible slot changes must then satisfy $\Delta n_1 = -(\Delta n_2 + \Delta n_3)$ and $\Delta n_2 = \Delta n_3$.[13] For the sake of clarity, consider a slot re-allocation such that $\Delta n_1 = 2$, $\Delta n_2 = -1$ and $\Delta n_3 = -1$, which involves two destination markets and four slots, including two peak and two off-peak slots. The airport uses the slots of two peak/peak and off-peak/off-peak destinations and re-allocate only one peak slot to each destination. Doing this, it increases the number of passengers by $2(q_{lh} + q_{hl})$ and decreases it by $2q_{hh}$ and $2q_{ll}$. The comparison yields:

$$(q_{lh} + q_{hl}) - (q_{hh} + q_{ll}) = \frac{(s_h - s_l)[s_h s_l - (2s_h - s_l)\phi]}{3s_h s_l (4s_h - s_l)} > 0 \quad \text{for} \quad \phi < \phi_1.$$

As a result, when the airport reaches its capacity, it always benefits from allocating peak/off-peak slot configurations. This effect can be visualized in Figure 3 comparing $q_{lh} + q_{hl}$ and $q_{hh} + q_{ll}$.

The formal solution of the program (1.27) is derived in Appendix B as it follows:

(i) $n_1 = \min\{M, N\}, n_2 = 0, n_3 = N - n_1$ if $\phi < \phi_2$;

(ii) $n_1 = M$, $n_2 = 0$ and $n_3 = N - M$ if $\phi_2 < \phi < \phi_1$ and $N \geq M$;

[13]The constraints yield $n_1 = 2N - M - 2n_3$ and $n_2 = M - N + n_3$.

(iii) $n_1 = N - n_2, n_2 = M - N, n_3 = 0$ if $\phi_2 < \phi < \phi_1$ and $M > N$.

When the airport is below capacity ($M > N$) and imposes a low fee ($\phi < \phi_2$), it grants a single peak slot for all destinations ($n_1 = N$ in (i)). This reflects the above incentive to set peak/off-peak slot configurations. However, in this case, some peak slots are *inefficiently* discarded whereas they have a value to all passengers. As mentioned above, by differentiating the departure time the airport increases the number of "low cost" passengers. Thus the airport does not internalize the passengers' value. By contrast, for higher fees, it grants a first peak slot to all destinations and a second one to $M - N$ destinations ($n_1 = N + (N - M)$ and $n_2 = M - N$ in (iii)). In this case flight fares are too high to attract the "low cost" passengers and the airport prefers offering the best departure times to the maximum number of passengers.

When the airport is at capacity ($N \geq M$) and imposes a low fee ($\phi < \phi_2$), it gives a single peak slot for the maximum number of destinations and allocates the rest of destinations into the off-peak slot pool ($n_1 = M$, $n_3 = N - M$ in (iii)). The airport makes exactly the same decision for higher fees ($n_1 = M, n_3 = N - M$ in (ii)).

The foregoing discussion can be summarized in the following proposition.

Proposition 1 *Suppose all destination markets are served by duopoly airlines and the airport is private. For $M \leq N$, the airport uses all available peak slots and implements the "peak/off-peak" configuration in each destination market. For $M > N$ and*

- *$\phi \in (0, \phi_2]$, the airport does not use all available peak slots (inefficiency), and the configuration is "peak/off-peak" in each destination market;*
- *$\phi \in [\phi_2, \phi_1)$, the airport uses all available peak slots, and implements $(M - N)$"peak/peak" and $(2N - M)$ "peak/off-peak" configuration.*

Figure 4 describes the equilibria in the space (ϕ, M). When $\phi \in [\phi_2, \phi_1)$, the airport favors peak/peak allocations and as a consequence no peak slots would be optimally left unused. For per-passenger fees smaller than ϕ_2, the result depends on the relationship between peak slots M and number of destinations N. In a very busy airport where available peak slots are scarce relative to total demand ($M \leq N$), it is optimal to allocate all available peak slots. When peak slots are not scarce relative to the number of destination markets ($M > N$), a private airport would leave a number of peak slots unused when the pre-determined fee is small, thereby resulting in allocative inefficiency. In reality, such behavior may be expressed by misreporting true airport handling capacity. This is in line with De Wit and Burghouwt (2008), who find that efficient slot use can be affected by capping available slots through capacity declaration.

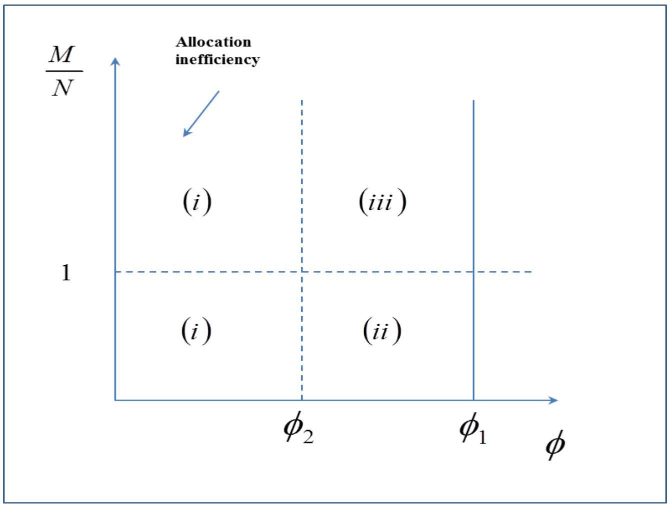

Figure 4: Equilibria: duopoly airlines and private airport

1.3.3 Airline operating costs

For the sake of completeness, we end the section by discussing the case where airlines have non-zero operating costs $c > 0$. The analysis can be developed in a similar vein as before, where the airline marginal cost is now $c + \phi$ rather than ϕ only. Naturally, in both configurations the volume of passengers is larger without operating cost. The conclusion drawn from the comparison of peak-off peak configuration also applies here. It follows that, with positive operating costs airport profit is also smaller in each configuration. The condition required to guarantee positive passenger volumes in equilibrium is

$$0 < \phi < \phi_1' \equiv \frac{s_h s_l}{2s_h - s_l} - c,$$

while the threshold determining the preference between peak/peak and peak/off-peak is

$$\phi_2' \equiv \frac{s_h s_l}{6s_h - 2s_l} - c.$$

Therefore, the above proposition now reads with ϕ_1' and ϕ_2' substituting for ϕ_1 and ϕ_2. If the cost c is small enough so that $\phi_2' > 0$, the proposition presents the same configurations and the same issue of allocative inefficiency. The configuration peak/off-peak induces more passenger volume than configuration peak/peak so that the airport does not distribute all available peak slots and inefficiency arises. However, if c is large enough so that ϕ_2' becomes negative, all available peak slots are distributed and allocative inefficiency never arises.

1.4 Airline monopolies

Having examined competition between duopolists in each destination market, we shall now investigate the case in which each destination market is served by a single airline that acts as a monopolist operating two flights. As with the baseline model, we analyze the second stage in each possible configuration separately, while the analysis of the third stage remains unchanged.

Suppose that an airline is offered both a peak and an off-peak slot. Its profit is given by $\pi = \pi_{lh}^{f} + \pi_{hl}^{f'}$ where π_{lh}^{f} and $\pi_{hl}^{f'}$ are defined in (1.15) and (1.16). The airline solves the profit maximization problem with constraints on the positivity of outputs:

$$\max_{q_{lh}^{f}, q_{hl}^{f'}} \pi = \left[s_l(1 - q_{lh}^{f} - q_{hl}^{f'}) - \phi\right] q_{lh}^{f} + (s_h - s_l q_{lh}^{f} - s_h q_{hl}^{f'} - \phi) q_{hl}^{f'} \tag{1.33}$$
$$\text{s.t. } q_{lh}^{f} \geq 0,\; q_{hl}^{f'} \geq 0.$$

The maximum profit (see Appendix C) is obtained for

$$q_{lh}^{m} = 0, \tag{1.34}$$
$$q_{hl}^{m} = \frac{s_h - \phi}{2 s_h}, \tag{1.35}$$

where the superscript m stands for "monopoly". The monopolistic airline allocates all seats in the peak flight. Indeed, it does not have any advantage to decrease consumers' value by offering a flight off peak. This result is due to the fact that the cost of boarding a passenger (in terms of per-passenger fee) is linear and equivalent between a peak and an off-peak flight. Given the same (marginal) per-passenger fee, the airline prefers to put all seats in the peak flight. In the real world, this implies that the monopoly airline assigns a "big" airplane flying on that destination in peak time rather than put two "small" airplanes flying one in the peak and the other in the off-peak slot. Plugging $q_{hl}^{m} = \frac{s_h - \phi}{2 s_h}$ into p_{hl}^{m} yields

$$p_{hl}^{m} = \frac{s_h + \phi}{2}. \tag{1.36}$$

Consider next the case where the monopoly airline is offered two slots at the same time for the same destination.[14] A small (unmodeled here) fixed cost per aircraft movement will entice the airline to operate only one aircraft on one slot. We denote the slot

[14] This configuration is mainly made for the sake of comparison. It is certainly the case in configurations where there are two (morning and evening) peak slots per day. The case where the airline merges the two flights is left for future research.

type (h or l) the monopolist obtains by i. The monopolist's profit is given by:

$$\pi_i = [(1-q_i)s_i - \phi]\, q_i, \text{ with } i \in \{h, l\}.$$

Taking the first-order condition for number of seats we get the equilibrium number of seats the monopoly would provide:

$$q_i^m = q_{hl}^m \equiv \frac{s_i - \phi}{2s_i}, \tag{1.37}$$

which is the same result as (1.35) for $i = h$. The aircraft capacity is larger at peak time: $q_h^m > q_l^m$. To ensure interior solutions, suppose condition $0 < \phi < s_l$ is satisfied. Plugging (1.37) into (1.24) and (1.10) yields:

$$p_i^m = \frac{s_i + \phi}{2}. \tag{1.38}$$

which is the same result as (1.36) for $i = h$. It is easy to check that peak flights are more expensive and transport more passengers.

The airport allocation problem simplifies to allocating peak and off-peak slots to the monopoly airlines:

$$\max_{m_1, m_2} [m_1 q_h^m + m_2 q_l^m]\, \phi$$

subject to

$$\begin{aligned} &m_1 + m_2 = N \\ &m_1 \leq M \\ &0 \leq m_1, m_2 \leq N. \end{aligned}$$

where m_1 and m_2 are the number of flights in the peak and off-peak slots. Since $q_h^m > q_l^m$, the solution is (i) $m_1 = N$ if $N < M$; (ii) $m_1 = M$ otherwise. The airport fills the peak slots until capacity is reached.

Proposition 2 *Suppose all destination markets are served by monopoly airlines. Then, airlines operate one flight per destination and the private airport uses all available peak slots.*

Proposition 2 shows that, if the destination market is served by a monopoly airline, the optimal slot allocation is to assign one peak slot to each destination market, while no off-peak flights operate. The intuition lies in the fact that the monopoly airline has no incentive in exploiting an off-peak slot, given that same marginal cost as operating during peak hours.

1.5 Public airport

In this section, we investigate the case of a pubic, welfare-maximizing airport. This allows us to obtain some insights on how the airport's ownership influences slot allocation. In this regard, social welfare W is represented by the sum of airport' profits Π, passenger surplus CS and airlines' profits:

$$W = \Pi + CS + n_1 \left(\pi_{hl} + \pi_{lh} \right) + 2 \left(n_2 \pi_{hh} + n_3 \pi_{ll} \right).$$

Since airport and airlines operating costs are normalized to zero, airport profits come from total per-passenger fees, whereas airline profits are the ticket income less total per-passenger fees paid to the airport. In turn, passenger surplus is represented by the total gross utility generated from flying minus all ticket payments. Since monetary transfers between airlines and airport cancel out, and so do transfers between passengers and airlines, then social welfare equals the sum of passengers' gross utility in all N destination markets. Thus W can be rewritten as:

$$W = n_1 \left(\int_{v_{lh}}^{v_{hl}} v s_l dv + \int_{v_{hl}}^{1} v s_h dv \right) + n_2 \int_{v_{hh}}^{1} v s_h dv + n_3 \int_{v_{ll}}^{1} v s_l dv, \tag{1.39}$$

where v_{lh}, v_{hl}, v_{hh} and v_{ll} are given by (1.1), (1.2), (1.3) and (1.4). Note that the surplus in the peak/peak (resp. off-peak/off-peak) configurations includes the value of all consumers from 1 to v_{hh} (resp. from 1 to v_{ll}) as all passengers take the same time slot.

In destination markets with the peak/offpeak configuration, the expression $\int_{v_{lh}}^{v_{hl}} v s_l dv$ represents the gross passenger surplus from taking the off-peak period flight, while $\int_{v_{hl}}^{1} v s_h dv$ is the gross passenger surplus from taking the peak period flight in the same destination market. In destination markets with the peak/peak configuration, the surplus is $\int_{v_{hh}}^{1} v s_h dv$,which is the second term on the right hand side of (1.39). The last term of (1.39) thus represents the gross passenger surplus in destination markets with off-peak/off-peak configuration.

The analysis of the second and third stage remains the same as in the baseline model. For notational simplicity we define $W_{lh} \equiv \int_{v_{lh}}^{v_{hl}} v s_l dv$, $W_{hl} = \int_{v_{hl}}^{1} v s_h dv$, $W_{hh} \equiv \int_{v_{hh}}^{1} v s_h dv$ and $W_{ll} \equiv \int_{v_{ll}}^{1} v s_l dv$, so that (1.39) becomes

$$W = n_1 \left(W_{lh} + W_{hl} \right) + n_2 W_{hh} + n_3 W_{ll}.$$

We will consider first the case with duopoly airlines and then the case with monopoly airlines.

1.5.1 Airline duopolies

Putting (1.21), (1.22) and (1.26) together with (1.1), (1.2), (1.3) and (1.4) yields:

$$v_{lh} = \frac{s_h\left(s_l + 2\phi\right)}{s_l\left(4s_h - s_l\right)}, v_{hl} = \frac{2s_h + \phi}{4s_h - s_l}, v_{hh} = \frac{s_h - 2\phi}{3s_h}, v_{ll} = \frac{s_l - 2\phi}{3s_l}. \tag{1.40}$$

Substituting (1.40) into (1.39) and solving the integrals yields:

$$W_{hl} + W_{lh} = \frac{4\phi s_h s_l(s_l - 2s_h) + s_h s_l\left(12s_h^2 - 5s_h s_l + s_l^2\right) - \phi^2\left(4s_h^2 + s_h s_l - s_l^2\right)}{2s_l(4s_h - s_l)^2},$$

and

$$W_{ii} = \frac{2(\phi + s_i)(2s_i - \phi)}{9s_i}, \; i \in \{h, l\}.$$

Assuming $\phi < \phi^P \equiv \frac{s_l}{2}$ ensures that W_{ii} is positive. By comparing the number of seats obtained in each configuration, we get (see Appendix C) $W_{hh} > W_{hl} + W_{lh} > W_{ll} > 0$ and $\frac{(W_{hh} + W_{ll})}{2} < W_{hl} + W_{lh}$. We thus arrive at a situation similar to the private airport case with $\phi \in [\phi_2, \phi_1)$. In particular:

- $n_1 = M, n_2 = 0$ and $n_3 = N - M$ if $N \geq M$ (Case (ii)).
- $n_1 = N - n_2, n_2 = M - N, n_3 = 0$ if $M > N$ (Case (iii)).

The following proposition summarizes the features of the equilibrium.

Proposition 3 *Suppose all destination markets are served by duopoly airlines, and the airport is public. For $M \leq N$, the airport uses all available peak slots and favors "peak/off-peak" configuration. For $M > N$ the airport implements a mix of $(M - N)$ "peak/peak" and $(2N - M)$ "peak/off-peak" configurations.*

Figure 5 describes the equilibria in the case of public airport with duopoly destination markets. The results are qualitatively similar to the case with private airport for $\phi_2 < \phi < \phi_1$. However, now the public airport would use all available peak slots in any case, hence, inefficiency does not emerge when the airport is public. This can be explained as follows. The consumers lose when they are presented departure time away from their preferences. The public airport internalizes this loss and avoids empty peak slots. The private airport rather implement empty peak slots because it can attract more (low valuation) passengers, even though those passengers would prefer travelling at peak time.

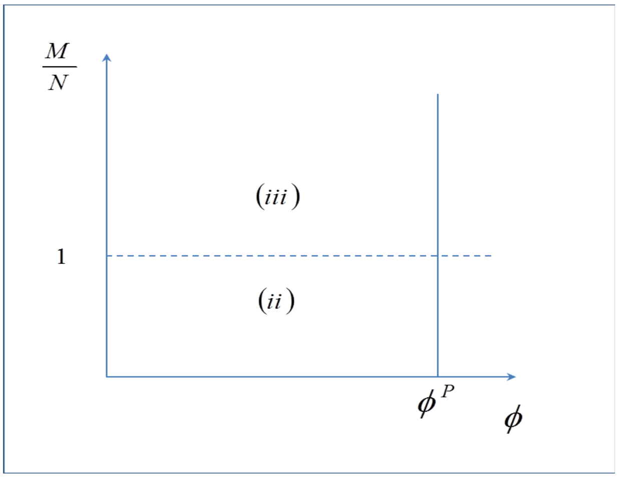

Figure 5: Equilibria: duopoly airlines and public airport

1.5.2 Airline monopolies

We now turn to the interplay between a public airport and airline monopolies. Compared to the private case, the market stage in each destination market does not change: given the opportunity of operating in peak slots, each monopoly airline uses one peak slot only. Thus W can be rewritten as:

$$W^m = m_1 W_h^m + m_2 W_l^m, \tag{1.41}$$

where $W_h^m = \int_{v_h}^1 v s_h dv$, $W_l^m = \int_{v_l}^1 v s_l dv$, m_1 and m_2 are the number of flights in the peak and off-peak slots, and v_h and v_l are the same as (1.3) and (1.4), respectively, but when only one flight operates. Putting $p_h^m = \frac{s_h + \phi}{2}$ and $p_l^m = \frac{s_l + \phi}{2}$ from (1.38) into (1.3) and (1.4), respectively, yields:

$$v_h = \frac{s_h + \phi}{2s_h}, v_l = \frac{s_l + \phi}{2s_l}. \tag{1.42}$$

Substituting (1.42) into (1.41) and solving the integrals yields:

$$W_i^m = \frac{(s_i - \phi)(3s_i + \phi)}{8s_i}, \; i \in \{h, l\},$$

where

$$W_h^m - W_l^m = \frac{(s_h - s_l)\left(3s_h s_l + \phi^2\right)}{8s_h s_l} > 0.$$

Since $W_h^m > W_l^m$, the solution is (i) $m_1 = N$ if $N < M$; (ii) $m_1 = M$ otherwise. The airport fills the peak slots until capacity is reached, as in the private case (with same intuition).

Proposition 4 *Suppose all destination markets are served by monopoly airlines. Then, airlines operate one flight per destination and the public airport uses all available peak slots.*

1.5.3 Compare social welfares of two market structures

The precedent analysis illustrates that monopoly airlines reduce their demand for slots, hence the airport congestion problem is solved by a shrinking of demand for peak slots. Moreover, efficiency is evaluated in terms of the degree of utilization of peak slots. We now look into the comparison of social welfare levels induced by these two market structures. The discussion can be divided into two cases: $M \leq N$ and $M > N$. When $M \leq N$, according to Proposition 3, there will be M peak/off-peak markets and $N - M$ off-peak/off-peak markets. While Proposition 4 suggests a number of M markets each filled with a peak slot, and $N - M$ markets each with a off-peak slot. The overall comparison problem then effectively reduces to a simpler version of comparing the social welfare of a peak/off-peak market $(W_{hl} + W_{lh})$, to that of a single peak slot market $\left(W_h^m\right)$; as well as an off-peak/off-peak market (W_{ll}) to a single off-peak slot market $\left(W_l^m\right)$. We could show:

$$W_{hl} + W_{lh} - W_h^m = \frac{4s_h^3\left(s_l^2 - 4\phi^2\right) + s_h^2\left(s_l^3 + 12s_l\phi^2\right) + 2s_h s_l^2\phi(s_l - 2\phi) + s_l^3\phi^2}{8s_h s_l (s_l - 4s_h)^2} > 0,$$

and

$$W_{ll} - W_l^m = \frac{5s_l^2 + 34s_l\phi - 7\phi^2}{72s_l} > 0,$$

for all $\phi < \frac{s_l}{2}$. Same argument and proof apply to the alternative case where $M > N$. Note that the comparison of different slots in duopoly with single peak slot in monopoly maintains, hence it suffices to look at same slots case only: $N - M$ peak/peak markets with same amount of single peak markets. It can be shown that duopoly yields higher social welfare:

$$W_{hh} - W_h^m = \frac{5s_h^2 + 34s_h\phi - 7\phi^2}{72s_h} > 0.$$

The intuition is straightforward on the grounds of both competition and passenger type. A duopoly of airlines that offer different slots attract both high and low type passengers; in addition, because two flights are partially substitutable, the competition curtails the airlines' market powers. The resulting social welfare in a duopoly setting hence exceeds that in a monopoly setting where only one flight is offered. Moreover, given the monopoly nature, it is intuitive that a duopoly providing homogenous goods would induce higher welfare than a monopoly that provides a single good.

1.6 Heterogeneous density

In this section, we assume heterogeneous density across destination markets. The point is to confirm that peak/off-peak slot configurations and allocative inefficiencies also occur when destination markets differ in sizes. For simplicity we study an economy with only two destination markets, $d \in D = \{1, 2\}$, that exhibit different densities: the small destination market 1 has a lower density δ_1 , the large destination market 2 has a higher density $\delta_2 > \delta_1$. We discuss three examples with decreasing peak slot scarcities. Namely, when the airport has either one, two or three peak slots for the two markets.

To begin with, we first look at the duopoly case where peak slots are highly scarce, where one peak slot is available compared to four slot demands, $M = 1$, $N = 2$. Let us label the small market flights by "11","12" $\in A(1)$ and the large market ones by "21","22" $\in A(2)$. There are two ways to allocate the peak slot: (1) one of the two airlines in the small destination market, and (2) one of the two airlines in the large destination market.

The demand functions for each airline can be in two possible configurations. First if the smaller destination market $d = 1$ has the peak slot we have

$$q^{11}_{lh} = \delta_1 \left(\frac{p^{12}_{hl} - p^{11}_{lh}}{s_h - s_l} - \frac{p^{11}_{lh}}{s_l} \right),\ q^{12}_{hl} = \delta_1 \left(1 - \frac{p^{12}_{hl} - p^{11}_{lh}}{s_h - s_l} \right),$$

and for destination market 2:

$$q^{21}_{ll} + q^{22}_{ll} = \delta_2 \left(1 - \frac{p^{21}_{ll}}{s_l} \right),$$

(where $p^{21}_{ll} = p^{22}_{ll}$). Second, if larger destination market $d = 2$ has the peak slot we get the same quantities where δ_1 substitutes for δ_2 and $(q^{11}_{lh}, q^{12}_{hl}, q^{21}_{ll}, q^{22}_{ll})$ is replaced by $(q^{21}_{lh}, q^{22}_{hl}, q^{11}_{ll}, q^{12}_{ll})$. In the similar manner with (1.19), (1.20) and (1.25), we could derive the optimal passenger volumes served by each airline and consequently each destination market in equilibrium. A comparison of the equilibrium passenger volumes in the two configurations yields:

$$q^{11}_{lh} + q^{12}_{hl} + q^{21}_{ll} + q^{22}_{ll} - \left(q^{11}_{ll} + q^{12}_{ll} + q^{21}_{lh} + q^{22}_{hl} \right)$$

$$= \frac{(\delta_1 - \delta_2) \left[s_l \left(9 + s_h + 2s_l \right) + 2\phi \left(s_h - s_l \right) \right]}{3s_l(4s_h - s_l)} < 0.$$

implying configuration 2 yields a higher number of passenger than configuration 1. The ranking in this simple framework suggests

Proposition 5 *Consider an economy with two duopolies with different density levels, a private airport and a single peak slot. Then the airport allocates the peak slot to one of the airlines in the large destination market, and inefficiency would not arise.*

We investigate next the case where peak slots have a moderate scarcity at the airport. In this setting there are two peak slots available for two destination markets, $M = N = 2$. It is straightforward to see that allocation "two peak slots to destination market 2" dominates allocation "two peak slots to destination market 1". Indeed, the large market has a bigger multiplier for density $\delta_2 > \delta_1$. This, together with the fact that off-peak/off-peak is strictly dominated by peak/peak and peak/off-peak, implies that there are two possible allocations: (1) two peak slots to the large destination market, and (2) one peak slot to each destination market. The demand functions for each airline in these two configurations are:

1. **Configuration 1.** Destination market 1 :

$$q_{ll}^{11} + q_{ll}^{12} = \delta_1 \left(1 - \frac{p_{ll}^{11}}{s_l}\right);$$

destination market 2:

$$q_{hh}^{21} + q_{hh}^{22} = \delta_2 \left(1 - \frac{p_{hh}^{21}}{s_h}\right).$$

2. **Configuration 2.** Destination market 1:

$$q_{lh}^{11} = \delta_1 \left(\frac{p_{hl}^{12} - p_{lh}^{11}}{s_h - s_l} - \frac{p_{lh}^{11}}{s_l}\right),\ q_{hl}^{12} = \delta_1 \left(1 - \frac{p_{hl}^{12} - p_{lh}^{11}}{s_h - s_l}\right);$$

destination market 2:

$$q_{lh}^{21} = \delta_2 \left(\frac{p_{hl}^{22} - p_{lh}^{21}}{s_h - s_l} - \frac{p_{lh}^{21}}{s_l}\right),\ q_{hl}^{22} = \delta_2 \left(1 - \frac{p_{hl}^{22} - p_{lh}^{21}}{s_h - s_l}\right).$$

Comparing the two configurations we obtain

$$q_{ll}^{11} + q_{ll}^{12} + q_{hh}^{21} + q_{hh}^{22} - \left(q_{lh}^{11} + q_{hl}^{12} + q_{lh}^{21} + q_{hl}^{22}\right) > 0$$

for

$$\phi > \phi_3 \equiv \frac{s_h s_l \left(\delta_1 + \delta_2\right)\left(s_h + 2s_l + 9\right)}{2\left(s_h - s_l\right)\left[\delta_2\left(3s_h - s_l\right) - \delta_1 s_h\right]}.$$

This result can be summarized as follows.

Proposition 6 *Consider an economy with two duopolies with different density levels, a private airport and two peak slots. Then, for $\phi < \min(\phi_1, \phi_3)$, the airport allocates one peak slot to each market; for $\phi_1 > \phi > \phi_3$, the airport allocates two peak slots to the large market, and inefficiency would not arise.*

When the per-passenger fee is sufficiently small, the allocation is fair and does not favor any market so that each destination is equally served. For high per-passenger fees, the denser market obtains all available slots. As a consequence, passengers in the small market have no chance to fly at peak hours, while passengers in the denser market cannot fly at off-peak hours. In either case, all available peak slots will be used.

Finally, we examine the example where peak slots are relatively abundant. In particular, there are three peak slots to be allocated to two markets, $M = 3$, $N = 2$. Given that configuration off-peak/off-peak is strictly dominated, we can set aside the situation where the airport leaves one slot unused and gives two slots to the big market. Indeed, the airport could be better off by giving the unused one to the small market. It follows that there are three plausible configurations: (1) two peak slots to market 2- one peak slot to market 1, (2) two peak slots to market 1- one peak slot to market 2, and (3) one peak slot to each market.

1. **Configuration 1.** Market 1 :

$$q^{11}_{lh} = \delta_1 \left(\frac{p^{12}_{hl} - p^{11}_{lh}}{s_h - s_l} - \frac{p^{11}_{lh}}{s_l} \right),\ q^{12}_{hl} = \delta_1 \left(1 - \frac{p^{12}_{hl} - p^{11}_{lh}}{s_h - s_l} \right);$$

   market 2:

$$q^{21}_{hh} + q^{22}_{hh} = \delta_2 \left(1 - \frac{p^{21}_{hh}}{s_h} \right).$$

2. **Configuration 2.** Market 1:

$$q^{11}_{hh} + q^{12}_{hh} = \delta_1 \left(1 - \frac{p^{11}_{hh}}{s_h} \right);$$

   market 2:

$$q^{21}_{lh} = \delta_2 \left(\frac{p^{22}_{hl} - p^{21}_{lh}}{s_h - s_l} - \frac{p^{21}_{lh}}{s_l} \right),\ q^{22}_{hl} = \delta_2 \left(1 - \frac{p^{22}_{hl} - p^{21}_{lh}}{s_h - s_l} \right);$$

3. **Configuration 3.** Market 1:

$$q^{11}_{lh} = \delta_1 \left(\frac{p^{12}_{hl} - p^{11}_{lh}}{s_h - s_l} - \frac{p^{11}_{lh}}{s_l} \right),\ q^{12}_{hl} = \delta_1 \left(1 - \frac{p^{12}_{hl} - p^{11}_{lh}}{s_h - s_l} \right);$$

market 2:

$$q_{lh}^{21} = \delta_2 \left(\frac{p_{hl}^{22} - p_{lh}^{21}}{s_h - s_l} - \frac{p_{lh}^{21}}{s_l} \right), \; q_{hl}^{22} = \delta_2 \left(1 - \frac{p_{hl}^{22} - p_{lh}^{21}}{s_h - s_l} \right).$$

Comparing the three configurations we obtain

$$\text{configuration } 1 > 2 > 3 \text{ when } \phi > \phi_4;$$
$$\text{configuration } 1 < 2 < 3 \text{ when } \phi < \phi_4,$$

with

$$\phi_4 \equiv \frac{s_h s_l \left(s_h + 2s_l + 9\right)}{2\left(s_h - s_l\right)\left(3s_h - s_l\right)}.$$

Therefore:

Proposition 7 *Suppose an economy with two duopolies with different density levels, a private airport and three peak slots. Then, for* $\phi < \min(\phi_1, \phi_4)$*, the airport allocates one peak slot to each market, and leaves one peak slot unused (inefficiency); for* $\phi_1 > \phi > \phi_4$*, the airport allocates two peak slots to the large market and one peak slot to the small market.*

Proposition 7 implies that when markets have different passenger densities, allocative inefficiency would arise if the per-passenger fee is sufficiently small. On the other hand, if the per-passenger fee is sufficiently high, the allocation outcome is efficient, with the denser market obtaining both peak slots and the smaller market obtaining one peak slot. Such allocation favors the denser market, which is a result of airport's profit maximizing behavior.

