How many bits should be reported in quantized cooperative spectrum sensing?

Nhan Nguyen-Thanh, Philippe Ciblat, Sina Maleki and Van-Tam Nguyen

Abstract—We introduce an algorithm for optimizing sensing parameters including the number of sensing samples and the number of reporting bits of a quantization-based cooperative spectrum sensing scheme in cognitive radio networks. This is obtained by maximizing the network throughput subject to a target detection probability. With Rayleigh fading and energy detector, the proposed algorithm simultaneously optimizes the number of sensing samples at a local node, the number of bits for quantizing local sensing data and the global threshold at a fusion center.

Index Terms—cognitive radio, spectrum sensing, cooperative, multibit decision, quantization, sensing-throughput tradeoff.

I. INTRODUCTION

Cognitive radio (CR), which enables secondary access to licensed bands, is a promising candidate for enhancing the utilization of the scarce spectrum resource in future communication systems. A secondary user can be permitted to use licensed spectrum, provided that it does not interfere with any primary users. This means that CR should be able to exploit spectrum holes by detecting them and using them in a cognitive manner. A widespread approach for characterizing the spectrum usage of primary systems is the so-called spectrum sensing [1], [2].

Spectrum sensing at terminals may not provide sensing results as accurate as required because of deep shadowing or fading. To deal with this problem, a fusion center (FC) collects sensing information from multiple terminals to eventually obtain a more reliable decision. This method is called cooperative spectrum sensing [3]–[5]. Main works related to cooperative sensing dealt with the design of local sensing algorithm, the combination of the local parameters at the FC (see [1] and references therein). In contrast, only a few works have been devoted to the optimization of the whole secondary system, especially by finding the trade-off between the duration of the sensing step and that of the data transmission step [6]–[8]. An efficient way to exhibit this trade-off is to maximize the throughput [6] with respect to the sensing duration with 1-bit hard decision and conventional fusion rules. However, the duration for reporting local information from each CR to the FC, which is linearly related to the quantizer resolution of the local decision, has never been optimized. The reporting step for 1-bit hard decision and even soft decision has been taken into account only through sensing performance [3], [8]–[10]. Obviously, if the reporting time is too short and so carries a degraded version of the local information (in the worst case, 1 bit), the sensing decision at the FC may not be reliable and the whole system may not perform well. In contrast, if the reporting time is too long, the time devoted to data transmission may be too short and the required data rate may not be fulfilled. As a consequence, this paper deals with the number of bits allowed for quantizing local sensing information. This means that our work has a strong connection to the problem of selecting hard decision or soft decision in [5], [8].

The trade-off between the sensing process length and the utilization channel time is investigated by formulating an optimization problem of maximizing the network throughput under the constraint of primary system protection requirement. The algorithm to find the optimal number of sensing samples and the optimal number of reporting bits is proposed.

II. SYSTEM MODEL

We consider a CR network with $K$ users. The CR network utilizes opportunistic spectrum access for sharing spectrum bands with primary systems. Cooperative sensing is adopted to detect primary users. The cooperative sensing scheme includes two steps. The first step consists of spectrum sensing of CR users. The second step is sensing result reporting to the FC, which makes a final decision on the primary user state.

In this work, we consider the energy detection method for the first step because of its simple implementation and its robustness to unknown information of the source signal and channel fading [11], [12]. For the second step, the reporting is done through a control channel with a fixed limited bandwidth [9], [13]. Since every methods of orthogonal multiple access, e.g., Time Division Multiple Access (TDMA), Frequency Division Multiple Access, etc., offer the same spectral efficiency, they can be used equivalently. For the sake of simplicity of the presentation and without lost of generality, we consider TDMA scheme and a soft data fusion rule with multi-bit local decisions at the FC. The structure of the operation frame is illustrated in Fig. 1.

