Calculation of the pitch with the Cano method

When using a lens of radius $R$ on top of a flat substrate, both inducing uniform planar alignment, the pitch of the cholesteric liquid crystal can be calculated from the radii $r$ of the circular defects according to the following equation, which can be determined from geometrical considerations:

$$\frac{p}{2} = \sqrt{R^2 - r_n^2} - \sqrt{R^2 - r_{n+1}^2}$$

<table>
<thead>
<tr>
<th>$r_n$ / µm</th>
<th>$\sqrt{R^2 - r_n^2}$</th>
<th>$p$ / nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>139</td>
<td>22999,581</td>
<td>360</td>
</tr>
<tr>
<td>166</td>
<td>22999,401</td>
<td>342</td>
</tr>
<tr>
<td>188</td>
<td>22999,229</td>
<td>307</td>
</tr>
<tr>
<td>206</td>
<td>22999,076</td>
<td>347</td>
</tr>
<tr>
<td>225</td>
<td>22998,902</td>
<td></td>
</tr>
</tbody>
</table>

Calculation of the wavelength reflected by the $N^*$ phase confined in cylindrical fibres

When confined inside a cylindrical fibre according to the geometry in Fig. 8c the cholesteric liquid crystal must adapt its helical director modulation to fit the cylinder diameter $d$, in the sense that an integer number of full helix turns (pitches $p$) fit in the cylinder, i.e.

$$d = np$$  \hspace{1cm} (1)

where $n$ is an integer. This relation is fulfilled by the natural pitch $p_0$ of the helix only in exceptional cases, hence the helix will be either compressed or expanded to fulfil (1) with a value of the pitch $p$ that is as close to $p_0$ as possible.

Under these constraints, the effective number of helix turns $i$ for a given natural pitch $p_0$ and a given cylinder diameter $d$ can be calculated as:

$$i := \text{Floor}\left((d+p_0/2) / p_0\right);\hspace{1cm} (2)$$

Note that for $d < p_0/2$ the helix is assumed to be unwound, yielding $i = 0$ in (2). For $d \geq p_0/2$ the effective pitch is thus $p = d/i$ with $i$ calculated according to (2), yielding selective reflection for light with a wavelength $d/i$ in the liquid crystal medium. We finally achieve the reflected wavelength in air (its refractive index approximated as 1) by multiplying this wavelength by the average refractive index of our cholesteric liquid crystal, $\lambda = n_{N^*} d/i$, where $n_{N^*}$ has been determined experimentally (see above) to be about 1.36.

The full functions for obtaining Figures 10 and 11, written in the script language (resembling Pascal) of the plotting and fitting software Pro Fit (Quantumsoft), are included below.
function ConstrainedPitch;
defaults
    \[a[1] := 340, \text{active,'p_0',1, INF;}\]
    \[a[2] := 1.36, \text{active,'n',1,2;}\]
var
    NumberOfPitches: integer;
    NaturalPitch, d, RefIndex: real;
begin
    d:=x;
    NaturalPitch:=a[1];
    RefIndex:=a[2];
    NumberOfPitches:= \text{Floor}((d+NaturalPitch/2) / NaturalPitch);
    \text{if} \text{NumberOfPitches}>0 \text{ then } y:= \text{RefIndex}d/\text{NumberOfPitches};
end;

function ConstrainedPitchFixd;
defaults
    \[a[1] := 500, \text{active,'d',1, INF;}\]
    \[a[2] := 1.5, \text{active,'n',1,10;}\]
var
    NumberOfPitches: integer;
    NaturalPitch, d, RefIndex: real;
begin
    NaturalPitch:=x;
    d:=a[1];
    RefIndex:=a[2];
    NumberOfPitches:= \text{Floor}((d+NaturalPitch/2) / NaturalPitch);
    \text{if} \text{NumberOfPitches}>0 \text{ then } y:= \text{RefIndex}d/\text{NumberOfPitches};
end;