1.7 Conclusion

We have explored the optimal slot allocation in the presence of airport congestion in a model where peak and off-peak slots are modelled as products of different qualities in a vertically differentiated setting. Allocative inefficiency emerges when the airport does not exploit all its slots. In particular in a private airport, allocative inefficiency may emerge if the airport is not too congested and the per-passenger fee is small enough. In a public airport, allocative inefficiency does not emerge. Furthermore we have found that the airport, regardless of its ownership, tends to give different slots to flights with same destination if the underlying destination market is a duopoly, and one single slot if the underlying market is served by a monopoly.

The current work raises avenues for future research. First, it confines to the study of a single airport. A more comprehensive analysis can be carried out by extending the current framework to intercontinental flights where departure and arrival airports are managed by separate regulators. In particular, the two airports may be subject to equivalent or different levels of congestion. A recent study by Benoot et al (2013) investigates the strategic interaction between intercontinental airport regulators that determine charge and capacity separately. Further investigating how the regulators coordinate on allocating peak slots, as well as the associated impacts on social welfares, would make the current work applicable to a broad hub-spoke network.

Second, this work limits attention to passengers that have the same lowest and highest marginal willingness to pay for quality across all destinations. Admittedly, in practice some destinations are dominated by leisure travellers while others by business travellers. What seems to be an interesting avenue for future research, is to consider markets with passengers that have various average marginal willingness to pay for quality. The extension can be undertaken to reflect different marginal rate of substitution between income and quality in destinations which are determined by the dominant types of travellers.

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Appendix

Appendix A.

For our empirical illustration, we examine the airline market structure and slot occupation. Towards this aims, we gather data from 15 US airport's websites on three consecutive weekdays May 18,19 and 20, 2015 for flights' departure information. Weekdays are chosen to exclude irregular influx for air travel happens on weekends. We adopt the definition for peak load according to website of O'Hare International Airport (ORD)[15] and apply to all 15 airports. Our dataset contains airport level observations on air traffic: departure airport, service airline, destination airport, departure time. Among the 15 primary airports in our dataset,[16] 5 airports are the most busiest airports in US by total passenger traffic, according to ACI (Airports Council International North America) ranking in calender year 2013. The other 10 mid-sized airports are taken arbitrarily from the range 30nd-60nd on the same rank, scattered to 9 federal states. We apply the below rules to filter improper observations: (1) delete all cargo, private jet charter, aircraft Rental Service, etc.; (2) for code sharing airlines, keep the operating airline and delete all other (code sharing) partner airlines. The total number of observations used for our analysis is 5990 departure activities, of which monopoly is the primary feature of airline market. Table 5 shows per airport market structure. Table 1, reported in the Introduction, lists the numbers of origin-destination routes operated by monopoly, duopoly and oligopoly, and the pattern of slot occupancy.

[15] ORD defines 8-9am, 15-16pm, 17-18pm, and 19-22pm as peak hours.

[16] FAA defines a primary airport as having more than 10,000 passenger boardings each year.

Airport	#Mon.	#Duo.	#Olig.	Rank(2013)
Atlanta International Airport (ATL)	148	33	25	1
Los Angeles International Airport (LAX)	46	31	32	2
O'Hare International Airport (ORD)	101	45	51	3
Dallas/Fort Worth International Airport (DFW)	144	20	18	4
Denver International Airport (DEN)	86	32	31	5
Kansas City International Airport (MCI)	29	11	3	35
Oakland International Airport (OAK)	24	7	0	36
John Wayne Airport (SNA)	17	3	1	38
Luis Muñoz Marín International Airport (SJU)	21	6	6	43
San Antonio International Airport (SAT)	24	5	2	45
Indianapolis International Airport (IND)	28	5	2	48
Kahului Airport (OGG)	10	4	5	51
Buffalo Niagara International Airport (BUF)	15	4	0	55
Jacksonville International Airport (JAX)	18	5	0	56
Eppley Airfield (OMA)	14	3	0	60

Table 5. Per airport market structure

A further deletion was made to exclude all flights between the 5 largest airlines, as they are mostly inter-hub connection flights; and high frequency flights with frequency above and equal to 7 flights towards one destination airport. We then construct an index to measure allocative discrimination:

$$I = \frac{\left|x_h^a - x_h^b\right| + \left|x_l^a - x_l^b\right|}{2} \in \{0, 0.5, 1\},$$

where the subscripts h and l denote peak and offpeak time slots. x_h^a is a dummy that takes value 1 if airline a obtains a peak slot, 0 if not. The same logic applies to n_h^b, x_l^a and x_l^b. Hence when $I = 1$,allocative discrimination is largest, while, when $I = 0$, allocative discrimination doesn't exist. A higher average value of this index indicates a higher magnitude of slot discrimination. We then find evidence consistent with our model prediction that allocative discrimination over 10 smaller airports is smaller than that of the larger 5 airports, see Table 2 in the Introduction.

Appendix B

Linear programming

In this section we describe the general problem of the airport without getting into the specific cases considered in the chapter. The result of each case will depend on the relationship between M and N and the level of per-passenger fee ϕ. This will be examined and linked to the general solution in the next section. We define as Q_1, $2Q_2$ and $2Q_3$ the equilibrium number of passengers in a peak/off-peak, peak/peak and off-peak/off-peak configuration, respectively, for a general problem. The airport has the following linear programming problem to solve

$$\begin{aligned}
&\max_{n_1,n_2,n_2} \Pi = n_1 Q_1 + 2n_2 Q_2 + 2n_3 Q_3 \ , \\
&\text{s.t.} \\
&n_1 + n_2 + n_3 = N \ , \\
&n_1 + 2n_2 \leq M \ , \\
&0 \leq n_1, n_2, n_3 \leq N \ .
\end{aligned}$$

Using $n_3 = N - n_1 - n_2$ we can re-write

$$\begin{aligned}
&\mathcal{P} \equiv \max_{n_1,n_2,n_2} \Pi = n_1 \left(Q_1 - 2Q_3\right) + n_2 (2Q_2 - 2Q_3) + 2NQ_3 \ , \\
&\text{s.t.} \\
&n_1 + 2n_2 \leq M \ , \\
&0 \leq n_1 + n_2 \leq N \ .
\end{aligned}$$

We get the following solution:

1. If $(Q_1 - 2Q_3) > 0$ and $(2Q_2 - 2Q_3) > 0$, and $Q_1 > 2Q_2$ then $n_1 = \min\{M, N\}$, $n_2 = 0$, $n_3 = N - n_1$;

2. If $(Q_1 - 2Q_3) > 0$ and $(2Q_2 - 2Q_3) > 0$, and $Q_2 + Q_3 > Q_1$ then $n_2 = \min\{M/2, N\}$, $n_1 = 0$, $n_3 = N - n_2$;

3. If $(Q_1 - 2Q_3) > 0$ and $(2Q_2 - 2Q_3) > 0$, $2Q_2 > Q_1 > Q_2 + Q_3$ and $N \geq M$, then $n_1 = M$, $n_2 = 0$ and $n_3 = N - M$;

4. If $(Q_1 - 2Q_3) > 0$ and $(2Q_2 - 2Q_3) > 0$, $2Q_2 > Q_1 > Q_2 + Q_3$ and $M > N > M/2$, then $n_1 = 2N - M$, $n_2 = M - N$ and $n_3 = 0$;

5. If $(Q_1 - 2Q_3) > 0$ and $(2Q_2 - 2Q_3) > 0$, $2Q_2 > Q_1 > Q_2 + Q_3$ and $M/2 > N$, then $n_1 = 0$, $n_2 = M/2$ and $n_3 = N - M/2$.

6. If $(Q_1 - 2Q_3) < 0$ and $(2Q_2 - 2Q_3) > 0$, then $n_2 = \min\{M/2, N\}$, $n_1 = 0$, $n_3 = N - n_2$.

7. If $(Q_1 - 2Q_3) < 0$ and $(2Q_2 - 2Q_3) < 0$, then $n_1 = n_2 = 0$ and $n_3 = N$.

8. If $(Q_1 - 2Q_3) > 0$ and $(2Q_2 - 2Q_3) < 0$, then $n_1 = \min\{M, N\}$, $n_2 = 0$, $n_3 = N - n_1$.

Note that Solution 5. is not applicable because we assumed $M < 2N$.

Applications

The relevant results of the linear programming depend on the structure of the economy (public/private airport, and duopoly markets), the relationship between peak slots M and destinations N, and the level of per passenger fees ϕ. What follows helps to understand which solution applies to each case considered in the chapter.

Duopolies and private airport

For the duopoly case, let $Q_1 = q^f_{lh} + q^{f'}_{hl}$, $Q_2 = q^f_{hh}$ and $Q_3 = q^f_{ll}$. We know that $Q_1 > 2Q_2 > 2Q_3$ for $\phi < \phi_2$ and $2Q_2 > Q_1 > 2Q_3$ for $\phi_2 < \phi < \phi_1$. Also, $Q_1 > Q_2 + Q_3$. If $\phi < \phi_2$ we get $Q_1 > 2Q_2$, so that checking Section 1.7 result 1. applies: $n_1 = \min\{M, N\}, n_2 = 0, n_3 = N - n_1$;

If $\phi_2 < \phi < \phi_1$, we get:

i. if $N > M$, then $n_1 = M$, $n_2 = 0$ and $n_3 = N - M$;

ii. if $M > N > M/2$, then $n_1 = 2N - M$, $n_2 = M - N$ and $n_3 = 0$;

Checking the corresponding results in Section 1.7, this yields the solution:

- $n_1 = \min\{M, N\}, n_2 = 0, n_3 = N - n_1$ if $\phi < \phi_2$ or if $\phi_2 < \phi < \phi_1$ and $N > M$;
- $n_1 = 2N - M, n_2 = M - N, n_3 = 0$ if $\phi_2 < \phi < \phi_1$ and $M > N$.

Duopolies and public airport

For the duopoly case, let $Q_1 = B_{hllh}$, $Q_2 = B_{hh}/2$ and $Q_3 = B_{ll}/2$. We know $2Q_2 > Q_1 > 2Q_3$ if $\phi < \phi^P_1$ and $2Q_2 > 2Q_3 > Q_1$ if $\phi > \phi^P_1$. Also, $Q_2 + Q_3 > Q_1$ for $\phi > \phi^P_4$.

Checking the corresponding results in Section 1.7, this yields the following solution: for $\phi > \phi^P_1$, we have (result 6.) $n_2 = \min\{M/2, N\}$, $n_1 = 0$, $n_3 = N - n_2$. For $\phi^P_1 > \phi > \phi^P_4$, we have (result 2.) $n_2 = \min\{M/2, N\}$, $n_1 = 0$, $n_3 = N - n_2$. For $\phi < \phi^P_4$, then the relationship between destinations and peak slots matters. For $M > N$, the solution (result 4.) is $n_1 = 2N - M$, $n_2 = M - N$ and $n_3 = 0$. For $N > M$, result 3. occurs, according to which $n_1 = M$, $n_2 = 0$ and $n_3 = N - M$.

Appendix C

Monopoly airlines

The Lagrangian of problem (1.33) and its derivatives write as

$$\begin{aligned} L = & \left[s_l (q^f_{lh} - q^f_{lh} q^f_{lh} - q^{f'}_{hl} q^f_{lh}) - \phi q^f_{lh} \right] \\ & + (s_h q^{f'}_{hl} - s_l q^f_{lh} q^{f'}_{hl} - s_h q^{f'}_{hl} q^{f'}_{hl} - \phi q^{f'}_{hl}) \\ & + \lambda^f_{lh} q^f_{lh} + \lambda^{f'}_{hl} q^{f'}_{hl} \\ \frac{\partial L}{\partial q^f_{lh}} = & \, s_l (1 - 2q^f_{lh} - 2q^{f'}_{hl}) - \phi + \lambda^f_{lh} \\ \frac{\partial L}{\partial q^{f'}_{hl}} = & \, s_h - 2s_l q^f_{lh} - 2s_h q^{f'}_{hl} - \phi + \lambda^{f'}_{hl} \end{aligned}$$

where $\lambda_{lh} \geq 0$ and $\lambda_{hl} \geq 0$ are the Khun-Tucker multiplicators. The Hessian matrix is

$$H = \begin{bmatrix} -2s_l & -2s_l \\ -2s_l & -2s_h \end{bmatrix},$$

with determinant $4s_l (s_h - s_l) > 0$. Therefore H is definite positive and we have unique maximum.

The unique root of $\dfrac{\partial L}{\partial q^f_{lh}} = \dfrac{\partial L}{\partial q^{f'}_{hl}} = 0$ is given by

$$\begin{aligned} q^f_{lh} &= -\frac{1}{2s_l (s_h - s_l)} \left(\phi s_h - \phi s_l - s_h \lambda^f_{hl} + s_l \lambda^{f'}_{hl} \right), \\ q^{f'}_{hl} &= \frac{1}{2 (s_h - s_l)} \left(s_h - s_l + \lambda^{f'}_{hl} - \lambda^f_{hl} \right) \end{aligned}$$

The maximum solution $\lambda^f_{lh} \geq 0$, $q^f_{lh} \geq 0$, $\lambda_{lh} q^f_{lh} = 0$, and $\lambda^{f'}_{hl} \geq 0$, $q^{f'}_{hl} \geq 0$, $\lambda^{f'}_{hl} q^{f'}_{hl} = 0$. Suppose $\lambda^f_{lh} = 0$ and $\lambda^{f'}_{hl} = 0$ while $q^f_{lh} \geq 0$ and $q^{f'}_{hl} \geq 0$. Then, we get $q^f_{lh} = -\frac{1}{2} \frac{\phi}{s_l} < 0$ and $q^{f'}_{hl} = \frac{1}{2}$, which is impossible for $\phi > 0$. Hence the two flights f and f' are not operated together. Suppose $\lambda^f_{lh} > 0$ and $\lambda^{f'}_{hl} = 0$ while $q^f_{lh} = 0$ and $q^{f'}_{hl} \geq 0$. Then, we get $q^{f'}_{hl} = \frac{1}{2s_h} (s_h - \phi)$ and $\lambda^f_{hl} = \phi \frac{s_h - s_l}{s_h} > 0$,which is possible for $s_h > \phi$. Suppose $\lambda^f_{lh} = 0$ and $\lambda^{f'}_{hl} > 0$ while $q^f_{lh} \geq 0$ and $q^{f'}_{hl} = 0$. Then, we get $q^f_{lh} = \frac{1}{2s_l} (s_l - \phi)$ and

$\lambda_{hl}^{f'} = -(s_h - s_l) < 0$, which is impossible. Suppose $\lambda_{lh}^{f} > 0$ and $\lambda_{hl}^{f'} > 0$ while $q_{lh}^{f} = 0$ and $q_{hl}^{f'} = 0$. Then, we get $\lambda_{lh}^{f} = -s_l + \phi$ and $\lambda_{hl}^{f'} = -s_h + \phi$, which is impossible for $\phi < s_l$.

Public airport with duopoly airlines

By comparing the number of seats obtained in each configuration, we get:

$$W_{hh} - W_{ll} = \frac{2(s_h - s_l)(\phi^2 + 2s_h s_l)}{9s_h s_l} > 0,$$

$$W_{hh} - W_{hl+lh} = \frac{(s_h - s_l)\left(s_h^2 s_l (20s_h + s_l) + 4s_h s_l \phi (2s_h + s_l) + \phi^2 \left(36s_h^2 - 19s_h s_l + 4s_l^2\right)\right)}{18s_h s_l (s_l - 4s_h)^2} > 0,$$

$$W_{hl+lh} - W_{ll} = \frac{(s_h - s_l)\left(108s_h^2 s_l + s_l \left(8s_l^2 - 4s_l\phi - 13\phi^2\right) + s_h \left(28\phi^2 - 65s_l^2 - 8s_l\phi\right)\right)}{18s_l (s_l - 4s_h)^2} > 0,$$

Hence we obtain $W_{hh} > W_{hl+lh} > W_{ll}$ for all $0 < \phi < \frac{s_l}{2}$.

Next, we evaluate the differences in the allocation of two peak slots:

$$W_{hh} + W_{ll} - 2W_{hl+lh} = \frac{(s_h - s_l)\left(-s_l s_h \left(44s_h^2 - 33s_h s_l + 4s_l^2\right) + 4s_l s_h \phi (2s_h + s_l) + \phi^2 \left(4s_h^2 - 3s_l s_h + 2s_l^2\right)\right)}{9s_h s_l (s_l - 4s_h)^2} < 0$$

for all $0 < \phi < \frac{s_l}{2}$. Hence, $W_{hh} + W_{ll} < 2W_{hl+lh}$.

Chapter 2

Slot Allocation at Congested Airport with Endogenous Fee

2.1 Introduction

In the past decade, growth in air traffic continues to outstrip runway and other passenger handling infrastructure development. As a result, critical shortages of infrastructure capacity have been experienced by many airports worldwide. In response to airport capacity constraint, some policy makers tempt to micromanage airline flight schedules at busy operating hours. More seemly approaches include congestion pricing and slot allocation. A slot, by the definition of IATA, is a permit that allows the access to the full range of airport infrastructure necessary for departure or landing at a certain airport, within a specified time frame. Economic issue has been raised on how to allocate scarce airport slots in an efficient way. The existing literature on slot allocation is sparse. Barbot (2004) models slots for airline activities as products of either high or low quality, and carriers choose the number of flights they operate. Her work however ignores the cap of peak hour capacity. Verhoef (2008) and Brueckner (2009) compare the pricing and slot policy regimes, but their approach are not directly relevant to the issues studied in the current chapter.

Chapter 1[1] formally examines airport's slots allocation behavior in the context of a vertical differentiation model, showing that inefficiency may arise at a private airport if the airport is not heavily congested and the per-passenger fee is small enough, while with a public airport it does not emerge. Moreover, Chapter 1 is concerned about a fully regulated private airport, where airport charge is predetermined by policymakers

[1] Picard et al (2015).

or authorities. It is worth noting that we use the term regulated to refer to exogenous fee throughout, conversely the term unregulated refers to endogenous fee. One may argue that a fully regulated airport seems an unpalatable assumption in some circumstances. In effect, in recent years there has been a tendency of moving towards less government involvement, as reflected by advocating dismantlement of regulation and less-stringent price monitoring. The current chapter is a natural extension of the previous chapter. We attempt to direct emphasis towards an unregulated fee structure and shed lights on the potential impact of deregulation on pricing practices. We add additional complexities to the framework of the previous chapter by accommodating endogenous fee setting, in order to offer greater realism in relation to the changing regulatory policy in flex, and in turn derive more insightful policy implications. We find that the allocative inefficiency,[2] a possible outcome at a regulated private airport, would vanish at a unregulated private airport. Though in an unregulated environment the airport charge would never be set to a low level. For a public airport, instead of charging fee, a subsidy may well be required to reach first-best outcome. Additionally, when the destination market is served by a monopoly airline, a subsidy is desired due to monopoly airline's market power effect.

The remainder of this chapter is organized as follows. We will briefly introduce the basic modeling constructed in chapter 1 in section 2.2. The analysis of simultanous fee setting and slot allocation for a private airport would be carried out in section 2.3, and for a public airport would be described in section 2.4. Section 1.7 sets forth conclusion.

2.2 Model

It is instructive to first revisit the basic model setting of the previous chapter. According to them, a private airport have connections to N independent destination markets $d \in D = \{1, 2, ..., N\}$, each destination is served by 2 separate airlines. A flight is denoted as f: $f \in \mathcal{F} = \{1, 2, ..., 2N\}$. The model deals with only single trip departing flights, with endpoint airports being uncongested. Furthermore, quality differential is characterized only by the departing time and there are two travel periods, namely peak and off-peak. A peak period represents the time window that consists of the most desirable travel times in a day, whilst an off-peak period contains all the rest time intervals. In order to address the problem of peak slots congestion, the off-peak period is assumed to be uncongested, i.e., airport capacity can serve all flights within off-peak

[2] Allocative inefficiency is defined as the situation where not all available peak slots are used in Chapter 1.

period time intervals. Conversely, the peak period is congested. All potential passengers acknowledge and agree over peak load hours (denoted as subscripts h) are more preferable than the off-peak load hours (denoted as subscripts l) at equal price, i.e., slot qualities s_l and s_h are exogenously perceived: $s_h > s_l > 0$. We assume in each market the airlines engage in quantity competition. We denote $p^f_{ii'}$ as the price charged by an airline f that takes off at slot i while its competitor on the same destination takes off at slot i', $i, i' = \{l, h\}$. Similarly, $q^f_{ii'}$ denotes the number of passenger served by this flight. Demand in each destination is generated from a unit mass of passengers with a type parameter v, $v \in [0, 1]$, v being uniformly distributed with unit density.

Demand addressed to flight f is defined by the set of passengers who maximize their utility when flying with airline f, rather than flight f' to the same destination or refraining from flying. Accordingly, a potential passenger in the destination market d with the airlines f and f' has the following preferences:

$$U^d = \begin{cases} vs_i - p^f_{ii'} & \text{if she flies with airline } f \text{ in slot } s_i \text{ at price } p^f_{ii'}, \\ vs_{i'} - p^{f'}_{i'i} & \text{if she flies with airline } f' \text{ in slot } s_{i'} \text{ at price } p^{f'}_{i'i}, \\ 0 & \text{if she does not fly.} \end{cases}$$

The airlines choose the number of seats in order to maximize profits. Airline costs include airport per-passenger charge ϕ, while marginal operating costs are normalized to zero. The entry and exit of airlines are assumed away and fixed costs are sunk. Given these particular specifications, the profit of an airline f , as well as its counterpart f'[3], are given respectively by:

$$\begin{cases} \pi^f_{lh} &= \left(p^f_{lh}(q^f_{lh}, q^{f'}_{hl}) - \phi\right) q^f_{lh}, \\ \pi^{f'}_{hl} &= \left(p^{f'}_{hl}(q^f_{lh}, q^{f'}_{hl}) - \phi\right) q^{f'}_{hl}. \end{cases}$$

if slots are different in a destination, and

$$\begin{cases} \pi^f_{ii} = \left(p^f_{ii}(q^f_{ii}, q^{f'}_{ii}) - \phi\right) q^f_{ii}, \\ \pi^{f'}_{ii} = \left(p^{f'}_{ii}(q^f_{ii}, q^{f'}_{ii}) - \phi\right) q^{f'}_{ii}. \end{cases}$$

if slots are same in a destination.

Denote the total number of peak slots as M. The airport chooses per-passenger charge ϕ plus the slot allocation to maximize its profit

$$\max_{\{i,i'\}} \Pi = \Sigma^N_{d=1} \phi \left(q^f_{ii'} + q^{f'}_{i'i}\right),$$

[3]To be more succint, since destinations are independent, i.e., demand generated in a destination does not affect demand generated in other destinations, when we say compete we refer to the competition in the same destination.

subject to the peak slot capacity constraint

$$\#h \leq M.$$

Under this constraint, the overall allocated peak slots can not exceed the total number of available peak slots, and that peak capacity could not accommodate all flights $M < 2N$. To avoid a cumbersome discussion of ties, we assume that M is even. Allocative inefficiency is defined as below.

Definition 1 *Allocative inefficiency describes the situation when at least one peak slot is not used.*

The timing of the game is as follows. In the first stage the airport simultaneously sets charge ϕ and allocates peak and off-peak slots. In the second stage airlines choose number of seats to supply $q^f_{ii'}$ based on slot allocation. In the third stage passengers in each destination decide whether to fly with a peak period airline, an off-peak period airline, or not fly at all. The equilibrium concept is the subgame perfect equilibrium by backward induction.

2.2.1 Passengers

In the third stage, a passenger decides whether to fly and, if so, the time of flying. If two flights in a market obtain different slots, the indifferent passenger's type ($\bar{v}$) is characterized by $\bar{v}s_l - p^f_{lh} = \bar{v}s_h - p^f_{hl}$. Likewise, the marginal passenger's type ($\underline{v}$) is characterized by $\underline{v}s_l - p^f_{lh} = 0$. Thus airline markets are partially covered. Additionally, if two flights in a single market obtain slots of same quality, the indifferent passenger coincides with marginal passenger, whose type (v_0) is defined by $v_0 s_l - p^f_{ll} = 0$, or $v_0 s_h - p^f_{hh} = 0$, with $p^f_{ii'} = p^{f'}_{i'i}$ in this case. Three market configurations may arise at equilibrium: (i) the two airlines obtain peak/off-peak slots; and both obtain either (ii) off-peak slots, or (iii) peak slots. They are characterized by the following inverse demand functions, respectively:

$$(i) \begin{cases} p^f_{lh} &= s_l \left(1 - q^f_{lh} - q^{f'}_{hl}\right), \\ p^{f'}_{hl} &= s_h \left(1 - \frac{s_l}{s_h} q^f_{lh} - q^{f'}_{hl}\right), \end{cases} \tag{2.1}$$

$$(ii)\ p_{ll} = s_l \left(1 - q^f_{ll} - q^{f'}_{ll}\right),$$

$$(iii)\ p_{hh} = s_h \left(1 - q^f_{hh} - q^{f'}_{hh}\right).$$

2.2.2 Airlines

In the second stage, airlines set their optimal supply of seats.[4] According to (2.1), in configuration i), airlines' profits are:

$$(i)\quad \begin{cases} \pi^f_{lh} = & \left[s_l(1-q^f_{lh}-q^{f'}_{hl})-\phi\right]q^f_{lh}, \\ \pi^{f'}_{hl} = & (s_h - s_l q^f_{lh} - s_h q^{f'}_{lh} - \phi)q^{f'}_{hl}. \end{cases}$$

Airlines choose the number of seats to maximise profits, for any given ϕ. The first-order conditions are:

$$\frac{\partial \pi^f_{lh}}{\partial q^f_{lh}} = -\phi + (1-q^f_{lh}-q^{f'}_{hl})s_l - q^f_{lh}s_l = 0, \tag{2.2}$$

$$\frac{\partial \pi^{f'}_{hl}}{\partial q^{f'}_{hl}} = -\phi + s_h - 2q^{f'}_{hl}s_h - q^f_{lh}s_l = 0. \tag{2.3}$$

Solving (2.2) and (2.3) simultaneously with respect to q^f_{lh} and $q^{f'}_{hl}$ yields:

$$q^f_{lh} = \frac{s_h s_l - \phi\,(2s_h - s_l)}{(4s_h - s_l)s_l},$$

$$q^{f'}_{hl} = \frac{2s_h - \phi - s_l}{4s_h - s_l}.$$

To ensure interior solutions in terms of seats, i.e., $q^f_{lh}, q^{f'}_{hl} > 0$,the condition $0 < \phi < \phi_1 \equiv \frac{s_h s_l}{2s_h - s_l}$ is assumed to hold. Note that $q^f_{lh} < q^{f'}_{hl}$ for all $0 < \phi < \phi_1$:

$$p^f_{lh} = \frac{s_h(2\phi + s_l)}{4s_h - s_l},$$

$$p^{f'}_{hl} = \frac{2s_h^2 + s_h(3\phi - s_l) - \phi s_l}{4s_h - s_l},$$

being always positive.

In configuration ii) and iii), analogously the airlines face the demand

$$q^f_{ii} + q^{f'}_{ii} = 1 - \frac{p_i}{s_i}.$$

Airline profits are:

$$\begin{cases} \pi^f_{ii} = \left[(1-q^f_{ii}-q^{f'}_{ii})s_i - \phi\right]q^f_{ii}, \\ \pi^{f'}_{ii} = \left[(1-q^f_{ii}-q^{f'}_{ii})s_i - \phi\right]q^{f'}_{ii}. \end{cases}$$

[4]Assuming airlines simultaneously choose seat supply is equivalent to assuming a two stage sequential game where they choose capacity at the first stage and compete in flight fare at the second stage (Kreps and Scheinkman, 1983).