![Fig. 1. Frame structure of the CR network.](image)

The local spectrum sensing is a binary hypothesis testing problem as follows

$$y_k[n] = \begin{cases} w_k[n], & H_0, \\ h_k s[n] + w_k[n], & H_1 \end{cases} \quad k = 1, 2, \cdots, K$$

(1)
functions (cdf) of the test are thus computed by

\[ E \text{ the symbol variance } \]

and

\[ H \text{ under } \]

with variance

\[ w \]

where \( y \) \( P \) \( Q \)

is the block-fading channel gain between the primary user and the \( k \)-th CR user. \( N \) is the number of sensing samples, \( (N = f_s T_S) \), where \( f_s \) is the sampling frequency and \( T_S \) is the sensing time. \( H_0 \) and \( H_1 \) represent the hypotheses of the absence and the presence of primary signal, respectively. We consider that the channel is a slow Rayleigh flat fading with variance \( \sigma_w^2 \). The channel realization is generated independently frame by frame as done in [11], [12].

The test statistic of the energy detector is given by

\[ Z_k = \frac{\gamma_k}{\sigma_w^2} E_k \]

where \( \gamma_k = |h_k|^2 E_k / \sigma_w^2 \) is the instantaneous Signal-to-Noise Ratio (SNR) of the received signal at the \( k \)-th user with the symbol variance \( E_k \). Given \( h_k \), the cumulative density functions (cdf) of the test are thus computed by

\[
F_{z_k|H_0}(z | H_0) = P_N(z/2) \quad (2)
\]

\[
F_{z_k|H_1,h_k}(z | H_1, h_k) = 1 - Q_N\left(\sqrt{2N\gamma_k}, \sqrt{z}\right) \quad (3)
\]

where \( Q_N(\ldots) \) denotes the generalized Marcum Q-function, \( P_N(b) = \gamma(N, b)/\Gamma(N) \) with the gamma function \( \Gamma(\ldots) \) and the incomplete gamma function \( \gamma(\ldots, \ldots) \). As \( h_k \) is a Rayleigh channel, the SNR \( \gamma_k \) follows an exponential probability density function (pdf) given by

\[ f(\gamma_k) = \frac{1}{\gamma_k} \exp(-\gamma_k/\gamma_k), \quad \gamma_k = \frac{\sigma_w^2 E_k}{h_k} \]

which \( \gamma_k = \sigma_w^2 E_k / \sigma_w^2 \) is the average SNR received at the \( k \)-th user. Using Eq. (9) in [12] and Section 8.35 in [15], we obtain the cdf and the pdf of \( z_k \) under \( H_1 \) as follows.

\[
F_{z_k|H_1}(z | H_1) = P_N\left(\frac{z}{2}\right) - e^{-z\gamma_k/\gamma_k} \frac{\gamma_k}{2M_k} P_{\nu}\left(\frac{\gamma_k}{2M_k}, \nu, z\right) \quad (4)
\]

\[
f_{z_k|H_1}(z | H_1) = \frac{e^{-z\gamma_k/\gamma_k}}{2M_k N\gamma_k} \frac{\gamma_k}{2M_k} P_{\nu}\left(\frac{\gamma_k}{2M_k}, \nu, z\right) \quad (5)
\]

where \( M_k = 1 + 1/(N\gamma_k) \) and \( \nu = N - 1 \).

The cdf of \( z_k \) under \( H_0 \) is the same as that of Eq. (2). Since it is independent of the fading, its pdf is given by

\[ f_{z_k|H_0}(z | H_0) = \frac{z^{N-1} e^{-z/2}}{2^{N}\Gamma(N)}. \quad (6) \]

After the sensing period, each energy test is reported to the FC, where a squared-law combining is adopted [12], and the global test is then given by

\[ Z = \sum_{k=1}^{K} h_k z_k \overset{\text{H}_0}{\overset{\text{H}_1}{\sim}} \eta \quad (7) \]

where \( \eta \) is the decision threshold.