The first-order conditions with respect to the number of seats yield:

$$\begin{cases} -\phi - q_{ii}^{f} s_i + s_i(1 - q_{ii}^{f} - q_{ii}^{f'}) = 0, \\ -\phi - q_{ii}^{f'} s_i + s_i(1 - q_{ii}^{f} - q_{ii}^{f'}) = 0. \end{cases}$$

Solving for q_{ii}^{f} and $q_{ii}^{f'}$ lead to:

$$q_{ii}^{f} = q_{ii}^{f'} = \frac{s_i - \phi}{3s_i}. \tag{2.4}$$

Fares are derived via the inverse demand functions:

$$p_{ll} = \frac{s_l + 2\phi}{3}, \quad p_{hh} = \frac{s_h + 2\phi}{3}.$$

Equation (2.4) specifies that condition $0 < \phi < s_i$ must hold for $q_{ii}^{f}, q_{ii}^{f'} > 0, i \in \{h, l\}$. To ensure interior solutions, we assume $0 < \phi < s_i$ and $s_i > \phi_1$, it then follows that $0 < \phi < \phi_1$ is sufficient condition for q_{lh}^{f}, q_{hl}^{f}, and $q_{ii'}^{f}$ to be positive. Moreover, it can be verified that $q_{hl}^{f} > q_{hh}^{f} > q_{ll}^{f} > q_{lh}^{f}$ hold for all $0 < \phi < \phi_1$.

2.3 Private Airport

2.3.1 Duopoly airlines

The previous section looks at the last two stages of the game. The situation of per-passenger fee ϕ taken as given by the airport implies that the airport is subject to ex-ante government regulation. In this section, we investigate the first stage of the game with a private airport. In particular, the airport both sets ϕ and allocates slots so as to maximize its profits, respectively. The last two stages of the game do not change relative to the analysis conducted in Chapter 1: in the second stage, airlines compete in quantities, in the third stage, passengers buy (or not) one ticket for their destinations.

Recall that overal destination markets N can have three types of slot allocations. Let M denote the overall peak slot number, n_1 the number of destination markets with peak/off-peak configuration, n_2 destinations markets with two peak slots, and n_3 destinations markets with two offpeak slots. It follows that the airport allocation problem is given by:

$$\max_{n_1, n_2, n_3, \phi} \left(n_1 \left(q_{lh}^{f} + q_{hl}^{f'} \right) + n_2 \left(q_{hh}^{f} + q_{hh}^{f'} \right) + n_3 \left(q_{ll}^{f} + q_{ll}^{f'} \right) \right) \phi$$

subject to

$$
\begin{aligned}
& n_1 + n_2 + n_3 = N \\
& n_1 + 2n_2 \leq M \\
& 0 \leq n_1, n_2, n_3 \leq N \\
& 0 \leq \phi \ .
\end{aligned}
$$

The first equation regulates that aggregate number of various configuration markets to be equal to the number of existing markets. The second constraint restricts overall allocated peak slots not to exceed smaller than the number of available peak slots. The rest constraints are imposed to preclude negative fee and the number of each market.

As pointed out in the first chapter, when per-passenger charge is exogenous, the airport profit maximization problem coincides with the quantity maximization problem. When the charge is endogenous, the equilibrium airport allocation choice does not change relative to the baseline case, hence the allocation scheme derived for the exogenous fee setting still holds. Follow the lead of last chapter, we again denote $\phi_2 \equiv \frac{s_h s_l}{6s_h - 2s_l}$ as the critical point where passenger number in peak/peak market surpasses peak/offpeak market. Recall that Proposition 1 of last chapter stresses three cases for a private airport with duopoly airlines:

a. for $0 < \phi < \phi_2$, the configuration is "peak/off-peak"in each destination market. The airport uses all available peak slots when $M \leq N$; and does not use all available peak slots (inefficiency) when $M > N$;

b. for $\phi_2 < \phi < \phi_1$ and $M \leq N$, the airport uses all available peak slots and assigns all M peak slots to M markets, resulting in a mix of M "peak/off-peak"and $(N - M)$ "off-peak/off-peak" markets;

c. for $\phi_2 < \phi < \phi_1$ and $M > N$, the airport uses all available peak slots, and implements $(M - N)$ "peak/peak"and $(2N - M)$ "peak/off-peak" configuration.

More importantly, Proposition 1 states that for a private airport with duopoly airlines, if Case *a* occurs, then inefficiency arises at the mildly congested airports, while the heavily congested airports are exempted from the inefficiency concern. When the pre-determined fee is set at a relative high level, as depicted in Case *b* and *c*, allocation is always efficient.

Airport's profit in all three cases can be characterized as:

$$\Pi(\phi) = \begin{cases} \Pi_a(\phi) & \text{for } (\phi, N) \in [0, \phi_2] \times R^+, \\ \Pi_b(\phi) & \text{for } (\phi, N) \in [\phi_2, \phi_1] \times [M, \infty], \\ \Pi_c(\phi) & \text{for } (\phi, N) \in [\phi_2, \phi_1] \times [0, M). \end{cases}$$

Profit $\Pi(\phi)$ is linear in n_1, n_2 and n_3, therefore choosing each of n_1, n_2 and n_3 is equal to choosing the linear combination of the others.

With exogenous fees setting they could be reformulated as the following.

$$\Pi_a(\phi) = \left[\min\{M, N\}\left(q_{lh}^f + q_{hl}^{f'}\right) + (N - \min\{M, N\})\left(q_{ll}^f + q_{ll}^{f'}\right)\right]\phi,$$
$$\Pi_b(\phi) = \left[M\left(q_{lh}^f + q_{hl}^{f'}\right) + (N - M)\left(q_{ll}^f + q_{ll}^{f'}\right)\right]\phi,$$
$$\Pi_c(\phi) = \left[(2N - M)\left(q_{lh}^f + q_{hl}^{f'}\right) + (M - N)\left(q_{hh}^f + q_{hh}^{f'}\right)\right]\phi.$$

For each possible allocation choice, the airport (i) maximizes its objective function with respect to per-passenger fees ϕ, (ii) verifies if the optimal level of per passenger fees is consistent with the allocation choice, (that is, if ϕ lies in the range determined by the equilibrium allocation) and (iii) evaluates whether the optimal ϕ ensures an interior solution in the market stage ($\phi < \phi_1$). The ensuing steps are developed separately for each case. In the interest of brevity, many of the technical details as well as proofs are relegated to the Appendix.

- Case *a*

Case *a* describes $0 < \phi < \phi_2$, which can be classified into two possible situations according to whether $N > M$ or $N < M$. The analysis begins with the scenario $N > M$, where the airport's problem can be stated as:

$$\max_{\phi} \Pi_a(\phi) = \max_{\phi}\left[M\frac{s_h(3s_l - 2\phi) - s_l^2}{s_l(4s_h - s_l)} + (N - M)\frac{2(s_l - \phi)}{3s_l}\right]\phi. \tag{2.5}$$

The second-order condition fulfills so that $\Pi_a(\phi)$ is concave in ϕ. The optimal fee is determined by the first-order condition of (2.5) with respect to ϕ. Denote this optimal fee as ϕ_{a1}, then ϕ_{a1} can be written as:

$$\phi_{a1} = \frac{s_l\left[s_l(M + 2N) - s_h(M + 8N)\right]}{4\left[s_h(M - 4N) + s_l(N - M)\right]}. \tag{2.6}$$

The next step is to verify if ϕ_{a1} is consistent with the allocation considered, more precisely, whether the necessary condition to ensure equilibrium allocation ($0 < \phi_{a1} < \phi_2$)

holds. It can be verify that the optimal per-passenger fee falls out of the parameters' range which supports this allocation pattern:

$$\phi_{a1} > \phi_2. \tag{2.7}$$

Therefore the optimal fee is ϕ_2 in this case. Next we examine the scenario of $N < M$, where the airport's problem is formulated as:

$$\max_{\phi} \Pi_a(\phi) = \max_{\phi} \; N \frac{s_h(3s_l - 2\phi) - s_l^2}{s_l(4s_h - s_l)} \phi.$$

The second-order condition is satisfied, and the optimal per-passenger fee, denoted as ϕ_{a2}, can be derived from first-order condition:

$$\phi_{a2} = \frac{s_l\,(3s_h - s_l)}{4s_h}.$$

In order for ϕ_{a2} to be the optimal fee, it is essential to ensure that ϕ_{a2} lies inside its allowed range: $0 < \phi_{a2} < \phi_2$. Likewise, upon checking ϕ_{a2} and ϕ_2, we discover:

$$\phi_{a2} > \phi_2,$$

the optimal per-passenger fee violates the parameters' allowable range that sustains this allocation pattern. For this scenario the optimal fee is again ϕ_2. The analysis can be summerized in the below Lemma.

Lemma 1 *Suppose all markets are served by duopoly airlines, and the airport is private. When airport could determine the charge imposed on airlines, allocative inefficiency would not occur.*

Case *a* describes the case where fee is set to be low in the regulated environment. Lemma 1 highlights the contrast of unregulated environment to the regulated environment in terms of allocation pattern: that allocative inefficiency completely vanishes if fee is endogenously determined by the airport. A low level of fee is not sustainable at the unregulated airport. The intuition is as follows. Within the context of exogenous fee, airport has only one instrument to manipulate: the peak slot allocation. Hence in maximizing profit, airport distorts the allocation pattern from social optimum level. Whilst when the fee becomes endogenous, airport has two substitutable instruments to extract airlines' surplus: fee and slot allocation. It can either phase out a low level of fee, or leave some peak slots unused, or use both instruments. In this case, the airport manipulates the fee rather than allocation, therefore the allocation inefficiency is precluded.

- Case b

We now turn to Case b where $\phi_2 < \phi < \phi_1$ and $N > M$. The analysis is identical to the scenario corresponding to $N > M$ in Case a, with airport problem stated by (2.5) and optimal fee by (2.6), respectively. In order to distinguish optimal fee for different cases, let ϕ_b denote the optimal fee under the current case, obviously $\phi_b = \phi_{a1}$, and it follows from (2.7) that $\phi_2 < \phi_b$. To complete the analysis for Case b, it remains to be checked whether $\phi_b < \phi_1$. We show that

$$\phi_b < \phi_1$$

for either $M < M_b \equiv \frac{2Ns_l(4s_h-s_l)}{6s_h^2-7s_hs_l+s_l^2}$ and $s_l < \frac{(5-\sqrt{17})}{2}s_h$; or $s_l > \frac{(5-\sqrt{17})}{2}s_h$. For later reference, we introduce the notion of peak slot scarcity $\frac{M}{N}$. Peak slot scarcity represents the ratio of the number of available peak slots to the number of markets. We could express peak slot scarcity under Case b as

$$\boxed{\frac{M_b}{N} = \frac{2s_l\left(4s_h - s_l\right)}{6s_h^2 - 7s_hs_l + s_l^2},} \tag{2.8}$$

with M_b indicating the maximum number of peak slots that an optimal fee ϕ_b could sustain. Under such specification, $\frac{M_b}{N}$ represents the maximum peak slot scarcity that supports an optimal fee equal to ϕ_b.

- Case c

Finally we examine Case c, which sketches $\phi_2 < \phi < \phi_1$ and $N < M$. The airport's problem is now:

$$\max_{\phi} \Pi_c(\phi) = \max_{\phi} \left[(2N - M)\, \frac{s_h(3s_l - 2\phi) - s_l^2}{s_l(4s_h - s_l)} + (M - N)\, \frac{2(s_h - \phi)}{3s_h} \right] \phi.$$

Solving first-order condition for ϕ yields the equilibrium fee, denoted as ϕ_c :

$$\phi_c \equiv \frac{s_hs_l\left[s_hM - 10s_hN - s_lM + 4s_lN\right]}{4\left[4s_hs_l(N-M) + 3s_h^2(M-2N) + s_l^2(M-N)\right]}.$$

One can check that ϕ_c lies in the valid range for parameter values $M < M_c \equiv \frac{2Ns_h(2s_h+s_l)}{(10s_h-3s_l)(s_h-s_l)}$ and $\frac{(5-\sqrt{17})}{2}s_h < s_l < \frac{2}{3}s_h$; or $s_l > \frac{2}{3}s_h$. Finally, we denote the maximum peak slot scarcity which sustains an optimal fee equals to ϕ_c as $\frac{M_c}{N}$. The appropriate expression of $\frac{M_c}{N}$ is then:

$$\boxed{\frac{M_c}{N} = \frac{2s_h\left(2s_h + s_l\right)}{\left(10s_h - 3s_l\right)\left(s_h - s_l\right)}.} \tag{2.9}$$

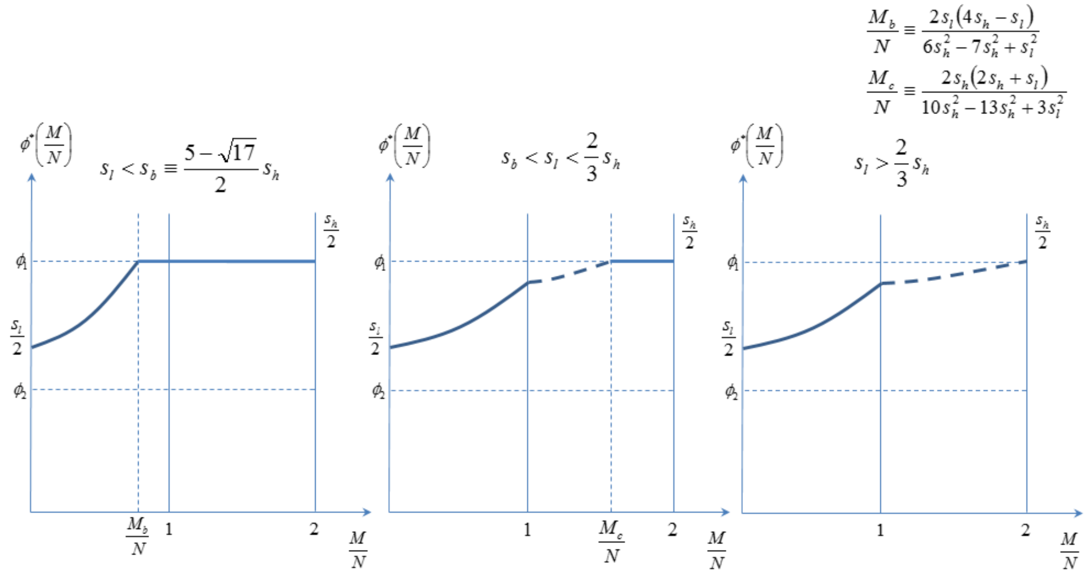

Figure 1: Optimal Per-passenger Fee

The following proposition summarizes the equilibrium per-passenger fee:

Proposition 1 *Suppose all markets are served by duopoly airlines, and the airport is private. Then the airport never sets the per-passenger fees below the critical point where passenger number in peak/peak market surpasses peak/offpeak market. Moreover, the optimal per-passenger fee is*

$$\phi^*\left(\frac{M}{N}\right) = \begin{cases} \phi_1 & \text{for } \frac{M_c}{N} < \frac{M}{N} < 2, \text{ and } s_l \in \left(\frac{(5-\sqrt{17})}{2} s_h, \frac{2}{3} s_h\right), \\ & \text{or } \frac{M_b}{N} < \frac{M}{N} < 2 \text{ and } s_l \in \left(0, \frac{(5-\sqrt{17})}{2} s_h\right); \\ \phi_c & \text{for } 1 < \frac{M}{N} < \frac{M_c}{N} \text{ and } s_l \in \left(\frac{(5-\sqrt{17})}{2} s_h, \frac{2}{3} s_h\right), \\ & \text{or } 1 < \frac{M}{N} < 2 \text{ and } s_l > \frac{2}{3} s_h; \\ \phi_b & \text{for } 0 < \frac{M}{N} \leq \frac{M_b}{N} \text{ and } s_l < \frac{(5-\sqrt{17})}{2} s_h, \\ & \text{or } 0 < \frac{M}{N} < 1 \text{ and } s_l \in \left(\frac{(5-\sqrt{17})}{2} s_h, s_h\right), \end{cases}$$

where $\frac{M_b}{N}$ and $\frac{M_c}{N}$ are given by (2.8) and (2.9), respectively.

Figure 1 depicts the equilibria, see Appendix B for the derivation of the graph and a discussion of peak slot scarcity. Proposition 1 immediately generates intuitively appealing comparison with exogenous fee framework and moreover, is crucial in the later comparison with a public airport. Unlike in a regulated fee situation, when the airport has the power of setting per-passenger fees, it never chooses a too low per-passenger fee (Case *a*) and essentially no peak slot would be left unused. In contrary to the exogenous fee case, the airport now has two instruments at disposal. It may use its dominant position to either exploit slot allocation or raise fee above the social optimal level, or both. Proposition (1) has stated that it would choose to distort fee rather than allocation. Note that distortion refers to the shrink of the range of fee. To be more precise, when fee is regulated, the range of fee is $(0, \phi_1)$; while when fee is endogenous, the range of fee is (ϕ_2, ϕ_1) .

2.3.2 Monopoly airline

We next examine the case of private airport with monopoly airlines.The equilibrium configuration with exogenous fees are (for details see section 1.4 of Chapter 1**):** monopoly airlines operate one flight per destination and the private airport uses all available peak slots. In light of this statement, $m_1 = \min\{N, M\}$, $m_2 = \max\{N - M, 0\}$, where m_1 and m_2 are the number of peak and off-peak slot markets, respectively. Moreover, the condition ensuring interior solutions, i.e., positive passenger volumes in all markets is $0 < \phi < s_l$. The ensuing analysis will be devided into two parts, depending on whether $M < N$ or $M \geq N$. We will discuss the two cases separately.

- If $\min\{N, M\} = M : m_1 = M$, $m_2 = N - M$. The second-order derivative is negative, details are placed in Appendix C. Solving for ϕ from first-order derivative yields:
$$\phi_1^{M*} = \frac{N s_h s_l}{2\left(N s_h - M s_h + M s_l\right)}.$$
Next we check that ϕ_1^{M*} lies in the admissible parameter range of $\phi : (0, s_l)$ when $s_h < 2s_l$; or $s_h \geq 2s_l$ and $M \leq \frac{N s_h}{2(s_h - s_l)}$. On the contrary, ϕ_1^{M*} lies outside the admissible parameter range when $s_h > 2s_l$ and $M \geq \frac{N s_h}{2(s_h - s_l)}$. The optimal fee depends on a combination of quality difference and relative scarcity of peak slots. When the quality difference is small, the optimal fee is ϕ_1^{M*}; while when the difference is large, the optimal fee is ϕ_1^{M*} if airport is not busy, and s_l if the airport is busy.

- If $\min\{N, M\} = N : m_1 = N, m_2 = 0$. We first check that the second-order condition holds, then solve for ϕ, yields:
$$\phi_2^{M*} = \frac{s_h}{2}. \tag{2.10}$$
When the quality difference is large $(s_h > 2s_l)$, the optimal fee lies beyond the permissable region: $\phi_2^{M*} > s_l$; while on the contrary, when the quality difference

is small $(s_h < 2s_l)$, the optimal fee lies within the region: $\phi_2^{M*} < s_l$. Therefore, the optimal fee is s_l when the quality difference is large and ϕ_2^{M*} when the quality difference is small. The ongoing analysis can be summarized as:

Proposition 2 *Suppose all markets are served by monopoly airlines, and the airport is private. Conditional on peak slot scarcity and parameter values, the airport sets* $\phi^* = \min\{s_l, \phi^{M*}\}$.

2.4 Public airport

In this section, we investigate the first stage of the game in the context of a public airport. The airport authority now represents a social maximizer and is concerned about social welfare, which is the sum of airport' profits, passenger surplus and airlines' profits. Since airport and airlines operating costs are normalized to zero, airport profits come from total per-passenger fees, whereas airline profits are the ticket income less total per-passenger fees paid to the airport. In turn, passenger surplus is represented by the total gross utility generated from flying minus all ticket payments. Since monetary transfers between airlines and airport cancel out, and so do transfers between passengers and airlines, then social welfare equals the sum of passengers' gross utility in all $2N$ destination markets.

Letting the number of destination markets with peak/off-peak allocation be denoted by n_1, with peak/peak allocation by n_2, and offpeak/offpeak allocation by n_3. Furthermore, in destination markets with the peak/offpeak configuration, the passenger type who is indifferent between taking the peak hour flight and the offpeak hour flight is denote as v_{hl}, and the passenger type indifferent between taking the offpeak hour flight and not flying is denoted as v_{lh}. The expression $\int_{v_{lh}}^{v_{hl}} vs_l dv$ thus gives the gross passenger surplus from taking the off-peak period flight, while $\int_{v_{hl}}^{1} vs_h dv$ is the gross passenger surplus from taking the peak period flight in the same destination market. Likewise, in a peak/peak (resp. offpeak/offpeak) destination market, let's denote the indifferent passenger as v_{hh} (resp. v_{ll}),the gross passenger surplus is thus $\int_{v_{hh}}^{1} vs_h dv$ (resp. $\int_{v_{ll}}^{1} vs_l dv$).

Total welfare, denoted as W, then emerges:

$$W = n_1 \left(\int_{v_{lh}}^{v_{hl}} vs_l dv + \int_{v_{hl}}^{1} vs_h dv \right) + n_2 \int_{v_{hh}}^{1} vs_h dv + n_3 \int_{v_{ll}}^{1} vs_l dv,$$

where $v_{hl} = \dfrac{p_{hl}^{f'} - p_{lh}^{f}}{s_h - s_l}, v_{lh} = \dfrac{p_{lh}^{f}}{s_l}, v_{hh} = \dfrac{p_{hh}^{f}}{s_h}$ and $v_{ll} = \dfrac{p_{ll}^{f}}{s_l}$, the derivation has been presented in section 1.2 of Chapter 1. We will consider first the case with duopoly airlines and then with monopoly airlines.

2.4.1 Duopoly airlines

Under exogenous fees and $M \leq N$, the airport allocates all peak slots according to peak/off-peak configuration, while for $M > N$ the airport implements $(M - N)$ "peak/peak" and $(2N - M)$ "peak/off-peak" configurations. In both cases, all peak slots are used. Moreover, the condition

$$\phi < \phi^P = \frac{s_l}{2}$$

must hold to ensure interior solutions. We will compute for the optimal fee in the first place, then check whether the fee lies in the allowed range.

- Case 1. $N \geq M$.

In this scenario, the solution is $n_1 = M$, $n_2 = 0$, $n_3 = N - M$. As explained before, the public airport maximizes welfare, which is equivalent to gross passenger surplus in altogether three market configurations. Under this case, the gross passenger surplus refers to the sum of gross passenger surplus in peak/offpeak configuration and offpeak/offpeak configuration markets:

$$\max_{\phi} W(\phi) = M\left(\int_{v_{lh}}^{v_{hl}} vs_l dv + \int_{v_{hl}}^{1} vs_h dv\right) + (N - M)\int_{v_{ll}}^{1} vs_l dv.$$

We solve for optimal fee, denoted as ϕ_1^{P*}, from first-order condition:

$$\phi_1^{P*} = \frac{2s_l\left(M\left(34s_h^2 - 17s_h s_l + s_l^2\right) - N(s_l - 4s_h)^2\right)}{M\left(28s_h^2 - 41s_h s_l + 13s_l^2\right) - 4N(s_l - 4s_h)^2}. \tag{2.11}$$

The sign of ϕ_1^{P*} depends on the relative magnitude of M. Deviating from the case of a private airport, the optimal fee for a public airport can be negative, for some parameter value that M might take. It is hardly surprising that subsidies may be required to reach the first-best welfare outcome. We check that optimal fee takes the form of a charge when M is sufficiently small, and a subsidization otherwise, as shown below:

$$\phi_1^{P*} = \begin{cases} > 0 & \text{when } M < \underline{M}, \\ < 0 & \text{when } \underline{M} < M \leq N. \end{cases}$$

where $\underline{M} \equiv \frac{\left(16s_h^2 - 8s_h s_l + s_l^2\right)N}{34s_h^2 - 17s_h s_l + s_l^2} < N$. The case of a positive ϕ_1^{P*} needs more illustration therefore we focus on a positive ϕ_1^{P*}. By some computation it can be verified that:

$$\phi_1^{P*} < \phi^P.$$

Apparently ϕ_1^{P*} lies inside the feasible region, therefore the optimal fee in this case is indeed ϕ_1^{P*}.

- Case 2. $N < M \leq 2N$.

In this instance, the solution is $n_1 = 2N - M$, $n_2 = M - N$, $n_3 = 0$ in the exogenous case. The public airport maximizes gross passenger surplus in peak/offpeak configuration and peak/peak configuration markets combined:

$$\max_{\phi} W(\phi) = (2N - M)\left(\int_{v_{lh}}^{v_{hl}} vs_l dv + \int_{v_{hl}}^{1} vs_h dv\right) + (M - N)\int_{v_{hh}}^{1} vs_h dv. \tag{2.12}$$

The optimal fee, denoted as ϕ_2^{P*}, is characterized by the first-order condition and can be written as:

$$\phi_2^{P*} = \frac{-2s_h s_l\left(34Ms_h^2 - 17Ms_h s_l + Ms_l^2 - Ns_l^2 - 52Ns_h^2 + 26Ns_h s_l\right)}{36Ms_h^3 - 72Ns_h^3 - 55Ms_h^2 s_l + 46Ns_h^2 s_l + 23Ms_h s_l^2 - 14Ns_h s_l^2 - 4Ms_l^3 + 4Ns_l^3}. \tag{2.13}$$

We are now concerned about the sign of ϕ_2^{P*} as well as whether ϕ_2^{P*} lies inside the feasible region, i.e., $\phi_2^{P*} < \phi^P \equiv \frac{s_l}{2}$. We check that both the sign and the relationship to ϕ^P are contingent on M, the lengthy proof is relegated to Appendix 2.5. We hereby present three scenarios regarding ϕ_2^{P*} and the corresponding relative magnitude of M :

$$\begin{cases} \phi_2^{P*} < 0 & \text{when } M < \underline{\underline{M}}, \\ 0 < \phi_2^{P*} < \frac{s_l}{2} & \text{when } \underline{\underline{M}} < M < \overline{M}, \\ \phi_2^{P*} > \frac{s_l}{2} & \text{when } M > \overline{M}, \end{cases}$$

where $\overline{M} \equiv \dfrac{\left(280s_h^3 - 150s_h^2 s_l + 18s_h s_l^2 - 4s_l^3\right)N}{172s_h^3 - 123s_h^2 s_l + 27s_h s_l^2 - 4s_l^3}$ and $\underline{\underline{M}} \equiv \dfrac{\left(52s_h^2 - 26s_h s_l + s_l^2\right)N}{34s_h^2 - 17s_h s_l + s_l^2}$. Additionally, it is useful to examine that (see Appendix D):

$$\overline{M} > \underline{\underline{M}}.$$

When $M < \underline{\underline{M}}$, instead of imposing a charge, a subsidization equals to ϕ_2^{P*} should be implemented to correct for the market failure, because duopoly airlines undersupply passenger volume relative to social optimum outcome. When M lies between $\underline{\underline{M}}$ and $\overline{M}$, ϕ_2^{P*} locates in the interior of the feasible region and thus the optimal fee is precisely ϕ_2^{P*}. While when $M > \overline{M}$, the optimal price is $\frac{s_l}{2}$.

Combining the above analysis, the following results can be established:

Proposition 3 *Suppose all markets are served by duopoly airlines, and the airport is public.*

1. *If $N \geq M$, then airport subsidizes airlines when M is relatively large and charges a fee ϕ_1^{P*} when M is relatively small. All peak slots are used for M peak/offpeak and $(N-M)$ off-peak/off-peak configuration.*

2. *If $N < M$, then airport subsidizes airlines when M is small, charges $\phi_2^{P*}$for a moderate M, and $\frac{s_l}{2}$ for a large M. All peak slots are used for $(2N-M)$ peak/offpeak and $(M-N)$ peak/peak configuration.*

A negative fee implies that the duopoly airlines exert market power and undersupply passenger volumes, relative to social optimum level. The public airport acts as a social welfare maximizer, thus subsidizes airlines an amount equals to $|\phi_1^{P*}|$ in order to correct for this failure in seat supply.

We are now in a position to discuss two ranges of M, namely, $(\underline{M}, N)$ and $\left(N, \underline{\underline{M}}\right)$, within which airport subsidy is required. Both ranges result in the configuration comprises a relatively large number of peak/offpeak slot markets, and a small number of different slots (offpeak/offpeak for the former, and peak/peak for the latter). To see that, note that for the former case, when M increases, more offpeak/offpeak markets will be replaced by peak/offpeak configuration. As for the latter, likewise, when M increases, more peak/offpeak markets will be replaced by peak/peak configuration. We could thus argue that peak/offpeak configuration leads to a more pronounced undersupply behavior, relative to same slot configuration. The proposition implies that peak/offpeak configuration requires more subsidies than peak/peak or offpeak/offpeak. The intuition is, when airlines compete with different slots, their quality differs and the two could thus exert more market power, and distort the equilibrium quantity from first-best outcome to a greater extent than they would otherwise with same slots.

2.4.2 Airline monopolies

Under the exogenous per-passenger fee regime, if all markets are served by monopoly airlines, then a public airport allocates one peak slot to each monopoly until all peak slots are used up (see section 5.2 of Chapter 1). The market stage in each destination market does not change: given the opportunity of operating in peak slots, each monopoly airline uses one peak slot only.

$$\max_{\phi} W(\phi) = m_1 \int_{v_h}^{1} v s_h dv + m_2 \int_{v_l}^{1} v s_l dv. \tag{2.14}$$

Accordingly, the equilibrium allocation is $m_1 = \min\{N, M\}$, $m_2 = \max\{N - M, 0\}$, where m_1 and m_2 are the numbers of flights in the peak and off-peak slots, respectively. The airport problem can thus be reformulated as:

- Case 1. When $\min\{N, M\} = N$,

$$\max_{\phi} W(\phi) = N \cdot \int_{v_h}^{1} v s_h dv. \tag{2.15}$$

- Case 2. When $\min\{N, M\} = M$,

$$\max_{\phi} W(\phi) = M \cdot \int_{v_h}^{1} v s_h dv + (N - M) \int_{v_l}^{1} v s_l dv,$$

where $v_h = \frac{s_h + \phi}{2s_h}, v_l = \frac{s_l + \phi}{2s_l}$.

We could check that the optimal fees in the two cases, denoted as ϕ_1^{PM*} and ϕ_2^{PM*} respectively, can be derived from first-order conditions:

$$\phi_1^{PM*} = -s_h < 0,$$

$$\phi_2^{PM*} = \frac{N s_h s_l}{M (s_h - s_l) - N s_h} < 0,$$

where subscripts denote for Case 1 and 2, respectively. In order to maximize welfare, the public airport subsidizes monopoly airlines, otherwise they undersupply traffic from the social point of view. Moreover, it can be shown that

$$|\phi_2^{PM*}| > |\phi_1^{PM*}|.$$

Recall that Case 2 represents a mix of M peak slot market and $(N - M)$ offpeak slot market, while Case 1 represents N peak slot market, the above relationship implies that the airport subsidizes offpeak slot market more than peak slot market.

Proposition 4 *Suppose all markets are served by monopoly airlines. Then a public airport uses all available peak slots, and subsidizes monopoly airlines an amount equals to* $|\phi_2^{PM*}|$.

2.5 Conclusion

This chapter contributes to the understanding of a peak period congested airport's optimal fee setting behavior when it has power to allocate slots. It is a natural extension of the previous chapter, while explicitly incorporating the endogenous fee setting behavior into the previous framework. We have found that allocative inefficiency is completely precluded at a private airport if fee is endogenously chosen by the airport. In the context of exogenous fee, in order to maximize profit the airport has only one instrument to exploit, which is the peak slot allocation. In the current chapter the airport has one more instrument at disposal -the fee, and hence it could distort the fee instead of allocation.

Furthermore, at a public airport with duopoly airlines, subsidization may be called for when the fraction of markets having different slots is relatively large. Having different slots impairs competition between airlines, which enables airlines to exploit more market power. When is market is served by a monopoly airline, subsidization is definitely required to reach social optimum. The result is hardly surprising. Without competition, the monopoly airline exploits market power at the cost of a reduced passenger surplus. In order to correct for this distortion, the public airport should subsidize the monopoly airline so that it would supply the socially optimal level of seats.

For model tractability, the present work has intentionally abstracted away from the possibility that passenger groups could be heterogenous, in the sense that the lowest as well as highest willingness to pay for quality differ across markets. An interesting extension of the present analysis would be to consider some destinations are primarily filled with business travellers while others with leisure travellers. The two types entail different average, as well as distribution of marginal willingness to pay for quality.

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Appendix

Appendix A. Private airport with duopoly airlines

- **Case** a

For $N > M$, airport's profit maximization condition is

$$\frac{\partial \Pi_a(\phi)}{\partial \phi} = \frac{s_l\left[s_l(M+2N)+4\phi(M-N)\right]}{3s_l\left(s_l-4s_h\right)} - \frac{s_h\left[s_l(M+8N)+4\phi(M-4N)\right]}{3s_l\left(s_l-4s_h\right)} = 0. \tag{2.16}$$

The second-order condition writes as:

$$\frac{\partial^2 \Pi_a(\phi)}{\partial \phi^2} = \frac{4\left[s_h(M-4N)+s_l(N-M)\right]}{3s_l\left(4s_h-s_l\right)} < 0 \text{ for } N > M.$$

Since $\phi_{a1} = \frac{s_l[s_l(M+2N)-s_h(M+8N)]}{4[s_h(M-4N)+s_l(N-M)]}$, we have:

$$\phi_{a1} - \phi_2 = \frac{s_l\left[s_h^2(5M+16N)-6s_hs_l(M+2N)+s_l^2(M+2N)\right]}{4\left(3s_h-s_l\right)\left[s_h(4N-M)-s_l(N-M)\right]} > 0 \text{ for } N > M.$$

For $M > N$, the airport's problem is

$$\max_{\phi} \Pi_a(\phi) = \max_{\phi} \ N\frac{s_h(3s_l-2\phi)-s_l^2}{s_l(4s_h-s_l)}\phi.$$

The first order condition with respect to ϕ is:

$$\frac{\partial \Pi_a(\phi)}{\partial \phi} = -\frac{N\left[s_h\left(4\phi-3s_l\right)+s_l^2\right]}{s_l\left(4s_h-s_l\right)} = 0, \tag{2.17}$$

while the second order condition is

$$\frac{\partial^2 \Pi_a(\phi)}{\partial \phi^2} = -\frac{4Ns_h}{4s_hs_l-s_l^2} < 0.$$

Solving (2.17) for ϕ yields

$$\phi_{a2} = -\frac{s_l\left(s_l-3s_h\right)}{4s_h}.$$

Checking whether ϕ_{a2} lies in the feasible range of ϕ :

$$\phi_{a2} - \phi_2 = \frac{s_l\left(7s_h^2-6s_hs_l+s_l^2\right)}{4s_h\left(3s_h-s_l\right)} > 0 \text{ for } N > M.$$

Hence ϕ_{a2} falls beyond the allowable range.

- **Case** b

Since $\phi_b = \phi_{a1}$, it follows that:

$$\phi_b - \phi_1 = -\frac{s_l\left[6Ms_h^2 - s_h s_l(7M+8N) + s_l^2(M+2N)\right]}{4\left(2s_h - s_l\right)\left[s_h(M-4N) + s_l(N-M)\right]},$$

which is negative when $M < M_b \equiv \frac{2Ns_l(4s_h - s_l)}{6s_h^2 - 7s_h s_l + s_l^2}$ and $s_l < \frac{\left(5-\sqrt{17}\right)}{2}s_h$; or $s_l > \frac{\left(5-\sqrt{17}\right)}{2}s_h$. Moreover, $\phi_b - \phi_1 > 0$ when $s_l < \frac{\left(5-\sqrt{17}\right)}{2}s_h$ and $M > M_b \equiv \frac{2Ns_l(4s_h - s_l)}{6s_h^2 - 7s_h s_l + s_l^2}$.

- **Case** c

Finally, consider Case c, under which $\phi_2 < \phi < \phi_1$ and $N < M$. The airport's problem is now:

$$\max_{\phi} \Pi_c(\phi) = \max_{\phi}\left[(2N-M)\,\frac{s_h(3s_l - 2\phi) - s_l^2}{s_l(4s_h - s_l)} + (M-N)\,\frac{2(s_h - \phi)}{3s_h}\right]\phi.$$

The first order condition of $\Pi_c(\phi)$ with respect to ϕ is:

$$\begin{aligned}\frac{\partial \Pi_c(\phi)}{\partial \phi} = & \frac{s_h^2\left[12\phi(M-2N) - s_l(M-10N)\right] + 4\phi s_l^2(M-N)}{3s_h s_l\left(4s_h - s_l\right)} \\ & + \frac{s_h s_l\left[s_l(M-4N) + 16\phi(N-M)\right]}{3s_h s_l\left(4s_h - s_l\right)} = 0,\end{aligned}$$

while the second order condition is

$$\frac{\partial^2 \Pi_c(\phi)}{\partial \phi^2} = \frac{4\left[4s_h s_l(N-M) + 3s_h^2(M-2N) + s_l^2(M-N)\right]}{3s_h s_l\left(4s_h - s_l\right)} < 0,$$

for $M < 2N$. Solving first-order condition for ϕ yields the optimal fee, denoted as ϕ_c :

$$\phi_c \equiv \frac{s_h s_l\left[17s_h M - 26s_h N - 5s_l M + 8s_l N\right]}{4\left[4s_h s_l(N-M) + 3s_h^2(M-2N) - s_l^2(M-N)\right]}.$$

Like before, we verify if the optimal per-passenger fee is consistent with the allocation considered. We get

$$\phi_c - \phi_2 = \frac{s_h s_l\left[3s_h^2(M+6N) - 2s_h s_l(2M+7N) + s_l^2(M+2N)\right]}{4\left(3s_h - s_l\right)\left[4s_h s_l(M-N) + 3s_h^2(2N-M) - s_l^2(M-N)\right]} > 0,$$

for $M < 2N$. Therefore the optimal allocation and optimal per-passenger fees are consistent. To complete the analysis, we have an interior solution in the market stage for

$$\phi_c - \phi_1 = -\frac{s_h s_l \left[s_h^2(4N - 10M) + s_h s_l(13M + 2N) - 3Ms_l^2\right]}{4\left(2s_h - s_l\right)\left[4s_h s_l(M - N) + 3s_h^2(2N - M) - s_l^2(M - N)\right]} < 0,$$

for $M < M_c$ and $s_l \in \left(\frac{(5-\sqrt{17})}{2}s_h, \frac{2}{3}s_h\right)$; or $s_h > s_l > \frac{2}{3}s_h$.

Appendix B. Discussion of peak slot scarcity $\frac{M}{N}$

As shown before, the optimal fee is contingent on the number of available peak slots, which translates into a dependence on the peak slot scarcity. We now probe into the dependence of optimal fee ϕ on the peak slot scarcity $\frac{M}{N}$. To start with, let us first focus attention on $\phi\left(\frac{M}{N}\right)$ for $0 \leq \frac{M}{N} \leq 1$. Within this range of peak slot scarcity, note that

$$\phi_b|_{\frac{M}{N}=0} = \frac{s_l}{2},$$

where $\phi_2 < \frac{s_l}{2} < \phi_1$, while

$$\phi_b|_{\frac{M}{N}=1} = \frac{s_l\left(3s_h - s_l\right)}{4s_h}.$$

By evaluating the first and second derivatives of ϕ_b with respect to $\frac{M}{N}$ we get

$$\frac{\partial\phi_b}{\partial\left(\frac{M}{N}\right)} = \frac{3s_l\left(4s_h - s_l\right)\left(s_h - s_l\right)}{4\left[\left(\frac{M}{N} - 4\right)s_h - \left(\frac{M}{N} - 1\right)s_l\right]^2} > 0,$$

$$\frac{\partial^2\phi_b}{\partial\left(\frac{M}{N}\right)^2} = -\frac{3s_l\left(s_h - s_l\right)^2\left(4s_h - s_l\right)}{2\left[\left(\frac{M}{N} - 4\right)s_h - \left(\frac{M}{N} - 1\right)s_l\right]^3} < 0.$$

The second-order condition is fulfilled, which guarantees a unique maximization. We now address the case where $1 < \frac{M}{N} \leq 2$ (since our maintained definition of congestion assumed away $M \geq 2N$, the possibility of $\frac{M}{N} \geq 2$ is precluded). Evaluating ϕ_c at $\frac{M}{N} = 1$ and $\frac{M}{N} = 2$, respectively, yields:

$$\phi_c|_{\frac{M}{N}=1} = \phi_b|_{\frac{M}{N}=1}, \phi_c|_{\frac{M}{N}=2} = \frac{s_h}{2} > \phi_1 \text{ for } s_l < \frac{2}{3}s_h.$$

The first and second derivatives of ϕ_c with respect to $\frac{M}{N}$ are shown as:

$$\frac{\partial\phi_c}{\partial\left(\frac{M}{N}\right)} = \frac{3s_h s_l\left(8s_h^3 - 14s_h^2 s_l + 7s_h s_l^2 - s_l^3\right)}{4\left[3(g-2)s_h^2 - 4(g-1)s_h s_l + (g-1)s_l^2\right]^2} > 0,$$

$$\frac{\partial^2\phi_c}{\partial\left(\frac{M}{N}\right)^2} = -\frac{3s_h s_l\left(s_h - s_l\right)^2\left(3s_h - s_l\right)\left(8s_h^2 - 6s_h s_l + s_l^2\right)}{2\left[3(g-2)s_h^2 - 4(g-1)s_h s_l + (g-1)s_l^2\right]^3} < 0.$$

Appendix C. Private airport with monopoly airline

If $M < N, m_1 = M, m_2 = N - M$, the airport problem is:

$$\max_{\phi} \phi \left[M \cdot \frac{s_h - \phi}{2s_h} + (N - M) \cdot \frac{s_l - \phi}{2s_l} \right]. \tag{2.18}$$

The first order condition with respect to ϕ yields

$$\frac{\partial \Pi(\phi)}{\partial \phi} = -\frac{\phi M}{s_h} + \frac{2\phi M - 2\phi N + N s_l}{2s_l} = 0,$$
$$\Rightarrow \phi_1^{M*} = \frac{N s_h s_l}{2\left(N s_h - M s_h + M s_l\right)}.$$

The second-order condition is

$$\frac{\partial^2 \Pi(\phi)}{\partial \phi^2} = -\frac{M}{s_h} + \frac{M - N}{4s_l} < 0.$$

Checking for the range of ϕ_1^{M*} :

$$\phi_1^{M*} - s_l = -\frac{s_l}{2} \cdot \frac{N s_h + 2M\left(s_l - s_h\right)}{N s_h + M\left(s_l - s_h\right)},$$

which is positive for $s_h > 2s_l$ and $M \geq \frac{N s_h}{2(s_h - s_l)}$; and negative for $s_h < 2s_l$; or $s_h \geq 2s_l$ and $M \leq \frac{N s_h}{2(s_h - s_l)}$.

If $M \geq N, m_1 = N, m_2 = 0$,the airport problem is:

$$\max_{\phi} \frac{\phi N\left(s_h - \phi\right)}{2s_h}. \tag{2.19}$$

The first order condition with respect to ϕ yields

$$\frac{\partial \Pi(\phi)}{\partial \phi} = \frac{N}{2} - \frac{\phi N}{s_h} = 0,$$
$$\Rightarrow \phi_2^{M*} = \frac{s_h}{2}.$$

We next compare s_l and ϕ_2^{M*}:

$$\phi_2^{M*} - s_l = \frac{s_h}{2} - s_l, \tag{2.20}$$

the comparison of ϕ_2^{M*} and s_l is straightforward. Finally we check the second-order condition:

$$\frac{\partial^2 \Pi(\phi)}{\partial \phi^2} = -\frac{N}{s_h} < 0.$$

Appendix D. Public airport and airline duopolies

- **Case 1.** $N \geq M$.

In this instance, the allocation pattern is $n_1 = M$, $n_2 = 0$, $n_3 = N - M$. We have already stated airport maximization problem in subsection 2.4.1. The optimal fee ϕ_1^{P*} is characterized by the first-order condition:

$$\phi_1^{P*} = \frac{2s_l \left(M \left(34s_h^2 - 17s_h s_l + s_l^2\right) - N(s_l - 4s_h)^2\right)}{M\left(28s_h^2 - 41s_h s_l + 13s_l^2\right) - 4N(s_l - 4s_h)^2}. \tag{2.21}$$

The second order condition for a maximum is satisfied:

$$\frac{\partial^2 W(\phi)}{\partial \phi^2} = \frac{M \underbrace{\left(-4s_h^2 - s_h s_l + s_l^2\right)}_{<0}}{s_l\left(s_l - 4s_h\right)^2} - \frac{4(N-M)}{9s_l} < 0.$$

We show that:

$$\phi_1^{P*} - \frac{s_l}{2} = \frac{9Ms_l \underbrace{\left(12s_h^2 - 3s_h s_l - s_l^2\right)}_{>0}}{\underbrace{2\left(28Ms_h^2 - 41Ms_h s_l + 13Ms_l^2 - 64Ns_h^2 + 32Ns_h s_l - 4N\right)s_l^2}_{<0}} < 0$$

for $N \geq M$.

- **Case 2.** $N < M \leq 2N$.

The allocation pattern in this scenario is $n_1 = 2N - M$, $n_2 = M - N$, $n_3 = 0$. Airport's maximization problem is stated in (2.12) and ϕ_2^{P*} is derived in (2.13). We check that:

$$\phi_2^{P*} > 0 \text{ for } M > \underline{\underline{M}} \equiv \frac{\left(36s_h^2 - 18s_h s_l\right)N}{34s_h^2 - 17s_h s_l + s_l^2}.$$

$$\phi_2^{P*} < \frac{s_l}{2} \text{ when } N < M < \overline{M} \equiv \frac{\left(216s_h^3 - 54s_h^2 s_l - 18s_h s_l^2\right)N}{172s_h^3 - 17s_h s_l + s_l^2}.$$

To see the ranking of $\underline{\underline{M}}$ and $\underline{M}$, we show that:

$$\underline{\underline{M}} - \underline{M} = \frac{N \underbrace{\left(20s_h^2 - 10s_h s_l - s_l^2\right)}_{>0}}{34s_h^2 - 17s_h s_l + s_l^2} > 0,$$

for all $s_h > s_l > 0$ and $2N > M > N$.

It is thus clear that:

$$\underline{\underline{M}} > \underline{M}.$$

We also check that the second-order condition is negative:

$$\frac{\partial^2 W(\phi)}{\partial \phi^2} = \frac{(2N - M)\left(-4s_h^2 - s_h s_l + s_l^2\right)}{s_l\left(s_l - 4s_h\right)^2} - \frac{4(N - M)}{9s_h} < 0.$$

Chapter 3

Per-flight and per-passengers congestion pricing when airline quality differs

3.1 Introduction

Air traffic has grown rapidly in the past decade, the growth of traffic has outstripped airport capacity during some time windows of the day at many airports worldwide. Airport congestion, as a result, has increased relentless. For instance, from the year 2000 to 2007 alone, 20% of U.S. commercial flights experienced delay (Rupp (2009)). Moreover, with an expansion of demand due to the increase of income, as well as the growth of some developing countries, more airport infrastructure capacities will fail to keep pace with demand and congestion is expected to get worse in the coming decade. According to European Commission's estimation, half of the world's new traffic will come from Asia Pacific region in the next 20 years.[1] As a consequence, air traffic in Europe will roughly double by 2030, and that 19 key airports will be at saturation, with 50% passengers and cargo flights will be affected by congestion.

At many major airports, current capacities are unlikely to accommodate the estimated increase in traffic, and will fall short of flight activity demand. A persisting congestion problem is socially costly, and solutions are widely discussed. Constructing new runways and infrastructure may be one remedy, though the time-consuming planning horizon as well as gestation period make it hardly an effective short or interim solution. Alongside other factors that hamper increment of capacity including environ-

[1] See MEMO/11/857.

mental concerns (e.g. noise emission, global warming), political disagreement, public resistance,[2] etc. Given the many difficulties to alleviate the congestion problem, economists actively engaged in trying to alleviate this problem by introducing the congestion pricing mechanism.

The economic principles behind congestion pricing is derived from charging road travellers for the externality they create.[3] In the arena of road traffic, the theory of congestion pricing says that usage of a road is excessive since users don't consider the congestion they impose on other users. From the economic viewpoint, charging a toll based on a flight's contribution to congestion would relieve the congestion problem. However, when one extrapolates road-pricing framework to the context of airport, a clear distinction between the two markets should be noted: unlike road travellers, airlines are nonatomistic due to their market power. Agents should be charged atomistic toll in sofar as they are atomistic. Due to their self-internalization behavior, nonatomistic agents already take the internal congestion effects into account when choosing optimal prices and frequencies, optimal toll should contain only the cost they impose on other agents (Daniel (1995)). As a result, a reduction in congestion pricing should be made for airlines with market power (Brueckner (2002, 2005)).

With concerns regarding airline's market power, it then comes along naturally that subsidization may be required to reach the first-best welfare result (Pels and Verhoef (2004)). Important recent theoretical advances in mix of per-passenger and per-flight based airport charges include Silva and Verhoef (2013) and Czerny and Zhang (2015). Silva and Verhoef (2013) consider the mix of per-flight and per-passenger charges at a congested airport. According to them, the public airport is confronted with two sources of inefficiencies: congestion externalities and airline market power. To reach the first-best outcome, congestion externalities should be corrected via charging congestion pricing toll, and market power via subsidization. With all its advantages of clarity and universality, congestion pricing is widely endorsed as an efficient means to alleviate traffic problems.

In an application of a congestion pricing regime, one should bear in mind that a fundamental characteristic of nowadays airline markets entails vertical differentiation of products, because many airlines operating on the same origin-destination routes are not considered as homogeneous products. This is especially relevant to the rivalry

[2] In year 2012 alone, Fraport received 2.2 million complaints against aircraft noise, which amounts to 6,100 complaints per day.

[3] After Pigou (1920) and Knight (1924) initiated the concept, Vickrey (1969) promoted this mechanism and applied it on road traffic. Other important earliest works include Levine (1969), Carlin and Park (1970) and Morrison (1983).

between traditional carriers and low-cost carriers (LCC). Traditional carriers are typically distinguishable from LCC in terms of ticket fares and flight frequency. Though the rapid growth of LCC in the past decade has reshaped the world wide aviation industry, a relatively small attention has been devoted to congestion pricing for heteogenous carriers. This chapter aims to help fill the void by studying frequency and traffic competitions in a vertically differentiated duopolistic airline market. Setting in a single origin–destination route context, the framework captures the rivalry between vertically differentiated airlines, and provides a realistic specification in many airline markets.

Another motivation for studying per-passenger and per-flight stems from the divergence of oversight towards airport charge. Traditionally, airport charges are levied primarily through airlines. More specifically, charges are established based upon aircraft weight.[4] There has been a great deal of discussion in favor of transition from leving tolls through passenger rather than per-flight aeronautical activities (IATA (2010)). Given these opposing opinions, we wonder what difference would it make to levy passenger-based charge or flight-based charge, and which type of charge would be more favorable to the society.

The purpose of this chapter is to provide insights into the potential implementation of congestion pricing to asymmetric airlines, with the goal of helping to inform the regulators. We conduct comparative analysis of different competition regimes, traffic or ticket fare, between vertically differentiated duopoly airlines that serve a destination market, along with the corresponding first-best congestion price. As long as frequency is considered to be relating to service quality, the frequency setting choices of the duopoly airlines notably deviate from conventional wisdom. Under price competition, the low-quality airline internalizes less than self-imposed congestion, and the high-quality airline does not internalize congestion. The two airlines internalize more congestion delays under traffic competition. Moreover, if instead of duopoly airlines, the destination market is served by a monopoly airline, then he would undersupply frequency relative to the social optimum.

Follow the leads of Brueckner and Flores-Fillol (2007), we will use flight frequency as a proxy for airline quality. Controlling for other factors, a higher flight frequency increases the value of flying to the consumers, on the grounds that it reduces the difference between individual's desired departure time and actual departure time. The conventional airline industry wisdom contends that convenience of airline schedules, along with airfare, are often of paramount concern for consumers in ticket purchasing decision. Relating to practical observations, major carriers often times provide more

[4] For a thorough illustration see International Civil Aviation Organisation (ICAO, 2012).

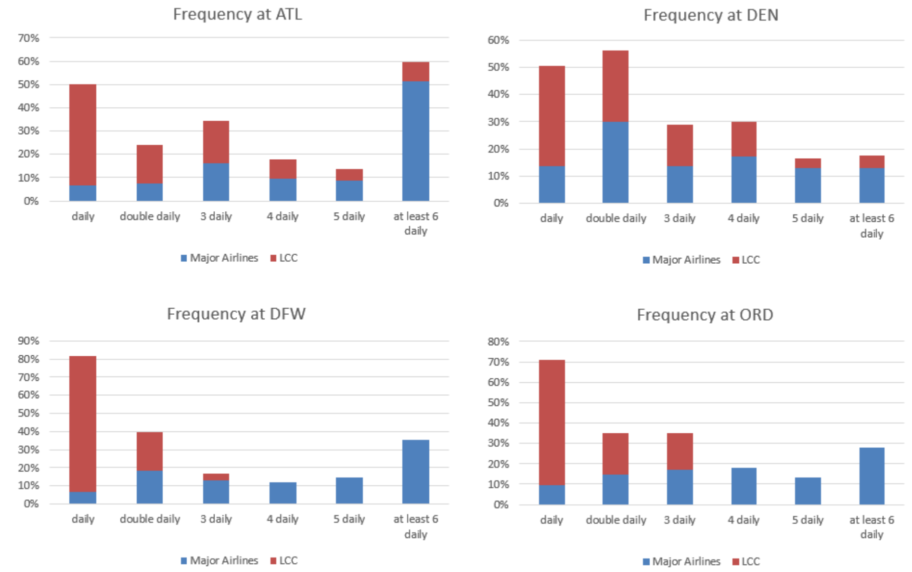

Figure 1: Frequency of traditional and low-cost carriers

frequent operations than their low-cost counterparts as they address primarily business travelers who attach higher values to the convenience of more frequent schedules.

In order to examine airline market structure and slot occupation, we gather data from four busiest airport's websites on a weekday (May 19, 2015) for flights' departures: Atlanta International Airport (ATL), Denver International Airport (DEN), Dallas/Fort Worth International Airport (DFW) and O'Hare International Airport (ORD).[5] Our dataset contains airport level observations on passenger air traffic: departure airport, service airline, and destination airport. We exclude all cargo flights, private jet charter and regional airlines. As for code sharing airlines, we record the operating airline and delete all other code sharing partner airlines. Figure 1 shows the portions of destinations served by traditional and low cost carriers separately that has frequency once, twice, three to above five times on a daily basis. As is clearly displayed, a high portion of low cost carriers destinations involve low frequency activities, while a small portion are served on a high frequent basis. On the contrary, major airlines offer more high frequent services.

Regarding airline level competition, modeling airline interaction in Cournot behav-

[5] The rank is according to ACI (Airports Council International North America) ranking in calender year 2013. LAX is ranked the second place, however since we focus on domestic destinations, while LAX has a high international passenger portion (53.8% of its total passenger number is international passenger), we consider it to be inproper for our purpose of study.

ior used to be the norm,[6] motivated by earlier empirical findings such as Oum et al (1993) and Brander and Zhang (1990). Quantity competition may be adequate when competing airlines offer homogeneous or horizontally differentiated products,[7] however when products are vertically differentiated, it can be well off the mark. Particularly, recent empirical studies by Fageda (2006) and Nazarenus (2011) suggest that price competition may be as pertinent for describing airline market as quantity competition. In light of this, this chapter analyses both traffic and price competition at airline level.

Closely related works include Silva and Verhoef (2013) and Brueckner and Flores-Fillol (2007). The current chapter is similar in its broad outlines to the framework used by the former, though we give explicit treatment for quality, which is absent in theirs. Our work has a flavor of the latter, but substantively distinct from their framework in the way we model quality, downstream passenger market and congestion cost. We find out that the two airlines oversupply flight frequeny under both types of competition, with the magnitude of overprovision being greater under price competition. More specifically, under price competition the high-quality airline does not internalize congestion delay at all, while its low-quality counterpart partially internalizes it. The findings of this work enables new insights into the competitve behavior of airline industry as well as congestion pricing charge, and may serve as a guidance on discussions of policy implications regarding traditional and budget airlines.

The remainder of the chapter is organized as follows. In Section 2 we introduce the model setting comprises of the airline side and the airport side. Section 3 is devoted to equilibrium outcome of airline competition in a laissez-faire context. Section 4 presents regulator's maximization problem. The extensions to monopoly airport is sketched in Section 5. And Section 6 presents concluding observations.

3.2 The model and notations

To keep the simplest possible focus on congestion internalization results, the model portrays a congested airport with a single origin–destination market served by a duopoly of airlines. This simple structural assumption allows to focus on most pertinent issues and achieve a clear exposition that brings out the line of the argument.[8] On the de-

[6] Theoretical papers assuming Cournot-type competition include, among others, Pels and Verhoef (2004), Zhang and Zhang (2006), Basso (2008), Brueckner (2005, 2009).

[7] A few works look into asymmetric airline competition. Brueckner (2009) assumes two airlines offer products at two fixed price levels and hence each faces perfect elastic demand. Basso and Zhang (2008) consider peak and off-peak slot as products of different qualities.

[8] Extensions to multiple independent destinations can be dealt with by adding a scale factor.

mand side, the construction of our framework is inspired by the vertical differentiated duopoly model initiated by Gabszewicz and Thisse (1979). Two airlines deliver different quality levels, as agreed by all consumers, and the revelant demand is generated from a unit mass of heteogenous consumers. In adopting this setting to the airline industry, we follow the convention in the literature to consider flight frequency as be the best proxy to service quality, on the grounds that higher frequency reduces passengers' waiting time for next flight.[9] Instead of incorporating peak and off-peak slots into quality consideration, we will consider a single congested period. Henceforth particular individual flights departure time are of no concern, the only pertinent element is frequency. Moreover, prices take the form of airfares.