III. QUANTIZED COOPERATIVE SENSING

Reporting a raw \( z_k \) requires time, bandwidth and energy. It is therefore relevant to communicate with the FC through a quantized version of the test statistic, which corresponds to work with a multi-bit decision at the local nodes. The real-valued (also called raw or soft) energy \( z_k \) is replaced with its \( B \)-bit quantized version in Eq. (7). The practical test at the FC then becomes

\[ Z^{(B)} = \sum_{k=1}^{K} z_k^{(B)} \overset{\text{H}_0}{\overset{\text{H}_1}{\sim}} \eta^{(B)} \quad (8) \]

where \( z_k^{(B)} = Q_k^{(B)}(z_k) \) is the quantized version of \( z_k \) and \( Q_k^{(B)} \) denotes a \( B \)-bit quantizer associated with the \( k \)-th user. Let \( M \) the number of quantization levels, then \( M = 2^B \). Let \( \{t_{k,i}\}^M_{i=0} \) and \( \{L_{k,j}\}^M_{j=1} \) the set of thresholds and the set of quantization levels for \( Q_k^{(B)} \), respectively. As the support of the pdf of \( z_k \) is \( \mathbb{R}_+ \), we have \( t_{k,0} = 0, t_{k,M} = +\infty \), and \( R_{k,i} = \{t_{k,i-1}, t_{k,i}\}, i = 1, \ldots, M - 1, \) and \( R_{k,i} \) denotes the \( i \)-th quantization region of the \( k \)-th user. The quantization level is usually the central point of the quantization region. Hence, we have

\[ L_{k,i} = \frac{1}{S_{k,i}} \int_{R_{k,i}} f_{z_k}(z) dz \quad (9) \]

where \( S_{k,i} = \int_{R_{k,i}} f_{z_k}(z) dz \) and \( f_{z_k} = \frac{\pi_0 f_{z_k|H_0} + (1 - \pi_0) f_{z_k|H_1}}{\pi_0} \) the probability of primary user inactivity.

The following quantizers are hereafter considered:

- **Uniform quantizer**: The quantization thresholds are given by \( t_{k,i} = t_{k,i-1} + \Delta_k, i = 1, \ldots, M - 1, \) where \( \Delta_k = t_{k,\text{max}} / M \). \( t_{k,\text{max}} \) is an artificial threshold for defining a maximum support of \( z_k \). Here, it is selected such that \( f_{0} f_{z_k|H_0} = 10^{-6} \).

- **Minimum mean square error (MMSE) quantizer** [16]: This quantizer aims at minimizing the quantization error. The levels and thresholds (with \( t_{k,i} = (L_{k,i} + L_{k,i+1})/2 \)) can be found by using Lloyd-Max algorithm.

- **Maximum entropy (ME) quantizer** [17]: The quantization thresholds \( t_{k,i} \) are obtained by forcing \( S_{k,i} = 1/M, \forall i = 1, \ldots, M \).

In order to perform the quantization and the dequantization, the local user \( k \) needs its quantization thresholds, and the FC needs the pdf of \( z_k \). If the coherence time of the statistics of \( z_k \) is large enough, the report of the pdf from the user to the FC will be rarely performed. When the report can not be implemented or when the statistics of \( z_k \) can not be archived, \( z_k \) can be considered as a uniformly distributed process, and so the quantizer with \( L_i = (i - 1/2)\Delta \) (where \( \Delta \) is a pre-defined term independent of the user) is well adapted.

To determine the threshold \( \eta^{(B)}(\ldots) \), the probability mass function (pmf) of \( Z^{(B)} \) under \( H_0 \) and \( H_1 \) is needed. Since the test at the FC, given by Eq. (8), is the sum of the \( K \) local independent tests, its pdf, denoted by \( f_{Z^{(B)}|H_j} \), is obtained by

\[ f_{Z^{(B)}|H_j} = f_{z^{(B)}_1|H_j} f_{z^{(B)}_2|H_j} \cdots f_{z^{(B)}_K|H_j} \quad (10) \]

where * denotes the convolution operator, and \( f_{z^{(B)}_k|H_j} \) is the pmf of \( z^{(B)}_k \) under \( H_j \) and is given by

\[ f_{z^{(B)}_k|H_j}(\ell) = \sum_{i=1}^{M} S_{k,i|H_j} \delta(\ell - L_{k,i}) \quad (11) \]
with $S_{k,i|H_j} = \int_{\mathbb{R}_+} f_{z_k|H_j}(z)dz$ and $\delta(\bullet)$ is the Dirac delta function. Substituting (11) into (10) leads to

$$f_{Z^{(0)}|H_j}(\ell) = \sum_{i_1 \ldots i_K = 1}^N S_{1,i_1|H_j} \ldots S_{K,i_k|H_j} \delta(\ell - L_{1,i_1} \ldots - L_{K,i_k}).$$

So $f_{Z^{(0)}|H_j}$ is a pmf, where the $q$-th level is denoted $L_q$. Thus,

$$f_{Z^{(0)}|H_j}(\ell) = \sum_{q} \psi_q|H_j \delta(\ell - L_q)$$

(12)

where $\psi_q|H_j$ is the probability of the level $L_q$ and is given by $\psi_q|H_j = \sum_{i_1 \ldots i_K \in N} S_{1,i_1|H_j} \ldots S_{K,i_k|H_j}$ with $L_q = \{i_1 \ldots i_K | L_{1,i_1} + \ldots + L_{K,i_k} = L_q \}$. The algorithms for computing $\psi_q|H_j$ and $L_q$ are presented in [18].

Given the pmf of $Z^{(B)}$, the false-alarm and the detection probabilities of the test can be expressed by

$$P_F(B, \eta^{(B)}) = \sum_{q L_q \leq \eta^{(B)}} \psi_q|H_0,$$

(13a)

$$P_D(B, \eta^{(B)}) = \sum_{q L_q \geq \eta^{(B)}} \psi_q|H_1$$

(13b)

IV. OPTIMAL QUANTIZED COOPERATIVE SENSING

According to [6], the normalized throughput of a CR network is approximately given by

$$R(N, B, \eta) \propto \left(1 - \frac{N}{Tfs} - \frac{KB}{Tfr} \right) \left(1 - P_F \right).$$

(14)

For the CR network with $K$ users, the throughput for the secondary user strongly depends on the cooperative sensing process, especially on the following parameters: the number of sensing samples, the number of reported bits and the optimal threshold of the global test. Therefore, optimizing these parameters to maximize the network throughput for a target detection probability $P_D^{(0)}$ is necessary. This optimization is then formulated as

$$[N_s, B_s, \eta_s] = \arg \max_{N,B, \eta} R(N, B, \eta), \quad \text{s.t. } P_D \geq P_D^{(0)}.$$  

(15)

For a certain integer value of $N$, the number of reported bits $B$, which is also an integer, is necessarily less than $B_{\text{max}}$ with $B_{\text{max}} = \lfloor (T - N/\text{fs}) f_r/K \rfloor$. In addition, $N < N_{\text{max}}$ with $N_{\text{max}} = T/\text{fs}$. Thus, the optimal solution can be obtained by a discrete search along with both $N$ and $B$. Therefore, for a given pair of $[N, B]$, the optimization in (15) leads to

$$\eta_s^{(N,B)} = \arg \min_{\eta^{(N,B)}} P_F(N, B, \eta^{(N,B)}) \quad \text{s.t. } P_D > P_D^{(0)}.$$  

(16)

Thanks to Eq. (13), it is equivalent to

$$\eta^*_s^{(N,B)} = \arg \min_{\eta^{(N,B)}} \sum_{q L_q \geq \eta^{(N,B)}} \psi_q|H_0$$

s.t. \quad \sum_{q L_q \geq \eta^{(N,B)}} \psi_q|H_1 \geq P_D^{(0)}.$$  

(17a)

Since the sums in Eqs. (17a) and (17b) decrease with respect to $\eta_s^{(N,B)}$, the optimal $\eta_s^{(N,B)}$ is equal to the maximum level $L_q$. Consequently, the algorithm for finding $\{N_s, B_s, \eta_s\}$ is given as Algorithm 1.