Subscripts are employed to distinguish between airlines $\{l, h\}$, where l indicates low-quality airline, and h high-quality airline. Hence, flight frequencies are denoted as f_l and f_h, with $0 < f_l < f_h$. Passengers differ in their valuations of product quality, the preference parameter is described by $v \in [0, 1]$, v being uniformly distributed with unit density.

3.2.1 The demand side

On the demand side, we first analyze the downstream passenger market. The generalized fare of travelling with airline i takes the form:

$$\theta_i = p_i + D(f_l + f_h) + g(f_i). \tag{3.1}$$

The first term p_i is the ticket fare. $D(\cdot)$ represents passenger's cost of congestion delays experienced from flying valued in monetary terms, and is identical to all passengers. It is a function of the total flight activities and is invariant to seats. Finally, $g(f_i)$ denotes the schedule delay cost faced by a passenger who travels with airline i, which depends only on the flight frequency of this airline f_i. Schedule delay cost represents the monetary value of time between passenger's most desired departure time and the actual departure time scheduled by the airline, and therefore is related to the expected gap between passengers' actual and desired departure time, which depends solely on the frequency chosen by the airline i. It is thus natural that $g(f_i)$ is decreasing in f_i: the higher the frequency, the smaller the gap, and therefore the higher the generalized fare.

As is customary in literature (see Brueckner (2004), Basso (2008), Silva and Verhoef (2013) for an account), we make standard assumptions that:

[9]See among others, Douglas and Miller (1974), Brueckner (2004), Brueckner and Flores-Fillol (2007) for an account.

$$\frac{dD}{df_i} > 0, \frac{d^2D}{df_i^2} > 0, \frac{dg(f_i)}{df_i} < 0, \frac{d^2g(f_i)}{df_i^2} > 0, \forall i. \tag{3.2}$$

This assumption basically requires that: 1) congestion delay increases with traffic volume, and the severity is more pronounced when the congestion level is already high. Moreover, the marginal contribution of placing one more fight on all existing flights is the same for the two airlines; 2) schedule delay cost is associated with own frequency alone. This cost decreases with own frequency, and the marginal gain from cost saving is more significant when the existing frequency is placed at a low level. To ensure the existence of interior solutions, we further assume that the convexity of $g(f_i)$ is sufficiently large, whilst the convexity of D is sufficiently small.[10]

The demand addressed to an airline is defined by the set of passengers who maximize their utility when flying with this airline. Passengers differ in their valuations of airline quality. Each passenger flies at most once, and purchases a flight ticket that maximizes her utility $vf_i - \theta_i$, $i = \{l, h\}$, unless this yields negative utility in which case she purchases nothing. Potential passengers take into account the generalized fare instead of solely ticket price. If a passenger does not fly, her reservation utility is zero. It follows that the higher the quality of an airline, the higher the utility obtained by passengers for a given ticket price. Formally, a potential passenger has the following preferences:

$$U = \begin{cases} vf_l - \theta_l & \text{if she flies with quality } f_l \text{ at price } \theta_l \\ vf_h - \theta_h & \text{if she flies with quality } f_h \text{ at price } \theta_h \\ 0 & \text{if she does not fly} \end{cases} .$$

All passengers flying with one of the two airlines suffer from same congestion cost and schedule delay cost, though their gross utility differs, given their various tastes. We define the passenger who is indifferent between flying with the low-quality airline and the high-quality airline as the indifferent passenger, and the passenger who is indifferent between flying with the low-quality airline and not flying at all as the final passenger. From the utility function, the indifferent passenger is determined by $\bar{v}f_l - \theta_l = \bar{v}f_h - \theta_h$, implying that for this single passenger, the marginal gain in comfortness from switching from the low-qualtiy airline to the high-qualtiy airline is precisely offset by the fare increase generated from such a switch. Finally, the final passenger $\underline{v}$ is characterized by

[10] Corner solutions, for instance one of the airline becomes inactive, may arise if we don't impose assumptions on the convexities of the two cost structures. Should that happen, the market would be served by a monopoly airline.

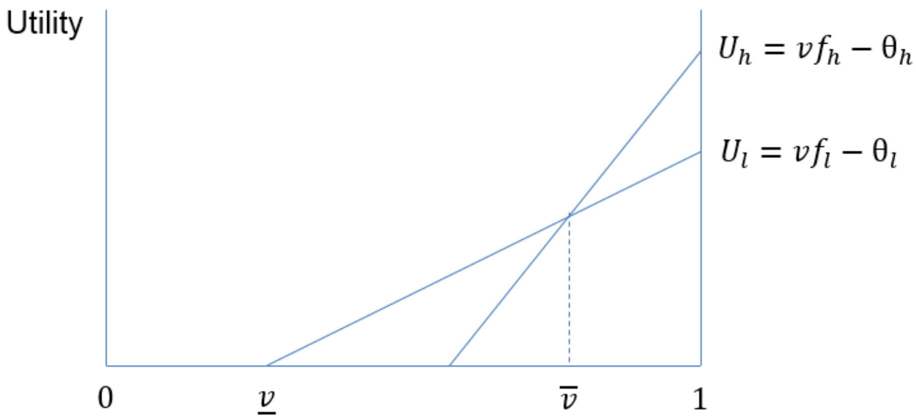

Figure 2: Passenger valuation of airline quality

$\underline{v} f_l - \theta_l = 0$. Hence the indifferent and final passenger has taste:

$$\bar{v} = \frac{\theta_h - \theta_l}{f_h - f_l}, \ \underline{v} = \frac{\theta_l}{f_l}. \tag{3.3}$$

Our specific formulation of the model refers to a market that is not fully covered. Hence with differentiated flights, the demand for flight h is $1 - \bar{v}$, while the demand for flight l is $\bar{v} - \underline{v}$. Moreover, there are $\underline{v}$ passengers that do not fly.[11] Equation (3.3) realistically alludes to $\theta_h > \theta_l$, which in turn implies $p_h + g(f_h) > p_l + g(f_l)$. Since $g(f_h) < g(f_l)$, it must hold that $p_h > p_l$. Airline h charges a ticket fare that is substantially higher than airline l, the ticket fare gap more than offsets the difference in schedule delay cost, as a result, the generalized price associated with airline h is greater than with airline l. Passengers valuation of airline quality is sketched out in Figure 2. Because v is uniformly distributed on $[0, 1]$ by assumption, the utility function gives rise to the demands that accompany differentiated airlines, denoted by $q_l(\theta_l, \theta_h)$ and $q_h(\theta_l, \theta_h)$ respectively:

$$\begin{cases} q_l(\theta_l, \theta_h) = \bar{v} - \underline{v} & = \dfrac{\theta_h - \theta_l}{f_h - f_l} - \dfrac{\theta_l}{f_l}, \\ q_h(\theta_l, \theta_h) = 1 - \bar{v} & = 1 - \dfrac{\theta_h - \theta_l}{f_h - f_l}. \end{cases} \tag{3.4}$$

Alternatively, the inverse passenger demands expressed as generalized fares can be

[11] Motta (1993) shows that Cournot competition can be studied only with partial market coverage, since the demand function can not be inverted with full market coverage.

derived:

$$\begin{cases} \theta_l &= f_l(1 - q_l - q_h), \\ \theta_h &= f_h - f_l q_l - f_h q_h. \end{cases} \tag{3.5}$$

Some clarifications should be made here. Here and throughout this chapter we will use the terms *traffic* and *passenger volume* (q_i) interchangeably, both refer to seats.

Combining (3.5) and (3.1) yield the inverse demand function associated with flight fare:

$$\begin{cases} p_l &= f_l(1 - q_l - q_h) - D(f_l + f_h) - g(f_l), \\ p_h &= f_h - f_h q_h - f_l q_l - D(f_l + f_h) - g(f_h). \end{cases} \tag{3.6}$$

3.2.2 The supply side

Having formulated the passenger behavior, we now analyze airline competition. First of all, assume the load factor to be identical across airlines, we allow airlines to choose aircraft size.[12] Because having idle seat capacity is not beneficial, it is natural that an airline will choose the aircraft size such that all seats are just filled. Airlines have three decision variables: traffic, aircraft size and frequency. Two of these are independent while the third is determined by the relation:

$$\boxed{\text{traffic} \equiv \text{aircraft size} \times \text{frequency.}}$$

More specifically, it should be noted that once aircraft size is simultaneously chosen by the duopolists at the second stage, it can not be modified afterwards, representing a form of commitment. Formulating in such way, choosing aircraft size and frequency would be equivalent to choosing traffic and frequency in Cournot game. We use τ to denote toll, and superscripts q, f to denote per-passenger and per-flight, respectively. Finally, given airport per-passenger and per-flight charges $\left(\tau_i^q, \tau_i^f\right), i \in \{l, h\}$, each airline chooses frequency and either traffic or ticket fare, depending on the nature of competition.

The last ingredient before constructing airline profit is cost structure. Follow Brueckner (2004) and Silva and Verhoef's (2013) lead, we make assumption that airlines' operating cost C_i is a function of aircraft size and frequency:

$$C_i = f_i \cdot (t_i^f + t_i^q \cdot \frac{q_i}{f_i}),$$

[12]Unlike Brueckner(2002), Pels and Verhoef (2004), Zhang and Zhang (2006), and Basso (2008), we do not assume "fixed proportions". Fixed proportion means the traffic per flight activity, i.e., product between load factor and aircraft size, is constant and identical among airlines. While in our model airlines choose aircraft size.

where t_i^q and t_i^f are per-passenger and per-flight operating cost, respectively. It follows that $\frac{q_i}{f_i}$ represents the aircraft size,[13] and the bracket term represents the per-flight operating cost. As is common with conventional wisdom, airline l has cost advantage over airline h, we thus normalize the operating costs of airline l to be zero for tractability, and employ t^q and t^f to airline h to reflect the cost differences. It is worth stressing that throughout this chapter congestion delay cost is borne by passengers alone. The argument is, as congestion delay cost is formulated as a monetary measure for passenger's discomfort from flight delay, the generalized fare that a passenger would accept for a given quality level is reduced. As a result, though this cost is born by consumers, the magnitude of its impact on airlines (via the generalized fare) and consumers are precisely identical. Suppose on the contrary, congestion cost is incorporated into airline cost function, this term would enter airline objective function in the same way as would passenger's congestion cost and hence would not bring new insights into the model.

With these specifications, the profit of airline l and h can now be constructed:

$$\begin{aligned} \pi_l &= q_l \cdot p_l - q_l \cdot \tau_l^q - f_l \cdot \tau_l^f, \\ \pi_h &= q_h \cdot p_h - q_h \cdot \left(\tau_h^q + t^q\right) - f_h \cdot \left(\tau_h^f + t^f\right). \end{aligned} \tag{3.7}$$

Airline's profit equals flight fare income gross of airport charge. We first address traffic competition then price competition.

The timeline of the model is as follows. In the first stage a duopoly of airlines compete in frequency and either quantities or airfares. In the second stage, consumers decide which airline to fly with, or not to fly at all. The equilibrium concept for this two-stage game is the subgame perfect equilibrium.

3.3 Equilibrium of airline duopoly market

Having introduced the basic model setting, this section investigates rivalry between the two airlines, each of which either simultaneously choose frequency and traffic (in Cournot competition), or frequency and ticket fare (in Bertrand competition) to maximize own profit.

[13] In this formulation of total cost, cost per flight is linear in the aircraft size (Swan and Adler (2006)).

3.3.1 Traffic competition

Under this market structure, airlines' strategic variables are traffic and frequency, and both airlines choose the two variables simultanously. Our specification of the relation *traffic* $\equiv$*aircraft size* $\times$ *frequency* displays an attribute of independency between strategic variables traffic and frequency, in the sense that when traffic is set, airlines can accommodate frequency choice by precisely choosing the proper aircraft size.

Reformulating (3.7) by using (3.1) and (3.5), together with the relation *traffic* $\equiv$*aircraft size* $\times$ *frequency*, we obtain airline profit as a function of frequency and traffic.

$$\pi_l = q_l \cdot \left(f_l(1-q_l-q_h) - D - g(f_l) - \tau_l^q\right) - f_l \cdot \tau_l^f, \tag{3.8}$$

$$\pi_h = q_h \cdot \left(f_h - f_l q_l - f_h q_h - D - g(f_h) - \tau_h^q - t^q\right) - f_h \cdot \left(\tau_h^f + t^f\right). \tag{3.9}$$

Equations (3.8) and (3.9) state that each airline's profit is affected by its own and its rival's frequencies. Intuitively, the rival's frequency choice influences its own profit via generalized fares, which comprises a congestion cost term borne equally by both airlines. With simultaneous choice, airline l chooses q_l and f_l to maximize (3.8), viewing q_h and f_h as parametric, and conversely for airline h. The first-order conditions are displayed as below.

$$\frac{\partial \pi_l}{\partial q_l} = -D - g(f_l) + f_l - q_h f_l - 2q_l f_l - \tau_l^q = 0, \tag{3.10a}$$

$$\frac{\partial \pi_h}{\partial q_h} = -D - g(f_h) + f_h - q_l f_l - 2q_h f_h - \tau_h^q - t^q = 0, \tag{3.10b}$$

$$\frac{\partial \pi_l}{\partial f_l} = \tau_l^f + q_l(D_l' + g_l' - (1-q_l-q_h)) = 0, \tag{3.10c}$$

$$\frac{\partial \pi_h}{\partial f_h} = \tau_h^f + q_h(D_h' + g_h' - (1-q_h)) = 0. \tag{3.10d}$$

The second-order conditions are satisfied by inspection. We will first discuss airlines traffic choice, followed by frequency choice, though the order of discussion does not imply a sequence of choices for strategic variables. The two airlines' best-response functions can be derived from first-order conditions.

$$q_l^*(q_h) = \frac{1}{2f_l}\left(\underbrace{-D - g(f_l) + f_l - \tau_l^q}_{>0}\right) - \frac{1}{2}q_h,$$

$$q_h^*(q_l) = \frac{1}{2f_h}\left(\underbrace{-D - g(f_h) + f_h - \tau_h^q - t^q}_{>0}\right) - \frac{1}{2}\cdot\frac{f_l}{f_h}\cdot q_l.$$

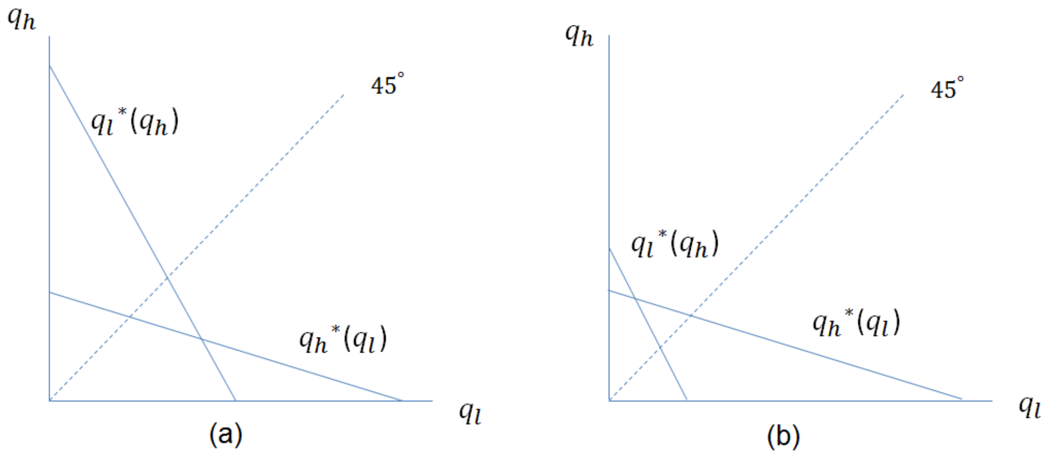

Figure 3: Quantity setting best-response functions

Both bracketed terms are positive by (3.6), and slopes of best-response functions are negative. The downward sloping best-response functions demonstrate that quantities are strategic substitutes. Recall that it has been assumed $f_l < f_h$, hence the slope of the low-quality airline's best-response function is steeper than the high-quality airline's, as $\frac{1}{2} > \frac{1}{2} \cdot \frac{f_l}{f_h}$. This means the low-quality airline responds more aggressively to any increase of traffic imposed by the high-quality airline, than the other way round. Having derived the slopes, yet the intercepts are less straightforward to see. Denote $\frac{1}{2f_l}\left(-D - g(f_l) + f_l - \tau_l^q\right) \equiv \Delta_l$ and $\frac{1}{2f_h}\left(-D - g(f_h) + f_h - \tau_h^q - t^q\right) \equiv \Delta_h$. Then the difference of Δ_l and Δ_h can be shown as:

$$\Delta_l - \Delta_h = \left(\frac{D}{2f_h} - \frac{D}{2f_l}\right) + \left(\frac{g(f_h)}{2f_h} - \frac{g(f_l)}{2f_l}\right) + \left(\frac{\tau_h^q + t^q}{2f_h} - \frac{\tau_l^q}{2f_l}\right).$$

Note that the first two terms are both negative. Keeping other things constant, the value of the third bracket is increasing in t^q. Recall that t^q is the per-passenger fee difference, when t^q is significant, it is more likely that $\Delta_l > \Delta_h$, a scenario corresponding to (a) in Figure 3. The intercepts of two best-response functions clearly indicate $q_l^* > q_h^*$ in equilibrium. On the contrary, when the cost difference is sufficiently small, $\Delta_l < \Delta_h$, a case depicted in (b) of Figure 3, in equilibrium $q_l^* < q_h^*$.

To derive the optimal flight fare, we rearrange first-order conditions and obtain:

$$p_l = \tau_l^q + f_l q_l, \tag{3.11}$$

$$p_h = \tau_h^q + t^q + f_h q_h. \tag{3.12}$$

The first RHS-term (first two) of (3.11) (resp., (3.12)) is airline's charge, whereas the second (resp., third) term is a pure markup that reflects market power effect.

Now we are interested in frequency setting. Again, we check that the second-order conditions are satisfied, which guarantees the existence of a unique and interior solution to the sub-game. After rearranging first-order conditions, we could derive the conditions under which the equilibrium frequencies should fulfill:

$$\tau_l^f = q_l(-D_l' - g_l' + \underbrace{(1 - q_l - q_h)}_{\geq 0}), \quad (3.13)$$

$$\tau_h^f + t^f = q_h(-D_h' - g_h' + \underbrace{(1 - q_h)}_{>0}). \quad (3.14)$$

At the airline optimum, the marginal frequency benefit, shown by the right-hand side of (3.13) and (3.14), is just offset by the airline's marginal frequency cost τ_l^f (or $\tau_h^f + t^f$ for airline h). The marginal frequency benefit comprises of marginal cost saving, $-D_i' - g_i'$; and marginal quality improvement benefit $1 - q_l - q_h$ (or $1 - q_h$ for airline h). Marginal cost saving gives the joint effects of an increase in congestion delay cost and a decrease in schedule delay cost. And the marginal quality improvement benefit shows the increment of monetary benefits of a flight to last passenger who is just willing to fly. Multiplying this benefit by own traffic yields the revenue increase. Clearly, congestion is only partially internalized. To see this, note that instead of multiplying by total traffic $q_l + q_h$, marginal frequency effects for the two airlines are multiplied by their own traffic. Hence in deciding frequencies, each airline only accounts for the congestion he induced to himself, a result in consistent with Brueckner (2002). More specifically, the internalization behavior is intrinsic and can be attributable to the exploitation of market power. The foregoing analysis can be summarized by the below proposition.

Proposition 1 *Under traffic competition, both airlines undersupply passenger volume; and internalizes only the congestion delay it imposes on itself.*

3.3.2 Comparative static analysis

We could now solve for equilibrium supply quantities taking τ_l^q and τ_h^q as given:

$$q_l = \frac{-2f_hD + f_lD + f_lf_h + f_lg(f_h) - 2f_hg(f_l) + f_l\tau_h^q - 2f_h\tau_l^q + f_lt^q}{f_l(4f_h - f_l)},$$

$$q_h = \frac{-D + 2f_h - f_l - 2g(f_h) + g(f_l) - 2\tau_h^q + \tau_l^q - 2t^q}{4f_h - f_l}.$$

The demand confronted by the two airlines depend directly on airport charge, congestion delay cost and schedule delay cost. We now dwell upon the question about how congestion cost and per-passenger cost difference would impact equilibrium quantities. The main results are presented below while the details are relegated to Appendix A.

$$\frac{\partial q_i}{\partial \tau_i^q} < 0, \frac{\partial q_i}{\partial \tau_j^q} > 0, \frac{\partial q_i}{\partial D} < 0, \frac{\partial q_l}{\partial t^q} > 0, \frac{\partial q_h}{\partial t^q} < 0.$$

These comparative static effects are displayed in the below lemma.

Lemma 1 *Under traffic competition, (i) an increase in the congestion delay cost reduces traffic; (ii) an increase in the per-passenger cost difference reduces airline h's seat, while increases airline l's traffic; (iii) each airline's congestion pricing reduces own quantity and increases the other's traffic.*

Recall that t^q stands for the per-passenger cost advantage airline l has over airline h, it is natural that the traffic supply of airline l increases with t^q, and conversely for airline h.

3.3.3 Assessment of market power in Cournot competition

Market power is defined as a firm's ability to charge and maintain a price that exceeds the marginal cost. A most immediate approach to measure market power focuses on the percentage markup of price over marginal cost as a fraction of price, as suggested by Lerner index. The Lerner index (L_i) signals the magnitude of price-setting discretion that steers away from zero profit marginal-cost pricing. It is an appropriate assessment for homogenous as well as differentiated product oligopolies. Discerning that direct application to vertically differentiated product market entail problems, we adapt Lerner index with special attention paid to treatments for congestion delay and schedule delay costs. These two terms enter demand function much as cost terms that affect actual price elasticity of demand, for which price is measured at generalized price level. Though theoretical relationship indicates that Lerner index corresponds to the inverse elasticity of demand faced by the firm, econometricians estimate firm's price elasticity of demand by constructing a full demand system for all interrelated products. As the attainable data is collected at actual demand elasticity level, we are interested in finding out how airline's price-cost margin is related to actual demand elasticity.

Let ε_l and ε_h denote the price elasticity of demand for airline l and h measured at ticket price respectively; and ε_l' and ε_h' measured at generalized price. It follows that

$$\begin{aligned} \varepsilon_l &= -\frac{\partial q_l}{\partial \theta_l} \cdot \frac{\partial \theta_l}{\partial p_l} \cdot \frac{p_l}{\theta_l} \cdot \frac{\theta_l}{q_l},\ \varepsilon_l' = -\frac{\partial q_l}{\partial \theta_l} \cdot \frac{\theta_l}{q_l}, \\ \varepsilon_h &= -\frac{\partial q_h}{\partial \theta_h} \cdot \frac{\partial \theta_h}{\partial p_h} \cdot \frac{p_h}{\theta_h} \cdot \frac{\theta_h}{q_h},\ \varepsilon_h' = -\frac{\partial q_h}{\partial \theta_h} \cdot \frac{\theta_h}{q_h}. \end{aligned} \tag{3.16}$$

Because (3.5) indicates $\frac{\partial \theta_l}{\partial p_l} = \frac{\partial \theta_h}{\partial p_h} = 1$, moreover our maintained assumption (3.3) indicates $\frac{p_l}{\theta_l} < 1$ and $\frac{p_h}{\theta_h} < 1$, we thus obtain $\varepsilon_l = \varepsilon_l' \cdot \frac{p_l}{\theta_l}$, $\varepsilon_h = \varepsilon_h' \cdot \frac{p_h}{\theta_h}$, which implies

$$\varepsilon_l < \varepsilon_l' \text{ and } \varepsilon_h < \varepsilon_h'.$$

The price elasticity of demand measured at ticket price level (ε_l and ε_h) is less elastic than the actual price elasticity of demand measured at the generalized price level (ε_l' and ε_h'). This says that consumers are less sensitive to a change in ticket price than to a change in generalized price. From a practical perspective, on the market level we typically observe ticket price elasticity, henceforth we argue that attainable data leads to an understatement of generalized price elasticity of demand.

Note also that upon some rearrangements of the first-order conditions with regard to traffic we obtain:

$$\begin{aligned} \frac{p_l - \tau_l^q}{p_l} &= \frac{f_l q_l}{p_l}, \\ \frac{p_h - \tau_h^q - t^q}{p_h} &= \frac{f_h q_h}{p_h}. \end{aligned} \tag{3.17}$$

The left-hand side term corresponds to price-cost margin in Lerner index. Because (3.5) indicates $\frac{\partial q_l}{\partial \theta_l} = \frac{\partial q_l}{\partial p_l} = -\frac{1}{f_h - f_l} - \frac{1}{f_l}$ and $\frac{\partial q_h}{\partial \theta_h} = \frac{\partial q_h}{\partial p_h} = -\frac{1}{f_h - f_l}$, we obtain:

$$\begin{aligned} \varepsilon_l &= \frac{f_h}{f_l\,(f_h - f_l)} \cdot \frac{p_l}{q_l}, \\ \varepsilon_h &= \frac{1}{f_h - f_l} \cdot \frac{p_h}{q_h}. \end{aligned}$$

Now we compare $\frac{1}{\varepsilon_l}$ and $\frac{1}{\varepsilon_h}$ to actual price-cost margins, as indicated on the right-hand side of (3.17). It is straightforward to see

$$\frac{p_l - \tau_l^q}{p_l} > \frac{1}{\varepsilon_l},\ \frac{p_h - \tau_h^q - t^q}{p_h} > \frac{1}{\varepsilon_h}.$$

The price-cost margin exceeds a conventional monopolist's markup.

3.3.4 Price competition

The precedent analysis has outlined the principles of congestion pricing in the context of seat competition. In this section we study airline market in the context of price competition. Under this market structure, airlines' strategic variables are airfare and frequency, and airlines choose the two variables simultanously. To start with, we reformulate airline's maximization problem as described in (3.7), and use equivalent formulation in terms of the levels of airfare and frequency. We check that second-order conditions are satisfied, and thereby we can solve for equilibrium from first-order conditions with respect to fares. According to (3.1) and (3.4), it can be derived that $\frac{\partial q_l}{\partial p_l} = -\frac{1}{f_l - f_h} - \frac{1}{f_l}$, $\frac{\partial q_h}{\partial p_h} = -\frac{1}{f_l - f_h}$, then the resulting equilibrium prices are

$$p_l = \frac{(f_h - f_l) f_l}{f_h} \cdot q_l + \tau_l^q, \tag{3.18}$$

$$p_h = (f_h - f_l) \cdot q_h + \tau_h^q + t^q. \tag{3.19}$$

The first RHS-terms of (3.18) and (3.19) are the positive markups which reflect market power effect, while the rest terms represent payments for per-passenger charge. The ability to maintain a markup over marginal cost is attributable to airlines market power in their differentiated products.

Finally, we dwell upon the frequency choice in price competition. Having checked that the second-order conditions are satisfied, the first-order conditions determine airline l and h's profit-maximizing choices of frequencies, f_l and f_h respectively, which fulfill:

$$\frac{\partial \pi_l}{\partial f_l} = 0 \Rightarrow \tau_l^f = -q_l \left(\left(1 - \frac{f_l}{f_h} \right) D_l' + g_l' - \frac{f_l(\theta_h - \theta_l)}{f_h(f_h - f_l)} - \frac{\theta_l (f_h - f_l)}{f_l f_h} \right), \tag{3.20}$$

$$\frac{\partial \pi_h}{\partial f_h} = 0 \Rightarrow \tau_h^f + t^f = -q_h \left(g_h' + \frac{\theta_h - \theta_l}{f_h - f_l} \right). \tag{3.21}$$

These conditions differ from traffic competition conditions (3.13) and (3.14) in two critical ways. First, the absence of congestion delay cost D_h' in (3.21) clearly states that airline h does not internalize the congestion incurred by its passengers. It should be stressed that the term *internalization* employed throughout this chapter refers specifically to airline's direct response to the change in congestion cost. Admittedly, when choosing optimal f_h, airline h takes into account the impact of its choice on airline l's choice of f_l, which in turn influences f_h. Nevertheless, such strategic interaction affects the optimal f_h indirectly and should be distinguished from the concept of internalization we adopt here. In particular, it is typical in the literature to apply the notion of internalization to account for frequency choice that is directly affected by congestion cost.[14]

Our finding seems somewhat surprising and runs counter to many existing models. To understand these results it is important to recall the nature of market stage where vertically differentiated airlines are involved. It is worth noting that by model specification, all passengers regardless of preference types, suffer from identical congestion cost D. In airline h's passenger market, the highest-valuation passenger has a fixed taste parameter normalized to 1 due to model assumption. Whereas the lowest-valuation passenger of airline h incurs identical congestion cost invarient to the flight he takes, as indicated by (3.1) and (3.4). Airline h hence has no incentive to internalize congestion cost as doing so would not attract additional passengers. On the other hand, it internalizes schedule delay cost, as this cost has direct impact on its derived demand.

The second difference is associated with the magnitude of partial internalization by airline l in the two market structures. An inspection of the coefficients associated with D_l' in (3.20) and (3.13) reveals that airline l internalizes more congestion delays under traffic competition relative to price competition, since $1 > 1 - \frac{f_l}{f_h} > 0$.

A summary of the above analysis leads to the below proposition.

Proposition 2 *1) Under price competition, both airlines charge markup for flight fare;*

2) the high-quality airline does not internalize congestion delay, whilst the low-quality airline partially internalizes it;

3) both airlines internalize more congestion delays under traffic competition.

3.3.5 Assessment of market power in price competition

In the analogous way as for traffic competition, we rewrite (3.18) and (3.19) to obtain price-cost margin:

$$\begin{aligned} \frac{p_l - \tau_l^q}{p_l} &= \frac{(f_h - f_l)f_l}{f_h} \cdot \frac{q_l}{p_l}, \\ \frac{p_h - \tau_h^q - t^q}{p_h} &= (f_h - f_l) \cdot \frac{q_h}{p_h}. \end{aligned} \tag{3.22}$$

A comparison of (3.22) and (3.17) suggests that for both airlines, the price-cost margins are higher under traffic competition than under price competition.

Lemma 2 *Both airlines charge higher markups on flight fare under traffic competition.*

[14]See Pels and Verhoef (2004), Brueckner and Verhoef (2010).

3.4 Regulator's maximization problem

In the preceding analysis, we investigated airline competition and the resulting pricing and frequency setting behavior taking airport charges as parametric. From now on, we examine a public airport who acts as a regulator, and pursuits first-best optimum by dictating airline choices of both frequency and seat. Assuming that the airlines view the tolls as parametric, as discussed in the introduction. The outcome is a first-best optimum and will serve as a benchmark.

We shall confine our attention to the case of fixed airport capacity and henceforth focus on short-term equilibrium. The reason for studying short-term equilibrium stems from real world practice, simply put, that expanding capacity often takes up long time horizon. Therefore, we contend that focusing on short-term equilibrium is more imperative and offers greater realism.

3.4.1 The first-best optimum

It is useful to begin by considering a base case where a public airport whose mandate is to maximize social welfare, and has perfect seat regulation on the airline markets. The maximization problem is formally characterized as

$$\max_{(q_l, q_h, f_l, f_h)} SW$$

As is customary in literature, welfare is the sum of net benefits for all agents: consumer surplus, airlines' profits and airport's profit.

$$SW = CS + \pi_l + \pi_h + \Pi. \tag{3.23}$$

Consumer surplus can be formulated as:

$$CS = \int_{\underline{v}}^{\bar{v}} v f_l dv + \int_{\bar{v}}^{1} v f_h dv - q_l \theta_l - q_h \theta_h.$$

Recall that $\underline{v}$ respresents the final passenger, and $\bar{v}$ the indifferent passenger, whose definitions are provided in (3.3).

Substitute the expressions of $\underline{v}$, $\bar{v}$, θ_h and θ_l according to (3.3) and (3.5) into the consumer surplus equation yields:

$$CS = \frac{1}{2}\left(f_h q_h^2 + 2 f_l q_l q_h + f_l q_l^2\right). \tag{3.24}$$

In addition, airlines profits are already specified in (3.7). Airport profit, denoted as Π, comprises of total charge income net of operating costs. Assume airport incurs constant per-passenger marginal operating cost which is denoted by c^q, as well as constant per-flight marginal operating cost c^f, with both c^q and c^f being invariant to airlines. Such assumption is backed by the estimation studies of cost functions conducted by Morrison (1983) and Pels et al (2003), who find out that airport runways display constant return to scale. Therefore Π can be expressed as:

$$\Pi = q_l(\tau_l^q - c^q) + q_h(\tau_h^q - c^q) + f_l(\tau_l^f - c^f) + f_h(\tau_h^f - c^f). \tag{3.25}$$

The welfare expression and first-order conditions are presented in the Appendix B. Solving explicitly for q_l and q_h from first-order conditions, we get a social planner's choice of quantities:

$$q_l^{fb} = \frac{-f_h g(f_l) + f_l g(f_h) + f_l t^q}{f_l(f_h - f_l)} - \frac{c^q}{f_l} - \frac{D}{f_l}, \tag{3.26}$$

$$q_h^{fb} = 1 + \frac{g(f_h) - g(f_l) + t^q}{f_l - f_h}. \tag{3.27}$$

The airport operating cost and congestion delay cost affect only airline l's traffic supply. The assignment of flight consumers between two airline, however, is irrelavant to these two costs, rather, it is affected by the cost difference. To guarantee interior solutions $1 > q_l^{fb}, q_h^{fb} > 0$, a necessary condition must hold:

$$\bar{t} > t^q > \underline{t}, \tag{3.28}$$

where $\bar{t} \equiv f_h - f_l - g(f_h) + g(f_l)$ and $\underline{t} \equiv \frac{(f_h - f_l)c^q + (f_h - f_l)D + f_h g(f_l) - f_l g(f_h)}{f_l}$. The literal interpretation is two folds. For one thing, cost gap between two airlines should be sufficiently large, otherwise the regulator would optimally drive out the low-quality airline and keep only the high-quality airline in the market. For the other, if on the contrary the cost gap is excessively large, then the low-quality airline enjoys a significant cost advantage and wins the whole market. A comparison of q_l^{fb} and q_h^{fb} can thus be established. It could be verified that:

$$q_l^{fb} > q_h^{fb} \text{ when } \bar{t} > t^q > \widehat{t},$$
$$q_l^{fb} < q_h^{fb} \text{ when } \widehat{t} > t^q > \underline{t},$$

where $\widehat{t} \equiv \frac{(f_h - f_l)D + f_h f_l - f_l^2 - 2f_l g(f_h) + f_h g(f_l) + f_l g(f_l) + (f_h - f_l)c^q}{2f_l}$. When cost gap is not substantial, it is socially optimal for the high-quality airline to serve more passengers than its low-quality counterpart, and vice versa. It is straightforward to see $\frac{\partial \underline{t}}{\partial c^q} = \frac{f_h - f_l}{f_l}$, $\frac{\partial \widehat{t}}{\partial c^q} = \frac{f_h - f_l}{2f_l}$, hence $\frac{\partial \underline{t}}{\partial c^q} > \frac{\partial \widehat{t}}{\partial c^q} > 0$. More interestingly, $\bar{t}$ is irrelevant to c^q.

Now we make an attempt to relate a change in c^q to the optimal quantities. With an increase of c^q, the feasible range of t^q to ensure internal solutions shrinks, i.e., the lower bound of $\underline{t}'$ is higher than before: $\underline{t}' > \underline{t}$. In turn, the two airlines' optimal seat curves intersect at a critical point $\widehat{t}'$ that moves along to the right of the previous one $\widehat{t} : \widehat{t}' > \widehat{t}$, though the increase in critical point is smaller in magnitude relative to the increase in lower bound. Therefore, in order to accomodate an increase in airport operating cost, the lower bound of cost difference should increase. Moreover, when that cost difference is given, a larger airport operating cost makes it more likely that the low-quality airline should serve more passengers, from a social optimal point of view. The argument is, provided that operating cost is already high, yet the social planner aims to reach both types of consumers, in making price-quality combination affordable to the low type consumers, the benefit of being cost-efficiency appears prominent, and thus the low-quality airline attracts more passengers.

In addition, the total passenger volume is:

$$q_l^{fb} + q_h^{fb} = \frac{f_l - g(f_l) - D - c^q}{f_l}.$$

The operating cost and congestion delay cost are inversely related to total frequency. While airline h 's schedule delay cost doesn't have an impact since this cost only plays a role in allocation of traffic. As for frequency choice, the socially optimal frequencies should fulfill:

$$\begin{aligned} \frac{\partial SW}{\partial f_l} &= 0 \Rightarrow c^f = -(q_h + q_l) \cdot D_l' - q_l g_l' + q_l \left(1 - \frac{1}{2} q_l - q_h\right), \\ \frac{\partial SW}{\partial f_h} &= 0 \Rightarrow c^f + t^f = -(q_h + q_l) \cdot D_h' - q_h g_h' + q_h \left(1 - \frac{1}{2} q_h\right). \end{aligned} \tag{3.29}$$

The socially optimal frequency is determined by taking into account the congestion delay costs imposed on two airlines together. Recall that with laissez-faire competition, airlines either don't internalize, or internalize only the congestion they impose on themselves. For that reason, both airlines should be charged for the part of congestion they do not internalize.

3.4.2 Congestion pricing

The problem with laissez-faire is that it steers outcome away from first-best. In order to achieve an efficient outcome regarding seat (or price) and frequency, the regulator charges a Pigouvian toll to bring the market to first-best outcome. Pigouvian toll should be imposed in the first stage and charges each airline for the congestion damage that it does not internalize. Since there is market power effect, which should be treated with a subsidy when considered in isolation, the resulting optimal toll might turn out to be negative. We will derive and interpret the congestion pricing in the ensuing analysis, for the cases of traffic and price competition separately.

Quantity competition at airline market

In the Cournot competition model, the ticket price is established to clear the market after traffic supply is chosen. Indeed, airport charges $(\tau_l^q, \tau_h^q, \tau_l^f, \tau_h^f)$ are determined in order to induce the optimal outcome in later stages. In particular, τ_l^q should hold for (**??**), whereas q_l should satisfy both (**??**) and (3.45). Hence, substitute (**??**) and (3.10b) into (3.45) and rearrange terms yields the per-passenger congestion pricing:

$$\begin{aligned} \tau_l^{q(fb)} - c^q &= -f_l q_l < 0, \\ \tau_h^{q(fb)} - c^q &= -f_h q_h < 0, \end{aligned} \tag{3.30}$$

where superscript fb denotes for first-best outcome. The first-best net per-passenger toll takes the form of subsidy if the right-hand side of the equation is negative, and charge if positive. In this case, it is clear that first-best outcome is attainable via a subsidization which corrects for market power markup. According to (3.30), the subsidization $f_l q_l < f_h q_h$ when $\frac{q_l}{q_h} < \frac{f_l}{f_h} < 1$; whilst $f_l q_l \geq f_h q_h$ when $\frac{q_l}{q_h} \geq \frac{f_l}{f_h} > 1$. That is, the degree of subsidization depends on relative supply seat.

Similiarly, regarding per-flight charge, we substitute (3.13) and (3.14) into (3.29) and obtain:

$$\begin{aligned} \tau_l^{f(fb)} - c^f &= q_h D_l' - \frac{1}{2} q_l^2, \\ \tau_h^{f(fb)} - c^f &= q_l D_h' - \frac{1}{2} q_h^2. \end{aligned} \tag{3.31}$$

First-best per-flight toll net of operating cost has two components: uninternalized congestion delay cost and a term related to own passenger volume. The sign of the right-hand side terms determines first-best congestion pricing should take the form of a toll or a subsidization. In this case it can be either of two. Since $D_l' = D_h'$, for ease of comparison let us denote $D' \equiv D_l' = D_h'$. When marginal congestion delay cost is small, for instance a polar case $D' = 0$, the right-hand side of equation is negative, which implies that laissez-faire mechanism unambiguously leads to undersupplies of frequency from the social perspective.

Now we attempt to compare $\tau_l^{f(fb)}$ and $\tau_h^{f(fb)}$.

$$\tau_l^{f(fb)} - \tau_h^{f(fb)} = \left(D' + \frac{1}{2}(q_h + q_l)\right)(q_h - q_l).$$

The first bracket is clearly positive, the second bracket is contingent on the comparison of q_h and q_l. If $q_h > q_l$, then $\tau_l^{f(fb)} > \tau_h^{f(fb)}$; whilst if $q_h < q_l$, then $\tau_l^{f(fb)} < \tau_h^{f(fb)}$.

Proposition 3 *Under seat competition, in order to reach first-best outcome, social planner subsidizes airlines for passenger, and may subsidize or charge airlines for flight frequency.*

Having derived congestion pricing, we now proceed to compare airlines profits in the first-best outcome. The proof is presented in Appendix B.

Price competition at airline market

According to the expressions of p_l and p_h given by (3.6), conditions for flight fares equilibrium can be reformulated as

$$f_l(1 - q_l - q_h) - D - g(f_l) - \tau_l^q - \frac{(f_h - f_l)f_l}{f_h} \cdot q_l = 0,$$
$$f_h - f_h q_h - f_l q_l - D - g(f_h) - \tau_h^q - (f_h - f_l) \cdot q_h = 0.$$

Substitute the above two equations into (3.45) generates the first-best per-passenger toll for price competition:

$$\begin{aligned} \tau_l^{q(fb)} - c^q &= -\frac{f_l(f_h - f_l)}{f_h} q_l < 0, \\ \tau_h^{q(fb)} - c^q &= -(f_h - f_l) q_h < 0. \end{aligned} \tag{3.32}$$

Analogously, we could compute first-best per-flight toll from (3.20) and (3.21), as

$$\tau_l^{f(fb)} = c^f + D_l' \cdot \left(q_h + \frac{f_l}{f_h} q_l\right) + q_l^2\left(\frac{f_l}{f_h} - \frac{1}{2}\right), \tag{3.33}$$

$$\tau_h^{f(fb)} = c^f + D_h' \cdot (q_h + q_l) + q_h\left(f_h - f_l - \frac{1}{2} q_h\right). \tag{3.34}$$

The first term of RHS of both equations are clearly positive, though the signs of second terms remain ambiguous. The low-quality airline partially internalizes congestion cost, hence the first-best congestion toll imposed on him is scaled down by a factor smaller than one. In addition, the high-quality airline essentially does not internalizes congestion cost at all, and hence should pay for the full congestion delay cost. Note also the sign of the second term of the RHS depends on the difference of f_h and f_l. The first-best congestion toll for airline l decreases with the frequency difference, and conversely for airline h increases with frequency difference.

From what has been shown above, it is explicit that for both market structures, a public airport subsidizes passengers, otherwise airlines would undersupply. As for frequency, the first-best charge or subsidization is less clear-cut. Prior analysis is summarized in the proposition below.

Proposition 4 *1) First best per-passenger subsidies are smaller for both airlines in price competition than in seat competition;*

2) first best per-flight charges imposed on the high-quality airline decreases with the quality difference, whilst on the low-quality airline increases with the quality difference.

3.5 Monopoly airline

An extension to monopoly airline is sketched out in this section. With the optimal congestion pricing in duopolistic airlines understood, attention now shifts to an alternate case where the origin–destination market is served by a nondiscriminating monopoly airline. This monopoly can not charge a different fare to each passenger, rather it is restricted to charge a uniform price to all passengers. The main issue at stake here is whether a monopoly airline would provide the same quality that would be available by a social planner.

The primitive model setting from prior analysis maintains here, except for the absense of interplay between airlines. In this respect, we look into a low quality airline acting as a monopoly, though the analysis and results continue to hold in the case of a high quality airline. Since the monopoly airline charges a single price, the previous notions of indifferent passenger and final passenger coincides. Deriving from utility function, the final passenger $\underline{v}'$ is now determined by the equation $\underline{v}' f_m - \theta_m = 0$, hence in this setting, the final passenger is specified as:

$$\underline{v}' = \frac{\theta_m}{f_m}. \tag{3.35}$$

The utility function generates the demand for the monopolistic airline, denoted as $q_m(\theta_m)$:

$$q_m(\theta_m) = 1 - \frac{\theta_m}{f_m}.$$

Alternatively, inverse demand functions in terms of generalized fare can be expressed as $\theta_m = f_m (1 - q_m)$, and the airfare can be written as

$$p_m = f_m (1 - q_m) - D(f_m) - g(f_m). \tag{3.36}$$

The monopoly airline chooses traffic and frequency to maximize profit. Denote its profit as π_m, upon checking that second-order conditions are fulfilled,[15] the equilibrium traffic and frequency can be characterized by the first-order conditions:

$$\begin{aligned} p_m &= \tau_m^q + f_m q_m, \\ \tau_m^f &= -q_m \left(D'_m + g'_m - (1 - q_m) \right). \end{aligned} \tag{3.37}$$

It is hardly surprising to see the monopoly airline sets a markup on flight fare. For a given quality, the social welfare is maximized with respect to seat when the flight fare is equal to marginal per-passenger cost. Airline profits, however, are maximized when there is a price markup. The failure on traffic is familiar which is attributable to monopoly's exploitation of market power over price.

3.5.1 Congestion toll

We now consider the choices of an social planner who has the power to dictate airline make choice on flight frequency along with passenger volume for the airport. Analogous to previous analysis, the planner aims to maximize social welfare. Given the market is served by a single airline, the passenger surplus expression now simplifies to:

$$CS = \int_{\underline{v}'}^{1} v f_m dv - q_m \theta_m,$$

where $\underline{v}'$ is equivalent to $1 - q_m$. Consumer surplus can be rewritten by plugging in equations (3.35) and (3.46):

$$CS = \frac{f_m q_m^2}{2}.$$

Airport profit comprises tolls collected from the monopoly airline alone, which takes the form:

$$\Pi = q_m(\tau_m^q - c^q) + f_m \left(\tau_m^f - c^f \right).$$

[15] see Appendix C.

Social welfare is the sum of passenger surplus and airport profit, see Appendix C. The equilibrium seat supply of a monopoly airline could be derived from first-order condition with regard to seat:

$$q_m^{fb} = \frac{-D\left(f_m\right) - g(f_m) + f_m - c^q}{f_m}. \tag{3.38}$$

In a similiar fashion as in the duopoly case, first-best congestion toll under the monopoly case is derived by substituting equation (3.37) into the above first-order conditions:

$$\tau_m^q - c^q \;=\; -q_m f_m < 0, \tag{3.39}$$

$$\tau_m^f - c^f \;=\; -\frac{q_m^2}{2} < 0. \tag{3.40}$$

To generate social optimum outcome, the social planner should subsidize the monopolistic airline in both per-passenger and per-flight charges. This argument translates into the contention that an unregulated monopoly undersupplies both frequency and passenger volume relative to the social optimum. The underprovision of passenger is more familiar, while the undersupply of frequency is less straightforward. The result departs markedly from conventional literature on congestion pricing, which states that a monopolistic airline would internalize congestion it imposes on itself, hence there is no scope for congestion pricing.[16] An alternative explanation for the distortion of frequency is illuminated by the argument associated with the inability of uniform monopoly price to convey information about marginal passenger's valuation of frequency $\frac{\partial \theta_m}{\partial f_m}$.[17] Note that with our model specifications, for given q_m, the marginal valuation of frequency falls when one more passenger is attracted to fly: $\frac{\partial^2 \theta_m}{\partial q_m \partial f_m} < 0$. As a result, the airline sets frequency too low for given q_m.

We turn now to the comparison of traffic and frequency supply under regimes of monopoly and duopoly. Even though the sign of (3.31) is ambiguous, a comparison is still valid. A closer inspection of (3.40) in reference to the regime of duopoly (3.31)[18] indicates that the first term of (3.31), which is positive, doesn't appear in (3.40), clearly suggests the congestion toll is unambiguously smaller under monopoly regime. The intuition is as follows. Given that frequency represents quality, for duopoly airlines frequency choice has the direct effects on demand in the sense that an increasing in frequency precludes some low valuation passengers to travel with this airline.

[16] See Brueckner (2002).

[17] See Spence (1975) for a more complete discussion.

[18] A comparison with Pigouvian tax for price competition is analogous, and the primary result generated from quantity competition case holds for price competition case as well.

Moreover, several indirect consequences include: 1) increase own congestion cost, which is negatively related to own demand; 2) reduce own schedule delay cost which obviously has a positive impact on demand; 3) increase rivalry's congestion cost, which in turn has a positive effect on its own demand. The last point is relevant only to duopoly airlines. In this respect, duopoly airlines have extra incentives to place higher frequency relative to a monopoly, for the purpose of increasing the rival's congestion cost.

The characterization of monopoly airlines' supply behavior along with the partial effects are highlighted in the below proposition.

Proposition 5 *The monopoly airline undersupplies both traffic and frequency relative to the social optimum. In order to reach social optimum, airport should subsidize both.*

3.5.2 Comparison with duopolistic airlines

Though we could not compute equilibrium seat and quality for the monopoly and duopoly airlines without proposing structural forms, it is still feasible to establish comparison between the two market structures. We are interested in the questions such as, if the monopolistic airline has a fixed quality, either f_l or f_h, would he supply more than the duopolistic airlines with same quality level? Or the two combined? The ensuing analysis is devoted to answer these questions.

Note that the first-best supply seat evaluated at quality level l and h, denoted as $q_m^{fb}(f_l)$ and $q_m^{fb}(f_h)$ respectively, are characterized as:

$$\begin{aligned} q_m^{fb}(f_l) &= \frac{-D(f_l) - g(f_l) + f_l - c^q}{f_l}, \\ q_m^{fb}(f_h) &= \frac{-D(f_h) - g(f_h) + f_h - c^q}{f_h}. \end{aligned} \tag{3.41}$$

It can be verified that

$$q_m^{fb}(f_l) > q_l^{fb},$$

where q_l^{fb} is given by (3.26). The proof is relegated to Appendix D. Analogously we verify that $q_m^{fb}(f_h)$ is greater than the first-best supply seat of the high-quality airline, q_h^{fb} :

$$q_m^{fb}(f_h) > q_h^{fb}.$$

We then proceed to compare the aggregate market seat. Aggregate seat in the duopoly market, denoted as q^{fb}, is attained by summing up (3.26) and (3.27):

$$q^{fb} = \frac{f_l - D(f_l + f_h) - g(f_l) + c^q}{f_l}.$$

From inspection of the above expression it is obvious that the first-best supply seat is irrelavant to cost difference. We could show that:

$$\min\left(q_m^{fb}\left(f_l\right), q_m^{fb}\left(f_h\right)\right) > q^{fb}.$$

The analysis can be summarized in the below Lemma.

Lemma 3 *If the monopoly airline could take either low or high quality that are chosen by the duopoly airlines, then he serves more passengers than the duopoly airlines combined, regardless of frequency.*

3.6 Conclusion

A description of congestion pricing applied to price-quality differentiated airlines requires an understanding. In the airline industry, a proxy for quality characteristic is flight frequency. The present chapter is devoted to investigate the endogenous relation between airline choice of frequency and airfare. What we attempt to show is that when adopting first-best toll, the price-quality relationship should be taken into consideration.

This chapter presents a model of vertically differentiated airlines engaging in frequency and traffic competition. The analysis provides useful comparative-static predictions, and formally shows that under both quantity and price competition, airlines undersupply passenger volumes relative to first-best outcome. More importantly, we argue that first-best per-passenger congestion charge, which takes the form of subsidies in our context, are smaller for airlines in price competition than in quantity competition. The low-quality airline partially internalizes congestion delay cost, while his high-quality counterpart does not internalize congestion cost. An extension to monopolistic airline suggests that the monopolistic airline would undersupply flight frequency as well as passenger volume.

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Appendix

Appendix A

Traffic competition

Check for second-order condition:

$$\frac{\partial^2 \pi_l}{\partial q_l^2} = -2f_l < 0, \frac{\partial^2 \pi_h}{\partial q_h^2} = -2f_h < 0.$$

It can be easily verified that the second-order conditions with regard to frequency is negative:

$$\frac{\partial^2 \pi_i}{\partial f_i^2} = q_i \left(-D_i'' - g_i''\right) < 0.$$

Comparative statics

$$\begin{aligned}
\frac{\partial q_l}{\partial \tau_l^q} &= -\frac{2f_h}{f_l\left(4f_h - f_l\right)} < 0, \frac{\partial q_l}{\partial \tau_h^q} = \frac{1}{4f_h - f_l} > 0, \frac{\partial q_l}{\partial t^q} = \frac{1}{4f_h - f_l} > 0, \\
\frac{\partial q_h}{\partial \tau_h^q} &= -\frac{2}{4f_h - f_l} < 0, \frac{\partial q_h}{\partial \tau_l^q} = \frac{1}{4f_h - f_l} > 0, \frac{\partial q_h}{\partial t^q} = -\frac{2}{4f_h - f_l} < 0.
\end{aligned}$$

Price competition

The corresponding first order conditions for a maximum in price are:

$$\begin{aligned}
\frac{\partial \pi_l}{\partial p_l} &= q_l + \frac{\partial q_l}{\partial p_l}(p_l - \tau_l^q) = 0, \\
\frac{\partial \pi_h}{\partial p_h} &= q_h + \frac{\partial q_h}{\partial p_h}(p_h - \tau_h^q - t^q) = 0.
\end{aligned} \tag{3.43}$$

Appendix B. First-best optimum

Analysis of first-best optimum

Since airport's charge income cancels out against airlines' charge payment, plus airline's ticket income cancels out against passenger's ticket payment, social welfare is constructed as:

$$\begin{aligned}
SW(q_l, q_h, f_l, f_h) = \frac{1}{2}\left(f_h q_h^2 + 2f_l q_l q_h + f_l q_l^2\right) + q_l(-D - g_l(f_l) + f_l - f_l q_h - f_l q_l - c^q) \\
+ q_h(-D - g_h(f_h) + f_h - f_h q_h - f_l q_l - c^q - t^q) \\
- f_l c^f - f_h\left(c^f + t^f\right).
\end{aligned} \tag{3.44}$$

First-order derivatives associated with quantities read:

$$\frac{\partial SW}{\partial q_l} = -D - g\left(f_l\right) + f_l - q_h f_l - q_l f_l - c^q = 0, \tag{3.45}$$

$$\frac{\partial SW}{\partial q_h} = -D - g\left(f_h\right) + f_h - q_h f_h - q_l f_l - c^q - t^q = 0.$$

Analysis of profit comparison

By substituting (3.11), (3.12), (3.30) and (3.31) into (**??**), we obtain:

$$\pi_l = q_l \cdot f_l q_l - f_l \left(c^f + q_h D' - \frac{1}{2} q_l^2\right),$$

$$\pi_h = q_h \cdot f_h q_h - f_h \left(c^f + q_l D' - \frac{1}{2} q_h^2 + t^f\right).$$

In order to have interior solutions: $\pi_l, \pi_h > 0$, it should hold that $t_f < \frac{3q_h^2 - 2q_l D' - 2c^f}{2}$; $c^f < \frac{3q_l^2 - 2q_h D'}{2}$ for $q_h > q_l$ and $c^f \leq \frac{3q_h^2 - 2q_l D'}{2}$ for $q_h \leq q_l$. In other words, both per-flight cost difference and airport operating cost should be sufficiently small.

$$\pi_l - \pi_h = \frac{3f_l q_l^2 - 3f_h q_h^2}{2} + c^f\left(f_h - f_l\right) + \left(f_h q_l - f_l q_h\right) D' + f_h t^f.$$

A direct comparison of π_l and π_h is complex. For ease of exposition, we discuss two cases where $q_l > q_h$ and $q_l \leq q_h$. Imposing condition 3.28 to ensure interior solution, together with the conditions for $\pi_l > 0$ and $\pi_h > 0$, it can be verified that:

- if $q_h > q_l : \pi_l > \pi_h$ when $\frac{-2f_h c_f + 2f_l c_f - 2D' f_h q_l + 2D' f_l q_h + 3f_h q_h^2 - 3f_l q_l^2}{2f_h} < t_f < \frac{3q_h^2 - 2q_l D' - 2c^f}{2}$, and $\pi_l \leq \pi_h$ when $0 < t_f \leq \frac{-2f_h c_f + 2f_l c_f - 2D' f_h q_l + 2D' f_l q_h + 3f_h q_h^2 - 3f_l q_l^2}{2f_h}$.
- if $q_l \leq q_h : \pi_l > \pi_h$ when $f_h \leq \widehat{f_h}$ and $0 < t_f < \frac{1}{2}\left(3q_h^2 - 2c_f - 2q_l D'\right)$; or $f_h > \widehat{f_h}$ and $\widehat{t_f} < t_f < \frac{1}{2}\left(3q_h^2 - 2c_f - 2q_l D'\right)$; while $\pi_l \leq \pi_h$ when $f_h > \widehat{f_h}$ and $0 < t_f < \widehat{t_f}$, where $\widehat{t_f} \equiv \frac{-2f_h c_f + 2f_l c_f - 2f_h q_l D' + 2f_l q_h D' + 3f_h q_h^2 - 3f_l q_l^2}{2f_h}$, $\widehat{f_h} \equiv \frac{2f_l c_f + 2f_l q_h D' - 3f_l q_l^2}{2c_f + 2q_l D' - 3q_h^2}$.

Analysis C. Monopoly airline

Equilibrium quantity and quality chose by the airline

Assume the monopoly airline incurs zero operating cost, hence his profit is:

$$\pi_m = \left(f_m\left(1 - q_m\right) - D(f_m) - g(f_m) - \tau_m^q\right) q_m - f_m \tau_m^f. \tag{3.46}$$

First-order condition with regard to q_m yields:

$$\frac{\partial \pi_m}{\partial q_m} = f_m\left(1 - q_m\right) - D(f_m) - g(f_m) - \tau_m^q - f_m q_m = 0. \tag{3.47}$$

According to (3.36), we could write

$$p_m = \tau_m^q + f_m q_m. \tag{3.48}$$

Checking for second-order derivatives:

$$\frac{\partial^2 \pi_m}{\partial q_m^2} = -2f_m < 0,$$
$$\frac{\partial^2 \pi_m}{\partial f_m^2} = -q_m \left(D''_m + g''_m\right) < 0.$$

Hence there exists a unique and interior solution to the sub-game. The equilibrium condition for frequency can be derived analogously.