**Algorithm 1** Find $\{N_s, B_s, \eta_s\}$

1: for $n = 2$ to $N_{\text{max}}$
2: for $b = 1$ to $B_{\text{max}}$
3: Compute $f_{Z^{(0)}|H_j}$ for $j = 0, 1$ as in Eq. (12)
4: Let $q^{\text{max}}$ the number of levels in $f_{Z^{(0)}|H_j}$
5: $q \leftarrow q^{\text{max}}$
6: repeat
7: $q \leftarrow q - 1$
8: until $\sum_{q} \psi_q|H_j \geq P_D^{(0)}$
9: $\eta_s^{(n,b)} \leftarrow L_q$, compute $R(n, b, \eta_s^{(n,b)})$
10: end for
11: end for
12: $\{N_s, B_s, \eta_s\} \leftarrow \arg \max_{n,b} R(n, b, \eta_s^{(n,b)})$

The proposed algorithm should run only when channel statistics (actually the average received SNRs at local users) have changed. Algorithm 1, including the computation of $f_{Z^{(0)}|H_j}$, is performed at the FC. The preliminary parameters for the computation of $f_{Z^{(0)}|H_j}$, i.e., the thresholds $\{t_{k,i}\}_{i=0}^M$, the levels $\{L_{k,j}\}_{j=1}^M$ and the mass coefficients $\{S_{k,j}\}_{j=1}^M$, can be either computed at local users and then sent to the FC, or directly computed at the FC after having received the average SNR from the local users. In both cases, the FC finally sends the optimized quantizer’s configuration back to each local user.

Our work is valid for Rayleigh fading and energy detector. The extension for other fading channels is straightforward if $f_{z_k|H_j}$ is available in closed-form (e.g. energy detector along with a Nakagami channel [19]). When $f_{z_k|H_j}$ cannot be derived readily, the proposed algorithm can be adopted if the quantized version $f_{z^{(B)}|H_j}$ is achievable, e.g., based on numerical or empirical method, and stored in a lookup table.

V. NUMERICAL RESULTS

Unless otherwise stated, the CR network has 6 nodes and the average SNR values are -20, -18, -16, -14, -12, and -10 dB, the target probability of detection $P_D^{(0)}$ is 0.9, the frame length $T$ is 1ms, the sampling frequency $\text{fs}$ is 6MHz, and the reporting channel bandwidth $f_r$ is 100kHz. The variance of the Rayleigh channel is chosen according to the SNR value.

In Fig. 2, we plot the normalized throughput versus $B$ for different SNR configurations and $N = 500$. The normalized throughputs for all considered scenarios and quantization methods have the same shape and exhibit a maximum. When the number of reported bits is too small or too high, the throughput is low, due to the weak accuracy of the sensing or to the increase of the reporting time, respectively. We can see that the gaps between the maximum throughput points of the three quantizers are small, and the optimal numbers of reported bits for the three quantizers are close to each other.

In Fig. 3, we display the normalized throughput versus $N$ and $B$, when ME quantizer method is employed. The
SNR. The sensing time and hence the optimal global threshold depend more strongly on the SNR than on the reporting time.

VI. Conclusion

We maximized the throughput subject to a target detection probability with respect to the number of sensing samples and the number of reported bits. The proposed algorithm provides the method for selecting these parameters optimally. Reporting only a few bits is in general optimal.

Acknowledgment

The research leading to these results has received funding from the EC’s FP7/2007-2013 under Marie Curie Fellowship CORPA, from French industry ministry in Catrene CORPA project, and from Luxembourgish national funding FNR SENT.

References