Congestion pricing for the monopoly airline

Using $p_m = f_m (1 - q_m) - D(f_m) - g(f_m)$ and rearrange, social welfare can be written as:

$$SW = q_m \left(-D(f_m) - g(f_m) + f_m - f_m q_m - c^q\right) - f_m c^f + \frac{f_m q_m^2}{2}.$$

First-order conditions with respect to quantity and frequency read:

$$\frac{\partial SW}{\partial q_m} = -D(f_m) - g(f_m) + f_m - q_m f_m - c^q = 0,$$
$$\frac{\partial SW}{\partial f_m} = -q_m \left(D'_m + g'_m + \frac{1}{2} q_m - 1\right) - c^f = 0.$$

Appendix D. Comparison of monopoly and duopoly

$$q_m^{fb}(f_l) - q_l^{fb} = \frac{D(f_l + f_h) - D(f_l)}{f_l} - \frac{g(f_l)}{f_l} - \frac{g(f_h)}{f_h - f_l} + \frac{f_h g(f_l)}{f_l (f_h - f_l)} + 1 - \frac{t^q}{f_h - f_l}.$$

Upon some calculations, this can be rewritten as:

$$q_m^{fb}(f_l) - q_l^{fb} = \frac{D(f_l + f_h) - D(f_l)}{f_l} + \frac{g(f_l) - g(f_h)}{f_h - f_l} + 1 - \frac{t^q}{f_h - f_l}.$$

Clearly, the first term of the right-hand side of the equation is positive, as $D(f_l + f_h) > D(f_l)$. The second term is also positive since $g(f_l) > g(f_h)$. Recall that $f_h - f_l - g(f_h) + g(f_l) > t^q$ must hold to ensure interior solution, hence we have

$$1 - \frac{t^q}{f_h - f_l} > -\frac{g(f_l) - g(f_h)}{f_h - f_l}.$$

Thus we obtain

$$q_m^{fb}(f_l) - q_l^{fb} = \frac{D(f_l + f_h) - D(f_l)}{f_l} + \frac{g(f_l) - g(f_h)}{f_h - f_l} + 1 - \frac{t^q}{f_h - f_l} > 0.$$

Chapter 4

Uniform-price auction with endogenous supply: Should seller's reservation price be kept hidden?

4.1 Introduction

Airport noise is an externality. With the expansion of air traffic in the past decades, a lot of nearby local inhabitants have been painfully suffered from noise nuisance around airports. Those residents who live under the flight path of airports are affected by the sound of planes taking off and landing. Indeed, for most who do not live close to an airport, air transportation is exclusively a social benefits; whereas for the residents who live in the vicinity of an airport, flight noise is an imposition. Noise pollution adversely affects the lives of local inhabitants, which is supported by some medical research that suggests direct links between being exposure to aircraft noise and health. The public relations war with those waterfront residents are even fierce when the airport needs additional runway capacity to accommodate demand growth and seeks expansion. In recent years there is a calling for airport noise ombudsman who is responsible for advising on how best to compensate the residents. An independent ombudsman could play a fundamental role in establishing a fair and reasonable balance between demand for flight movements and noise control. A similiar body has already existed in Australia since 2010, and in France since 2009.

In economic literature, in order to compensate those who suffer from noise damage, some authors propose to assign property rights to the residents, see among others Brechet and Picard (2010). Inspired by their paper, the present chapter seeks to employ uniform-price auction to the airport noise emission licenses market. Within uniform-price auction regime, all trade is cleared at the same price. Uniform-price auction is widely used to allocate multiple identical objects, for instance Treasury bonds, online initial public offerings, electricity procurement, and emission permits such as EU ETS (CO2) and RGGI CO2. In these auctions, bids are instituted as demand schedules, airlines who make highest bids win, and all winners pay an identical price. In some respects such auctions seem to be analogous to Walrasian markets. If we take a Walrasian point of view, market clearing is ubiquitous as long as preference and cost functions are given. A Walrasian auction perfectly matches the demand and supply and as a result the uniform-price auction appears a practical auction format that resembles Walrasian auction. If trading airport noise license is conducted in a Walrasian fashion: airlines submit demand schedules, a benevolent auctioneer collects individual demand schedules and sum them up to produce an aggregate demand schedule, then noise victims find the market clearing price, and all airlines bidding higher than this pay for this price and obtain the amount of licenses indicated in their individual demand schedule. Nevertheless, the typical assumption on the large number of airlines is not innonuous and is not well suited for certain auction environments, in particular for the noise license market where airlines are non-atomistic airlines. For instance in Treasury auctions the top five airlines typically acquire almost one-half of the issue (Malvey and Archibald (1998)). Electricity and spectrum markets in general also exhibit high levels of concentration. When one or two sides of the market has (have) market power, the uniform-price auction differs substantively from Walrasian market in that airlines strategically misrepresent their true demand. The consequence is that price finding procedure departs from that of the Walrasian auction on the grounds that agents execute market power, which is particularly true when both sides are relatively concentrated oligopolies.

The susceptibility of uniform-price auction to collusive-seeming behavior has been extensively studied. Traditionally the major concern with uniform-price auction refers to the existence of low-price equilibria. Low-price equilibria was first observed by Wilson (1979), followed by ensuing studies including Back and Zender (1993), Engelbrecht and Kahn (1998), and Ausubel and Cramton (2002), among others. The arise of low-price equilibria is mainly generated by two factors. On one hand, bidders have incentive to shade bids, since a bid for an additional unit affect the price for units that are obtained earlier. On the other hand, there is no strategic role for the representative. Adding up the two effects which flow in the same direction, bidder's strategic demand curve lie beneath true demand curve for all positive quantities. Given the nature of airlines strategic manipulation behavior, in noise licenses market the existence of low price equilibria will be unfavorable to the residents.

More recently a varied literature looks at ways to minimize the extent of low-price equilibria, and introducing endogenous supply is one of them. Endogenous supply means the quantity supplied is not fixed a priori, rather seller is granted the right to determine supply quantity after collecting the submitted bid schedules. Literature on endogenous supply has been fast-evolving. Back and Zender (1993) show that fixing quantity beforehand and revealing it ex-ante is detrimental to the seller, whereas if seller can reduce supply after knowing aggregate demand schedule many collusive seeming equilibria would vanish. Lengwiler (1999) explores that in equilibrium all oligopolistic bidders overstate true demand at low price and understate true demand at high price when the seller is monopolistic. LiCalzi and Pavan (2005) propose that the seller commits to a supply curve that is more elastic than true supply. Taken together, a major conclusion of many studies pertaining to endogenous supply , by varying supply ex-post, low price equilibria is significantly reduced or even eliminated. Yet much remains to be learned about endogous supply format auction when the seller has a reservation price.

This chapter contributes to the traditional uniform price auction theory by considering endogenous supply format auction when the seller has a random reservation price. Random reservation price has been used, but not exclusive to natural-resource auctions.[1] The provisions for reservation price are showed in recent legislative proposals for emission trading. For example, calling for an auction reservation price to be introduced to the EU Emissions Trading Scheme (ETS) was highlighted when the traded price crashed to a record low. The statement is that the imposition of a reservation price is consistent with EU interest of dissuading polluters to emit. The US Regional Greenhouse Gas Initiative (RGGI) scheme has a reservation price in place, Belgium has a minimum price for renewable energy certificates.[2] In the random reservation price literature, Hendricks, Porter, and Wilson (1994) discuss the bidding behavior of asymmetrically informed airlines confronting a random reservation price under the common-value framework. Li and Perrigne (2003) consider a random reserve price model within independent-private-value context, and compute the winners' informational rents as well as the optimal reservation price.

The present work has several features:

[1] Random reservation price is also encountered in wine auctions, see Ashenfelter (1989); and Web auctions, see Bajari and Hortacsu (2000).

[2] Belgium Renewable Energy Fact sheet, 2007. Energy EU Europe's Energy Portal.

1. It discard the usual assumption of a large number of airlines (airlines are buyers), and accounts for duopolistic airlines. The single seller (seller is the single representative of residents) nevertheless acts as a monopoly. This framework is appealing in many applications when both sides of markets are relatively concentrated.

2. Specifically, the representative of residents retains flexibility in determining supply quantity after collecting bid schedules. She is allowed to decide supply quantity ex post, which is essentially equivalent to committing to maximize her ex post profit. Airlines, anticipating the adjustable supply scheme, will alter their bids accordingly.

3. Moreover, it examines airlines behavior when confronting with two distinct information settings: revealed and secret reservation price. In particular, we shed some light on how varing the amount of information available to airlines affect total amount of licenses traded, and in turn, representative's expected utility. Attempting to investigate the role that information revelation would play, we characterize equilibrium strategies for the two schemes separately. We find out that the information revelation decision depends upon both the relative magnitude of reservation price, and also policy maker's objectives. When the policy objectives involve reducing the amount of issued license to a certain target level, or maximizing social welfare, then it is best to reveal the reservation price before auction takes place when the reservation price is small. Conversely, when the reservation price is relatively high, it is optimal to keep reservation price secret ex ante.

Lengwiler (1999) is perhaps the closest in spirit to the present paper, though differences between the two is substantive. His work was the first to analyze adjustable supply, but in a restrictive setting where bids are limited to two prices. Ours aims to examine adjustable supply in a broader setting allowing for continuous price. Instead of discrete price-quantity pairs used in his model, the present paper takes a step further and adopts linear downward-sloping demand schedules.[3] Albeit its simple structure, linear demand schedule captures the essential features of continuous demand schedules, more importantly it fulfills another property of uniform-price auction, that airlines claim lower unit prices for larger quantities than for smaller quantities. Furthermore, in order to better address the paricular property of emission license market, seller's reservation price is imposed. In addition, in our formulation representative could either

[3] Linear demand schedule is adopted in Brechet and Picard (2010).

reveal or hide her reservation price, a distinct departure from complete information assumption. We demonstrate in generality that a airline who desires multiple units of goods has an incentive to understate his demand at all prices. Moreover, we show that this demand reduction necessarily leads to inefficiency from a social planner's perspective.

The rest of the paper is organized as follows. Section 4.2 presents the model and environment. Section 4.3 analyzes the case of announced reservation price. Section 4.4 determines the symmetric equilibrium strategic bidding behavior for the case of secret reservation price. We discuss the policy implications of the two information disclosure schemes from a social planner's perspective in Section 4.5.

4.2 Model

This paper considers an auction model with one seller and two homogenous bidders within the independent-private-value paradigm, i.e., each airline's valuation of emission licenses is independent of the other airline's. To begin with, we introduce notations and describe the basic model ingredients, then proceed to present strategies. Finally we set forth the timing of the auction.

4.2.1 Airlines demand and bid schedule

On the demand side, two (male) airlines i, j act as bidders. As is common in auction literature, each airline has a *value function* that is continuously differentiable, monotonically increasing and concave, denoted by $V_k(q, t_k),\ k = i, j$, where q is the amount of licenses each airline obtains, and t_k denotes the type of each airline. Formally, if type t_k of bidder k obtains $q \in [0, 1]$ units of the licenses at a price p, his surplus is given by

$$V_k(q, t_k) - pq. \tag{4.1}$$

Because in our formulation bidders are homogenous, they both acknowledge their own and the other's type from the outset: $t_i = t_j$. Simply put, the second argument in the value function t_k can be suppressed. The value function $V_k(q)$ patterned after Ausubel and Cramton (2002) has continuous derivatives with respect to q, which is effectively the marginal value function and is denoted by $v_k(q) \equiv \frac{dV_k(q)}{dq}$. $v_k(q)$ satisfies: $\frac{dv_k(q)}{dq} < 0$ for all $q \in [0, 1]$.

For the sake of analytical simplicity, we propose a linear marginal value function $v_k(q)$ that takes the form

$$v_k(q) = 1 - q.$$

Intuitively, the demand schedule is derived from marginal value function. Hence by imposing a structure as such, his true demand schedule, denoted by $\hat{d}_k^{-1}(q)$, can be explicitly identified:

$$\hat{d}_k^{-1}(q) \equiv \max\{1-q, 0\}.$$

Hereby we place a hat notation over d to distinguish the true demand schedule, in contrast to individual bid schedule which is denoted by $d_k(q)$. The bid schedule is written as:

$$d_k(p) = 1 - \frac{p}{\beta_k}, \tag{4.2}$$

At some points, our analysis is facilitated by inverting bid schedule as:

$$d_k^{-1}(q) = \beta_k(1-q). \tag{4.3}$$

The advantage of the particular choice of the marginal value structure is that it allows for an explicit computation and comparison of the outcomes of two information revelation schemes. Departing from true demand schedule, airlines simultaneously submit a continuous and downward sloping schedule that specifies his demand at each price. To distinguish airline's true and submitted demand schedules, we employ the term *demand schedule* to indicate true demand schedule, and *bid schedule* for submitted schedule. It is imperative to define the term *shade*.

Definition 1 *Shade means airline optimally shifts down his bid schedule relative to true demand schedule.*

More specifically, airline's bidding price for the first unit of license, equivalent to $d_k(q = 0)$, is called the highest bid. Thus, on grounds of literal interpretation, the highest bid exhibits to which extent an airline shades.

Aggregate bid schedule specifies the aggregate demand quantity confronting the representative at each price, which is the horizontal summation of two bid quantities at that price, and is denoted as $Q(p)$:

$$Q(p) \equiv d_i(p) + d_j(p).$$

Whereas a bid specifies a maximum price that airline i is willing to pay for the q_i-th unit, a strategy, denoted as $\beta_k(v_k(q))$, specifies a mapping from value function to bid schedule, $\beta_k(\cdot)$: $[0,1] \rightarrow [0, \beta_k]$. following the common in literature $\beta_k(\cdot)$ should be continuous and monotonic. The simplest exposition suggests an imposition of linear strategies $\beta_i(v_i(q)) = \beta_i \cdot v_i(q)$ and $\beta_j(v_j(q)) = \beta_j \cdot v_j(q)$. Since each airline knows his counterpart's valuation, which is equivalent to his, we safely drop the subscript i and j of $v_i(\cdot)$ function to save notation. Denote $\mathcal{S}_k$ as airline k's surplus, which is always non-negative,[4] therefore airlines would strategically choose $\beta_k \leq 1$ to maximize their surplus. Recall that shade behavior imposes $\beta_k < 1$, whereas $\beta_k = 1$ indicates a truthful bidding behavior. The problem for each airline is to determine β_k that maximizes his expected surplus. Rewriting (4.1) yields:

$$\max_{\beta_k} \mathcal{S}_k = \int_0^{q_k} \hat{d}_k^{-1}(\tilde{q})d\tilde{q} - p \cdot q_k, \qquad k \in \{i, j\},$$

where both p and q_k are functions of (β_i, β_j), but we omit the arguments here.

4.2.2 Representative's utility maximization

On the seller side, the single seller is a representative of a group of homogenous residents who live in the vicinity of an airport. The term homogenous alludes to their identical marginal disutility from being exposed to noise. As in Brechet and Picard (2010), the representative is responsible for participating in the auction on behalf of all residents. The representative offers a bunch of identical noise licenses to the two risk neutral airlines.

Assume representative's marginal disutility is constant and denoted as c. Hence c stands for marginal disutility evaluated in monetary term, which is analogous to a reservation price in standard auction theory.[5] Both terms will be used interchangeably in the ensuing analysis. Upon observing the bids and then establish aggregate bid schedule, the representative decides on market clearing price p, and in turn actual supply Q, to maximize her utility. A critical element should be emphasized that the representative should commit to stick to this price finding rule, i.e., market clearing price is found on the aggregate bid schedulet. With such a commitment in place, representative's knowledge about airline's true demand schedule doesn't play a role. Besides, once the representative receives the aggregate bid schedule, even if she could size up airlines real demand schedule through her reasonable calculation, she could not switch to other pricing rules, on the grounds of irreversible commitment. Finally, bids above and equal the stop-out price are awarded. In the uniform-price auction, airlines pay p for each unit of license they receive. In particular, if an airline's highest bid is lower than the stop-out price, then he would obtain zero license, a case demonstrated in Fig.1. Intuitively, if $c \geq 1$, both airlines obtain zero license with probability one and would

[4] The non-negativity implies $v_i(q) - \beta_i \cdot v_i(q) \geq 0$.

[5] A constant marginal cost of the seller has been assumed in Lengwiler (1999) and Damianov (2008), among others.

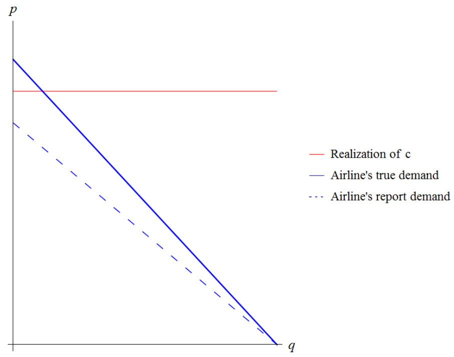

Figure 1: Shading behavior and the realization of c

thus exit the market. To ensure that both airlines are active in the auction, c is restricted to take value in the range $[0, 1)$.

A supply rule, denoted as $S(\cdot)$, defines a mapping from the aggregate bids into a supply quantity $Q \geq 0$, $S(\cdot) : \sum_{k=1}^{2} d_k(p) \to Q$.

When determining the market clearing price, and in turn supply quantity, the representative cares about aggregate demand schedule rather than individual demand schedule :

$$S(d_i(p), d_j(p), c) = S(Q(p), c) \forall p \geq 0.$$

4.2.3 Timing

The timing of the auction is demonstrated in Fig.2. In the first stage, airlines simultaneously submit bid schedules to the auctioneer. In the second stage, upon receiving submitted bid schedules and the realization of her reservation price, the representative decides on a supply quantity to maximize her utility. We distinguish between two types of information settings regarding the representative's reservation price: revealed and hidden. Acknowledging that the representative would choose a profit maximizing supply quantity,[6] airlines adjust their bids strategically so as to counterbalance the power of the representative. The payoff of each airline depends on his own and the other airline's bid, as well as the realization of representative's reservation price. The equilibrium concept in this complete information setting is subgame perfect nash equilibrium hence we will employ backward induction to analyze the equilibrium strategic shades of two airlines.

[6]The utility maximizing behavior of the representative serves as a commitment.

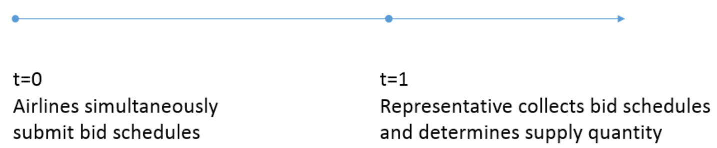

Figure 2: Timeline of the auction

4.3 Revealed reservation price

4.3.1 Equilibrium

Under a revealed reservation price scheme, the representative pre-announces a reservation price c, and commits herself to sell any amount of noise licenses no lower than c. A bidder obtains a positive number of licenses when only his highest bid exceeds the stop-out price. In this section we will characterize equilibrium bids when the policy is to reveal the reservation price.

To illustrate our analysis, we apply backward induction and first examine the representative's utility. Airlines' true demand schedules are known to themselves but not the to seller. Given that the two bid schedules take the form as indicated in (4.2), it follows that the aggregate bid schedule is $Q = 1 - \frac{p}{\beta_i} + 1 - \frac{p}{\beta_j}$. The representative is an utilitarian and considers the following utility function:

$$U = (p - c)Q, \tag{4.4}$$

which states that her utility is measured by the difference between total airline payments and disutility generated by noise. The representative behaves like a monopolist that maximizes her utility with respect to Q:

$$\max_Q U(Q) = \left(\frac{2 - Q}{\frac{1}{\beta_i} + \frac{1}{\beta_j}} - c \right) Q.$$

The first-order condition is both necessary and sufficient for setting forth the following optimal solution:

$$Q^*(\beta_i, \beta_j, c) = \frac{2\beta_i\beta_j - c \cdot \left(\beta_i + \beta_j\right)}{2\beta_i\beta_j}. \tag{4.5}$$

The above expression shows that the optimal quantity the representative would offer in the second stage is determined by her marginal disutility and airlines shading behaviors. It is straightforward that Q^* is decreasing in c, which shows a negative relation between marginal disutility and optimal supply quantity. This statement is in line with the standard monopoly theory.

Furthermore, the corresponding market clearing price $p(Q^*)$ is found on the aggregate bid schedule:

$$p(Q^*) = \frac{(2-Q)}{\dfrac{1}{\beta_i} + \dfrac{1}{\beta_j}} = \frac{2\beta_i\beta_j + c \cdot \left(\beta_i + \beta_j\right)}{2(\beta_i + \beta_j)}. \tag{4.6}$$

Given the above price $p(Q^*)$, airline k acquires $q_k^*\left(p(Q^*)\right)$ units of license, which is found on his own bid schedule at price $p(Q^*)$. It follows that

$$q_i^* = \frac{2\beta_i^2 - c(\beta_i + \beta_j)}{2\beta_i(\beta_i + \beta_j)}, q_j^* = \frac{2\beta_j^2 - c(\beta_i + \beta_j)}{2\beta_j(\beta_i + \beta_j)}. \tag{4.7}$$

At stage 2, airline i and j choose β_k to maximize their respective surplus S_k, $k \in \{i, j\}$:

$$\max_{\beta_k^*} S_k = \int_0^{q_k^*} (1 - \tilde{q})d\tilde{q} - p^* \cdot q_k^*, \tag{4.8}$$

Exploiting symmetry, calculation is facilitated by recognizing $\beta_i = \beta_j = \beta$. The standard first-order condition with respect to β leads to the following optimal condition: [7]

$$\frac{\partial S(\beta)}{\partial \beta} = \frac{1}{4\beta^2}\left(c - c^2 + \frac{c^2}{\beta} - \beta^2\right) = 0. \tag{4.9}$$

The above analysis can be summarized in the following proposition, the proof is provided in Appendix A.

Proposition 1 *For any given reservation price c,*

- *there exits a unique optimal strategy for each airline,* $\beta^* \in (\overline{\beta^*} = \sqrt{\frac{c-c^2}{3}}, 1)$*, which is the unique positive solution of polynomial*

$$F(\beta) = \beta^3 - (c - c^2)\beta - c^2 = 0; \tag{4.10}$$

[7]It is easy to check S_k is strictly concave in term of β, thus the first order condition is necessary and sufficient for the optimization.

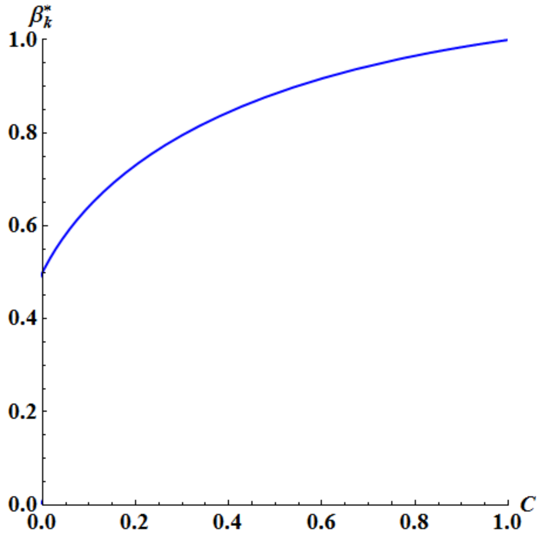

Figure 3: Optimal bidding with two bidders

- *representative's optimal price and suppy quantity are given by*

$$p^*(\beta^*) = \frac{c}{2} + \frac{\beta^*}{2},\ q^*(\beta^*) = \frac{1}{2} - \frac{c}{2\beta^*}; \tag{4.11}$$

- *given the optimal choice of β^*, representative's utility is characterized by*

$$U(\beta^*) = \frac{(\beta^* - c)^2}{2\beta^*}. \tag{4.12}$$

We check that the representative's utility is decreasing in the highest bid:

$$\frac{dU(\beta^*)}{d\beta^*} = \frac{1}{2} - \frac{c^2}{2\,(\beta^*)^2} < 0, \tag{4.13}$$

which implies that given a constant marginal disutility, the more truthful airlines reveal their demand schedules, the higher is the representative's utility.

Fig.3 illustrates the optimal bidding behavior of a two-bidder game. A closer inspection of the figure suggests that shading phenomena is an inevitable consequence of surplus maximizing decision at any price level. Recall that shade depicts a deviation from demand schedule to bid schedule, henceforth $1 - \beta$ can be interpreted as a measure of the magnitude of deviation. As indicated by Fig. 3, optimal bid β is monotonically increasing with c, or equivalently, shading behavior $(1 - \beta)$ is monotonically decreasing in c for all $c \in [0, 1)$. The intuition is as follows. Knowing that their behavior contributes to the choice of market clearing price, airlines strategically understate demand quantity at all price levels so as to affect price setting. By claiming to be a weaker bidder, airlines demand fewer units at all price levels, the representative chooses a market clearing price that is lower relative to a truthful bid situation. Airlines hence pay less for all inframarginal units, at the cost of obtaining fewer units than they could if they bid truthfully. The shading behavior results in a reduced supply quantity and a lower market clearing price.

4.3.2 Comparative static analysis

To illustrate the properties of the equilibrium, we undertake a comparative-static analysis. Aiming to find out how shading behavior affects the amount of licenses that airlines get in equilibrium, as well as the price they pay for each license, the first-order derivatives are checked. Mechanical calculation yields:

$$\frac{\partial p(Q^*)}{\partial \beta_i} = \frac{\beta_j^2}{(\beta_i + \beta_j)^2} > 0, \frac{\partial p(Q^*)}{\partial \beta_j} = \frac{\beta_i^2}{(\beta_i + \beta_j)^2} > 0. \tag{4.14}$$

The strict positive signs suggest that the less an airline shades, the higher per-license price he would pay in equilibrium.

$$\frac{\partial q_i^*}{\partial \beta_i} = \frac{2\beta_i^2 \beta_j + c(\beta_i + \beta_j)^2}{2\beta_i^2 (\beta_i + \beta_j)^2} > 0, \frac{\partial q_j^*}{\partial \beta_j} = \frac{2\beta_j^2 \beta_i + c(\beta_i + \beta_j)^2}{2\beta_j^2 (\beta_i + \beta_j)^2} > 0. \tag{4.15}$$

The strict positive signs of both expressions are intuitive, because a airline obtains more licenses in equilibrium if he bids more truthfully. This result is rather intuitive. By bidding more aggressively, airlines elicit a higher supply quantity from the representative side and pays a higher price for each license they obtains.

We proceed to derive the first-order derivative of q_k^* with respect to β_{-k}, which are:

$$\frac{\partial q_i^*}{\partial \beta_j} = -\frac{\beta_i}{(\beta_i + \beta_j)^2} < 0, \quad \frac{\partial q_j^*}{\partial \beta_i} = -\frac{\beta_j}{(\beta_i + \beta_j)^2} < 0. \tag{4.16}$$

The first-order derivative of q_k^* with respect to β_{-k} is negative, suggesting that an airline's truthfulness in bidding negatively affects its counterpart's obtained quantity in equilibrium. This result is hardly surprising. If airline j bids less truthfully, in the sense that he claims a smaller amount of licenses at all price levels, the representative would decide on a lower market clearing price compared to what he would choose if j bids true demand schedule. Therefore airline j's shading behavior is favorable to airline i, because i benefits from the impact of j's shading behavior on the representative's choice of quantity and in turn price.

In the ensuing analysis, we probe into the impact of c on the optimal choice of airline strategy β_i^*, supply price p_i^*, number of licenses traded q_i^* and representative utility $U(\beta_i^*)$.

- Take total differential of β_i^* with respect to c, the polynomial equation (4.10) reads
$$\frac{d\beta_i^*}{dc} = \frac{(1-2c)\beta_i^* + 2c}{3(\beta_i^*)^2 - c + c^2}. \tag{4.17}$$
Rearranging the numerator reveals that the numerator is non-negative: $(1-2c)\beta_i^* + 2c = \beta_i^* + 2c(1-\beta_i^*) \geq 0$. By further inspection of the denominator, we find out that $3(\beta_i^*)^2 - c + c^2 > 0$, the proof is presented in Appendix B. Hence,
$$\frac{d\beta_i^*}{dc} > 0. \tag{4.18}$$
The airline's optimal strategy β_i^* increases with c, which implies a higher reservation price induces less shading behavior.

- Revisiting equation (4.11), it is easily observable that $\frac{dp^*(\beta^*)}{dc} > 0$.

- In order to observe the impact of c on the number of licenses obtained by each airline, we now take total differential of $q_i^*$with respect to c:
$$\frac{dq_i^*}{dc} = \underbrace{\frac{\partial q_i^*}{\partial c}}_{\text{Direct Effect}} + \underbrace{\frac{\partial q_i^*}{\partial \beta_j^*}\frac{d\beta_j^*}{dc}}_{\text{Indirect Effect}}.$$

The total impact of c on the units of licenses obtained by each airline is ambiguous at the first sight: the direct impact is negative, shown by $\frac{\partial q^*}{\partial c} = -\frac{1}{2\beta^*} < 0$; while the indirect impact is positive $\frac{\partial q^*}{\partial \beta^*}\frac{d\beta^*}{dc} = \frac{c}{2(\beta^*)^2} > 0$. On one hand, the negative direct effects comes from airline k's downward sloping marginal valuation function. On the other hand, the positive indirect impact is intuitive. Note that each individual airline's optimal bidding strategy is decreasing in reservation price and increasing in total units of licences the other airline obtains. As a consequence, the other airline's bidding strategy is indirectly decreasing with reservation price. Interpreting the combined impact of the two effects is not straightforward, however, using (4.11) and (4.17), after careful manipulations we derive total effect of c on q_i^*, the proof is provided in Appendix C.
$$\frac{dq_i^*}{dc} = -\frac{1}{2\beta^*} + \frac{c}{2(\beta^*)^2}\left(\frac{(1-2c)\beta^* + 2c}{3(\beta^*)^2 - c + c^2}\right) < 0. \tag{4.19}$$

Thus, higher marginal disutillity leads to fewer licenses supplied in equilibrium. Direct impact outweighs indirect impact and decides the overall sign, which is reasonable: each airline's payoff is typically influenced more by his own strategic behavior than by his counterpart's.

- Likewise, total differential of $U(\beta_i^*, c)$ with respect to c is performed to unveil how c affects U.

$$\begin{aligned}\frac{dU(\beta_i^*,c)}{dc} &= \underbrace{\frac{\partial U(\beta_i^*,c)}{\partial c}}_{\text{Direct Effect}} + \underbrace{\frac{\partial U(\beta_i^*,c)}{\partial \beta_i^*}\frac{d\beta_i^*}{dc}}_{\text{Indirect Effect}} \\ &= \underbrace{\frac{c}{\beta_i^*} - 1}_{\text{Direct Effect}} + \underbrace{\frac{1}{2}\left(1 - \frac{c^2}{(\beta_i^*)^2}\right) \cdot \frac{d\beta_i^*}{dc}}_{\text{Indirect Effect}}.\end{aligned} \tag{4.20}$$

In exploring the sign of $\frac{dU(\beta_i^*,c)}{dc}$, note that c affects U in both direct and indirect ways. By our construction of the model, an airline wins a positive number of licenses only insofar as his highest bid exceeds representative's reservation price, hence $\beta > c$ holds over all feasible ranges of β and c. It follows that $\frac{c}{\beta^*} - 1 < 0$, the direct effect is negative. Following equation (4.4), the direct impact is immediate. Holding price and quantity constant, an increase in c induces a smaller utility.

Consider now the indirect effect. It is easily observable that the multiplier is positive: $\frac{1}{2}\left(1 - \frac{c^2}{(\beta_i^*)^2}\right) > 0$. Moreover, (4.18) impllies $\frac{d\beta_i^*}{dc} > 0$. Taken together, the indirect effect is positive, which is hardly surprising. For one thing, a higher c fosters more truthful bidding, because both airlines prefer to be served, and the feasible range for shading is more restricted if c is already high. For the other thing, shading less demonstrates a milder execution of market power, and is therefore favorable to residents. However, the overall effect appears less straightforward, and we verify that it depends on parameter values. The mathematical proof in established in Appendix D. The above analysis can be formalized by means of the following proposition.

Proposition 2 *The optimal shade β^* and equilibrium market clearing price are monotonically increasing in reservation price c, while the number of total licenses traded is decreasing in c. Representative's utility increases with c when $0 < c < \frac{1}{2}$ and $\beta_1 < \beta^* < 1$; while decreases with c when $0 < c < \frac{1}{2}$ and $0 < \beta^* < \beta_1$,or if $\frac{1}{2} < c < 1$; where β_1 is the positive root of $\beta^2 - c\beta - \frac{4c^2}{1-2c} = 0$.*

Proposition 2 portrays regions in the two-dimensional (β_1, c) parameter space for the overal impact of c on U to be either positive or negative, as shown in Fig. 4. When

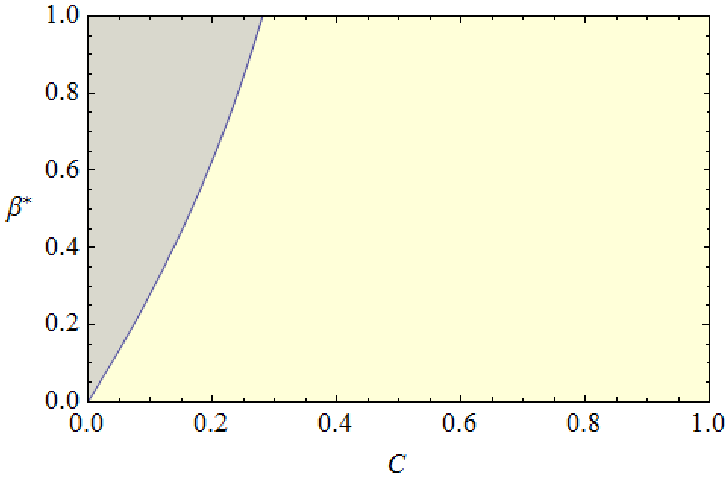

Figure 4: Impact of c on representative's utility (Grey +, Yellow -)

the representative has a high marginal disutility for being exposed to noise, her overall utility decreases with the marginal disutility. While when her marginal disutility is relatively small, her overall utility might increase in her marginal disutility, should the magnitude of shade being not significant.

4.4 Secret reservation price

4.4.1 Equilibrium

Up to this point we have discussed subgame perfect nash equilibrium of a two stage game in a complete information setting. From now on we reconsider the auction and probe into hidden reservation price.[8] We will discuss at length that changing information setting drastically alters our precedent conclusion. The primitive setting is essentially analogous to the preceding section, apart from c now being private knowledge to the representative. Airlines, however, know that c is uniformly distributed on support $[0, 1)$, whose cumulative distribution is denoted as $F(c)$, and probability distribution as $f(c)$. Though the realization of c is known only to the representative.

The timeline of the auction developed precedently still applies, with the only minor difference being here airlines know only the distribution of c at the outset of the game. Formally, at stage 1, based on the knowledge of $F(c)$, airlines submit bid schedules $d_k(p)$. At stage 2, representative decides on market clearing price and supply quantity,

[8] A secret reservation price is frequently used in France to sell timber, see Li and Perrigne (2003).

and airlines receive licenses as indicated by their submitted demand schedules.

To begin with, note that when c is hidden from the airlines, airlines acknowledge that the more they shade at stage 1, the more likely that his highest valuation shall be higher than the realization of c at stage 2, and he shall get zero license if this should happen. The assumption to exclude inactive airlines, i.e. $v(q = 0) > c$ is again imposed to make our model meaningful. In this circumstance, airline faces a tradeoff between benefiting from shading and running the risk of getting zero license. The latter case occurs if the realization of c turns out to be higher than his highest bid, a case depicted in Fig. 1.

Again we employ backward induction to characterize equilibrium bids. At stage 2, given the optimal supply price chosen by the representative, airlines' optimal quantities are derived in the similar fashion as in the previous section, which are written by (4.7).

Particularly, we demonstrate non-existence of asymmetric equilibria for the homogenous airlines and show for this case, airlines behavior can only be symmetric in equilibrium. At stage 1, airline i and j choose β_i and β_j simultaneously to maximize surplus:

$$\beta_i^* = \arg\max_{\beta_i} \int_0^1 \left(\int_0^{q_i} (1 - \tilde{q}) d\tilde{q} - p \cdot q_i \right) dF(c), \tag{4.21}$$

$$\beta_j^* = \arg\max_{\beta_j} \int_0^1 \left(\int_0^{q_j} (1 - \tilde{q}) d\tilde{q} - p \cdot q_j \right) dF(c),$$

with $0 < \beta_{i,j} \leq 1$. Note that the lower bound of outside integral being zero, implying that one or both of the airlines may win zero license.

As a point of departure assume there is asymmetric equilibrium in which the two airlines bids differently, we then check that asymmetric equilibrium does not hold. Without loss of generality, suppose airline i shades less than its counterpart $j : 1 \geq \beta_i > \beta_j \geq 0$. The aggregate demand structure is shown in Fig 5. The line $\beta_i AQ$ represents the horizontal sum of the two demand curves.The marginal revenue curve is given by the broken curve $\beta_i GFB$. Three cases should be examined separately.

Case 1. If constant marginal cost exceeds β_i, no airline is served.

Case 2. If constant marginal cost cuts the $\beta_i H$ portion, only one airline, namely airline i, is served.

Case 3. If constant marginal cost cuts the FB portion, then both airlines are served.

Apparently, of the three cases, Case 2 and 3 are more relevant to us. The piecewise

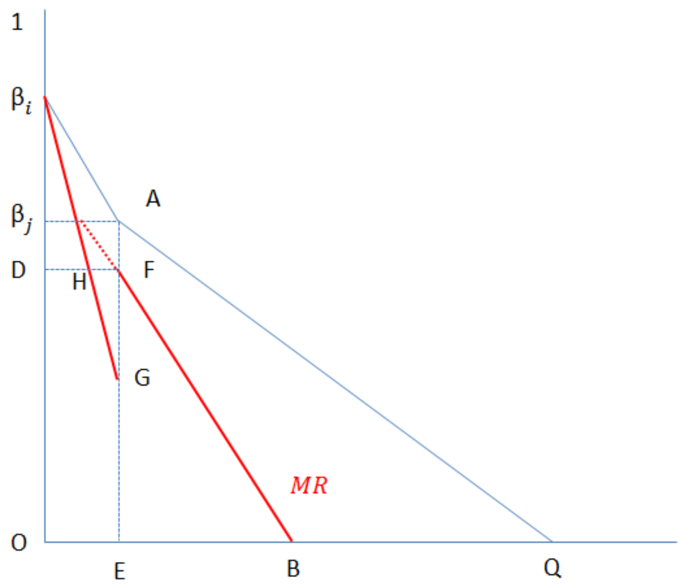

Figure 5: Aggregate demand of two asymmetric bidders

aggregate demand function can be characterized:

$$Q = \begin{cases} 0 & \text{if } c > \beta_i & \text{(Case 1)} \\ d_i(p) & \text{if } \beta_i \geq c > |OD| & \text{(Case 2)} \\ d_i(p) + d_j(p) & \text{if } c \leq |OD| & \text{(Case 3)}. \end{cases}$$

The kink (point A) has horizontal axis length equals to $|OE|$ and vertical axis length equals to β_j. Note that because the representative can not discriminate airlines, if marginal cost c lies in the price region $|\beta_j D|$, airline j's highest bid β_j exceeds c but the representative would set a price that exceeds β_j and serves i only.

Hence we could characterize each airline's utility, which breaks down to two parts. Denote airline's surplus as S, the number in the subscript for Case 2 or 3.

$$\beta_i^* = argmax\left(\int_0^{|OD|} S_{i3}dF(c) + \int_{|OD|}^{\beta_i} S_{i2}dF(c)\right), \tag{4.22}$$

$$\beta_j^* = argmax\left(\int_0^{|OD|} S_{j3}dF(c) + \int_{|OD|}^{\beta_i} S_{j2}dF(c)\right), \tag{4.23}$$

where

$$S_{i2} = \int_0^{q_{i2}^*}(1-\tilde{q})d\tilde{q} - p_i^* \cdot q_{i2}^*, S_{i3} = \int_0^{q_{i3}^*}(1-\tilde{q})d\tilde{q} - p^* \cdot q_{i3}^*,$$

$$S_{j2} = 0 \text{ by assumption, } S_{j3} = \int_0^{q_{j3}^*}(1-\tilde{q})d\tilde{q} - p^* \cdot q_{j3}^*.$$

Now it remains to derive $|OE|$ and $|OD|$. In case (2), i is the monopoly airline, and the marginal revenue function of i, denoted as MR_i, corresponding to line segment $\beta_i G$ on Fig. 5. It can be derived from i's demand function (4.3) and (4.4):

$$MR_i = \beta_i (1 - 2q_i).$$

In Case 3, i and j both obtains a positive number of licenses, and the marginal revenue function of i and j combined, denoted as MR_{ij}, which corresponds to line segment FB on Fig. 5, can be derived from i's demand function (4.6) and (4.5):

$$MR_{ij} = \frac{2 - 2Q}{\frac{1}{\beta_i} + \frac{1}{\beta_j}}. \tag{4.24}$$

Note that $|OE|$ equals inverse demand function of i ,which refers to (4.3), evaluated at $p_i = \beta_j$:

$$|OE| = 1 - \frac{\beta_i}{\beta_j}.$$

While $|OD|$ is determined by the intersection of line MR_{ij} and line AE, hence equals to MR_{ij} evaluated at $Q = |OE|$:

$$|OD| = \frac{2\beta_i^2 - \beta_i\beta_j}{\beta_i + \beta_j}.$$

Having derived $|OE|$ and $|OD|$, this next step is to derive airline surplus. Before proceeding, it is worth notifying that choosing $\beta_j = 0$ is dominated by choosing any positive β_j lies between [0,1]. If he chooses $\beta_j > 0$, his utility is $\max\{0, \int_0^{|OD|} S_{j3} dF(c)\}$, where $\int_0^{|OD|} S_{j3} dF(c) > 0$. On the contrary, if he chooses $\beta_j = 0$, then $|OD| = 0$, and $\int_0^{|OD|} S_{j3} dF(c) = 0$. Hence airline j would not submit $\beta_j = 0$, rather he would choose a β_j that maximizes his overal utility.

Under Case 2, airline i's obtained quantity, by backward induction, is deduced by setting $MR_i = c$:

$$\beta_i (1 - 2q_i) = c \Rightarrow q_{i2}^* = \frac{1}{2}\left(1 - \frac{c}{\beta_i}\right), \tag{4.25}$$

$$p_i^* = \beta_i (1 - q_{i2})\,|_{q_i = q_{i2}^*} = \frac{1}{2}(\beta_i + c).$$

Consequently i's maximization problem can be characterized as

$$S_{i2} = \int_0^{q_{i2}^*} (1 - \tilde{q}) d\tilde{q} - p_i^* \cdot q_{i2}^*. \tag{4.26}$$

Under Case 3, Airline i and j act as duopoly and choose β_{i3} and β_{j3} simultaneously to maximize surplus.

$$S_{i3} = \int_0^{q_{i3}^*} (1-\tilde{q})d\tilde{q} - p^* \cdot q_{i3}^*, \tag{4.27}$$

$$S_{j3} = \int_0^{q_{j3}^*} (1-\tilde{q})d\tilde{q} - p^* \cdot q_{j3}^*.$$

The expressions of p^*, q_{i3}^* and q_{j3}^* are given by (4.6) and (4.7).

Solving for (4.22) using S_{i2}, S_{i3} and S_{j3} as calculated above, yields

$$\beta_i = 0.86, \beta_j = 0.73.$$

The corresponding surplus are 0.065 and 0.05 for i and j respectively. We are now in a position to investigate the simultanous choice of two airlines. If both choose $\beta_i = \beta_j = 0.86$, then they end up with being in a symmetric duopoly case, the surplus for this case is 0.055 for each. While if both choose $\beta_i = \beta_j = 0.73$, the surplus for each is 0.057. The bidding behavior can be described in the below matrix.

		j	
		$\beta = 0.86$	$\beta = 0.73$
i	$\beta = 0.86$	$(0.055, 0.055)$	$(0.065, 0.05)$
	$\beta = 0.73$	$(0.05, 0.065)$	$(0.057, 0.057)$

It is straightforward to see that the cell corresponds to strategy $(0.86, 0.86)$ is the nash equilibrium. This result shows that if the residents marginal disutility is hidden to airlines, the airlines optimally submit bid schedules $0.86(1-q)$, a clear evidence for exertion of market power. That is, at any price airlines ask for fewer licenses compared to their true demand. Compare to the case where c is revealed, airlines shade to a lesser degree for a wide range of c, implies their submitted bid schedules are by and large closer to true demand. We conclude this section with the following equilibrium characterization result.

Proposition 3 *When both airlines have value schedules $v_i(q) = v_j(q) = 1-q$, and representative's reservation price is secret, airlines optimally bid $0.86(1-q)$.*

When c is secret, choosing an optimal β_k reflects a combination of two factors, namely probability of winning some licenses and payment in case of winning. On the one hand, by shading less, or in other words bidding more truthfully, airlines diminishes the risk of losing out the auction. This concern is more relevant when the realization of reservation price turns out to be relatively high in the second stage. However, his payment is comparatively higher if he shades less and is still served. On the other hand, shading less increases the payment, and in turn decreases the surplus an airline could get if the realization of reservation price is sufficiently low. Hence there is a tradeoff between probability of winning and payment in case of winning. Combining these two effects mitigates the magnitude of low price equilibria.

It is necessary to point out that the existence of a clear-cut outcome is in part led by our simple formulation of the problem. One should recognize the importance of the assumption of uniform distribution of c and airlines linear value schedules, which facilitate the derivation of a unique Nash equilibrium. The equilibrium outcome suggests that airlines scale down true demand function by a factor of 0.86. When other forms of distributions are imposed, more complex equilibria may be expected.

4.4.2 A comparison of revealed and hidden reservation price

A revealed reservation price can create greater certainty for airlines, and greater stability for the auction itself. However, when the reservation price is low, revealing it would entice airlines to exert market power and is thus detrimental to the residents. Having analyzed both the revealed and the hidden cases, we now turn to compare the outcomes of the two settings.

A closer inspection of Eq. (4.17) discloses that in the revealed case, $\beta^*(c)$ is monotonically increasing in c in the range $(0, 1)$. Due to the monotonicity relationship of $\beta^*(c)$ and c, we could find out a unique c that corresponds to the coefficient $\beta = 0.86,$ *i.e.*, a reservation price in the revealed case that induces same degree of shading in the unrevealed case. Recall Eq. 4.9, a scale factor evaluated at 0.86 is found out at $c = 0.44$. The implication is as follows. When representative's marginal disutility is sufficiently low, i.e., smaller than 0.44, in this circumstance if she announces her marginal disutility, airlines's bid would deviate significantly from his true demand schedules. On the contrary, if she hides her marginal disutility, she would foresee each airline report to a factor of 0.86 of their true demand. Conversely, when her marginal disutility is above 0.44, airlines shade less if marginal disutility is revealed. The decision to whether disclose or hide reservation price is up to the political objective of the ombudsman. If the ombudsman targets to achieve a license cap constraint, he would opt to hide the reservation price rather than announcing it when the reservation price $c > 0.44$. Whereas if he aims to encourage more truthfull bidding, he would announce the reservation price when $c > 0.44$. And similiar argument applies for $c \leq 0.44$.

4.5 A social planner's choice

The precedent sections have investigated and compared the equilibrium outcome of auction under two different information settings. Yet the preceding analysis are formulated out of the residents' perspective. In this section we are in a position to look for the social planner's choice. A social planner, for instance the city government or federal state where the airport locates, maximizes total welfare. As is conventional among literature, welfare is the summation of airline surplus and representative utility. Denote by q_k^w the amount of licenses obtained by each airline when the objective is to maximize welfare, and $\mathcal{W}$ the social welfare. Since the payment between airlines and representative cancels out, the social welfare equals airlines' gross surplus less representative's total disutility.

Thus $\mathcal{W}$ can be written as:

$$\max \mathcal{W} = 2\int_0^{q_k^w} (1-\tilde{q})d\tilde{q} - 2cq_k^w.$$

First-order condition yields

$$q_k^{w*} = 1-c.$$

Denote total supply quantity chosen by the social planner as Q^w. It follows that $Q^w = 2q_k^{w*} = 2(1-c)$.

Recall in the revealed c case, see (4.5), a utility maximizing representative optimally chooses $Q^* = \dfrac{-c\beta_i - c\beta_j + 2\beta_i\beta_j}{2\beta_i\beta_j}$. It can be verified that for all (c, β_i, β_j) :

$$Q^* < Q^w.$$

We conclude the above analysis in the following proposition:

Proposition 4 *A welfare maximizing social planner supplies more licenses than a representative in an auction.*

The fact that a welfare maximizing quantity exceeds the auction outcome suggests if the primitive policy objective is to curb noise pollution (rather than maximizing social welfare), then an auction mechanism performs better than appointing the allocation task to a public omnibusman. In practice this scenario is not rarely seen, for instance the imposition of Kyoto Protocol, among others. Governments face an emission ceiling and are compelled to restraint noise or a broad variety of other pollutions to below the target level, despite at times at the cost of impairing social welfare. Should this happen, the purpose of setting up the target may run counter to its initial goal. If, on the contrary, licenses are allocated by a public omnibusman, then the allocation may run counter to its initial purpose, and the target would not be met eventually.

4.6 Conclusion

The present analysis shows that when two airlines bid for noise emission licenses against a representative of airport nearby residents, both airlines optimally misreport their true demand in order to maximizes profits. The outcomes of two cases where the representative's reservation price is pre-announced and hidden, are presented. The following comparison of these two cases provides novel insights for the policy makers. On that grounds we suggest the present paper could be served as a guidance for policy makers in considering allocation approach to distribute emission licenses.

From the analysis, a number of issues emerge for future research. One basic assumption of our model relies on symmetric airlines and, although this is a common assumption in airline literature, a more realistic demand structure could be considered for studying interactions between asymmetric airlines. Indeed, studying airlines that are either symmetrically differentiated[9] or vertically differentiated[10] require a more elaborate specification of the demand schedules. Another natural extension of the present analysis involves other assumptions on the distribution of reservation price. Finally, the present paper has confined the analysis to a single airport for simplicity and to focus on the main insights. But each flight operation involves a take-off and a landing airport, thus network effects is also seen as a logical extension for future research.

[9]For instance, Air France vs. Lufthansa.
[10]For instance Lufthansa vs. Easyjet.

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Appendix

Appendix A. Proof of Proposition 1

From the definition of the polynomial, $F(\beta) = \beta^3 - (c - c^2)\beta - c^2$, we can easily state that 3rd degree polynomial has at least one real root. Furthermore, it is easy to check that $F(\beta = 0) = -c^2 < 0$ and $F(1) = 1 - c > 0$. Thus, there is at least one positive root for $F(\beta) = 0$ in the interval $\beta \in (0, 1)$.

In the following, we will show that this is the only positive root.

From the Descartes' rule of sign[11] , we can state that if there are only real roots, then there are one positive root and two negatives roots.

And if there is only one real root, then this root is positive and locates in interval $\beta \in (0, 1)$, and the other two are conjugate pair complex roots.

We denote this unique positive root by β^*.

Furthermore, given function $F(\beta)$ is strictly convex for $\beta \in (0, 1)$, the positive root β^* must lie in the increasing part of the convex curve. That is, $\beta^* \in (\overline{\beta^*}, 1)$, in which $\overline{\beta^*}$ is the minimum point of $F(\beta)$ and given by $\overline{\beta^*} = \sqrt{\frac{c-c^2}{3}}(< 1)$, such that $F'(\overline{\beta^*}) = 0$ and $F(\overline{\beta^*}) < 0$.

That finishes the proof of Proposition 1.

Appendix B. Proof of $\frac{d\beta^*}{dc} > 0$

We need to prove that

$$\frac{d\beta^*}{dc} = \frac{(1-2c)\beta^* + 2c}{3(\beta^*)^2 - c + c^2} > 0.$$

As stated before, the numerator is always positive. So we have to show that the denominator is also positive. Noticing that the denominator is the first order derivative

[11] Descartes' Rule of Signs is a method of determining the maximum number of positive and negative real roots of a polynomial. Let $P(x)$ be a polynomial with real coefficients written in descending order of x. Then it follows that :

(1) the number of positive roots of $P(x) = 0$ is either equal to the number of variations in sign of $P(x)$, or less than this by an even integer.

(2) Number of negative roots of $P(x) = 0$ is either equal to the number of variations in sign of $P(-x)$, or less than by an even integer.

of $F(\beta)$ evaluated at β^*, that is, $F'^*) = 3(\beta^*)^2 - c + c^2$. In the above proof of Proposition 1, we already showed that β^* can only appear at the increasing part of polynomial $F(\beta)$ for $\beta \in (0,1)$. In other words, we must have $F'^*) > 0$. That finishes the proof.

Appendix C. Proof of $\frac{dq^*}{dc} > 0$

Rewrite the total differential as following

$$\frac{dq^*}{dc} = -\frac{1}{2\beta^*} + \frac{c}{2(\beta^*)^2}\left(\frac{(1-2c)\beta^* + 2c}{3(\beta^*)^2 - c + c^2}\right) = \frac{1}{2\beta^*}\left[\frac{c}{\beta}\left(\frac{(1-2c)\beta^* + 2c}{3(\beta^*)^2 - c + c^2}\right) - 1\right].$$

Denote

$$H(\beta^*, c) = \frac{c}{\beta}\left(\frac{(1-2c)\beta^* + 2c}{3(\beta^*)^2 - c + c^2}\right) - 1.$$

From the fact that

$$(\beta^*)^3 - (c - c^2)\beta^* - c^2 = 0,$$

we have

$$3(\beta^*)^3 = 3(c - c^2)\beta^* + 3c^2 > 0.$$

Substituting the above into $H(\beta^*, c)$ and rearranging terms, we obtain

$$\begin{aligned}
H(\beta^*, c) &= \frac{\beta^*(c - 2c^2) + 2c^2}{3(c - c^2)\beta^* + 3c^2 - (c - c^2)\beta^*} - 1 \\
&= \frac{2c^2 + (c - 2c^2)\beta^*}{3c^2 + 2(c - c^2)\beta^*} - 1 \\
&= -\frac{c^2 + c\beta^*}{3c^2 + 2(c - c^2)\beta^*} \\
&< 0
\end{aligned}$$

Thus, $\frac{dq^*}{dc} < 0$. We finish the proof.

Appendix D. Proof the sign of $\frac{dU^*}{dc}$

(4.20) can be rewritten as

$$\begin{aligned}
\frac{dU(\beta_i^*, c)}{\partial c} &= \left(1 - \frac{c}{\beta_i^*}\right)\left[\frac{\left(\left(\frac{1}{2} - c\right)\beta_i^* + c\right)\left(1 + \frac{c}{\beta_i^*}\right)}{3(\beta_i^*)^2 - c + c^2} - 1\right] \\
&= \underbrace{\left(1 - \frac{c}{\beta_i^*}\right)}_{>0}\left[\frac{(\beta_i^* + c)\left(\left(\frac{1}{2} - c\right)\beta_i^* + c\right) - \beta_i^*\left(3(\beta_i^*)^2 - c + c^2\right)}{\underbrace{\beta_i^*\left(3(\beta_i^*)^2 - c + c^2\right)}_{>0}}\right]
\end{aligned}$$

Denote the numerator as $H: H \equiv (\beta_i^* + c)\left(\left(\frac{1}{2} - c\right)\beta_i^* + c\right) - \beta_i^*\left(3(\beta_i^*)^2 - c + c^2\right)$ .Then after some manipulations we get:

$$H(\beta_i^*, c) = \left(\frac{1}{2} - c\right)\left(\underbrace{(\beta_i^*)^2 - c\beta_i^* - \frac{4c^2}{1-2c}}_{=h}\right)$$

We discuss two cases when $0 < c < \frac{1}{2}$ and when $\frac{1}{2} < c < 1$ separately. First denote the second bracket in RHS as $h: h \equiv (\beta_i^*)^2 - c\beta_i^* - \frac{4c^2}{1-2c}$. It follows that $\Delta(h) = c^2 + \frac{16c^2}{1-2c} = c^2 \cdot \frac{17-2c}{1-2c}$. It can easily be verified that

- when $0 < c < \frac{1}{2} : \Delta(h) > 0$.

  To see this, note that when $0 < c < \frac{1}{2} : 1 - 2c > 0$ and $17 - 2c > 0$. In this case h has two real roots:

$$\beta_2 = \frac{c - \sqrt{\Delta(h)}}{2} < 0, \beta_1 = \frac{c + \sqrt{\Delta(h)}}{2} > 0.$$

Therefore,

$$\frac{dU(\beta_i^*, c)}{\partial c} < 0 \text{ when } 0 < c < \frac{1}{2} \text{ and } 0 < \beta_i^* < \beta_1$$
$$\frac{dU(\beta_i^*, c)}{\partial c} > 0 \text{ when } 0 < c < \frac{1}{2} \text{ and } \beta_1 < \beta_i^* < 1.$$

- when $\frac{1}{2} < c < 1 : \Delta(h) < 0$.

  This comes from the fact that when $\frac{1}{2} < c < 1 : 1 - 2c < 0$ and $17 - 2c > 0$. In this case h has no real roots, thus $h > 0, \forall c$. As a consequence $H(\beta_i^*, c) < 0$ and $\frac{dU(\beta_i^*, c)}{\partial c} < 0$.

Summerizing the above analysis:

$$\frac{dU(\beta_i^*, c)}{\partial c} \begin{cases} < 0 \text{ when } \begin{cases} 0 < c < \frac{1}{2} \text{ and } 0 < \beta_i^* < \beta_1 \\ \frac{1}{2} < c < 1, \end{cases} \\ > 0 \text{ when } 0 < c < \frac{1}{2} \text{ and } \beta_1 < \beta_i^* < 1. \end{cases}$$

We thus finish the proof.