Shape Optimization Directly from CAD: an Isogeometric Boundary Element Approach

Haojie Lian

Supervisors: Professor S.P.A. Bordas
Dr. P. Kerfriden
Dr. R.N. Simpson

Advanced Materials and Computational Mechanics Group
The School of Engineering, Cardiff University
1. Motivation

2. CAD techniques (B-splines, NURBS, T-splines)

3. Isogeometric boundary element methods in shape optimization

4. Conclusions
Mesh Burden in Shape Optimization

Meshing:
1. Time consuming.
2. Human intervention.
3. CAD recovery from mesh.

FEM optimization process

80% of the time!
1. Separate the CAD from the analysis

- Meshfree Methods (Belytschko et al. 1994)
- XFEM (Belytschko, et al., 1999)

2. Integrate the CAD and the analysis

- Isogeometric Analysis (Hughes et al., 2005)
Isogeometric Boundary Element Methods (IGABEM)

Use the same basis functions in CAD to discretize Boundary Integral Equations (BIE). (Simpson et al., 2010)

1. Seamlessly compatible with CAD due to the boundary representation.

2. No meshing and no CAD model recovery step throughout the optimization.

3. The control points can be naturally chosen as design parameters.

IGABEM shape optimization flowchart
B-splines

Knot vector

\[ x(\xi) = \sum_A N_A(\xi) P_A \]

NURBS curve

NURBS surface
B-spline Basis

B-spline basis

\[ N_{A,0}(\xi) = \begin{cases} 1, & \text{if } \xi_A \leq \xi < \xi_{A+1}, \\ 0, & \text{otherwise}, \end{cases} \]

\[ N_{A,p}(\xi) = \frac{\xi - \xi_A}{\xi_{A+p} - \xi_A} N_{A,p-1}(\xi) + \frac{\xi_{A+p+1} - \xi}{\xi_{A+p+1} - \xi_{A+1}} N_{A+1,p-1}(\xi). \]

Fig 1. B-spline basis (left) vs quadratic polynomials (right)

Linear independence.
The partition of unity.
Locally supported.
No Kronecker delta property.
Non-Uniform B-splines (NURBS) is obtained by

\[
x(\xi) = \sum_{A=1}^{n} R_{A,p}(\xi) P_A
\]

and

\[
R_{A,p}(\xi) = \frac{N_{A,p}(\xi) w_A}{\sum_{A=1}^{n} w_A N_{A,p}(\xi)} \quad \text{and} \quad P = \{x, y, z, w\}^T
\]

where \( R \) is the NURBS basis function, \( N \) is the B-spline basis function, and \( w \) is the weight.
T-splines

NURBS (source: Rhino3D website)

T-spline (source: Rhino3D website)

Fig. 1: NURBS mesh topology

Fig. 2: T-spline mesh topology
Bézier Extraction

Bézier extraction formulation (Borden, 2011 and Scott, 2012):

\[ \mathbf{N} = \mathbf{C}\mathbf{B} \]

\( \mathbf{N} \): NURBS basis functions.
\( \mathbf{C} \): Bézier extraction coefficient matrix (vary from element to element).
\( \mathbf{B} \): Bézier basis functions (the same for all elements).

1. Easy to be incorporated into the existing FEM/BEM code.
2. Accelerate basis function evaluation time.
3. The extraction coefficients do NOT depend on the control points.

Fig. 1: Bézier extraction (source: Scott, 2011, IJNME)
Boundary Integral Equation (BIE)

\[ C_{ij}(s)u_j(s) + \int_S T_{ij}(s, x)u_j(x)\,dS(x) = \int_S U_{ij}(s, x)t_j(x)\,dS(x) \]

Discretize BIE with CAD basis function \( R \),

\[ u_j(\xi) = R_A(\xi)\tilde{u}^A_j \]

\[ t_j(\xi) = R_A(\xi)\tilde{t}^A_j \]

After discretization, matrix equations are

\[ Hu = Gt \]

The main implementations challenges:

1. Singular Integrals.
2. Jump terms.
1. Regularized Boundary Integral Equation

\[ \int_S T_{ij} (s, x) [u_j (x) - u_j (s)] \, dS (x) = \int_S U_{ij} (s, x) t_j (x) \, dS (x) \]

No jump terms.
No strongly singular integrals.
Also available for sensitivity analysis.
Reduce to rigid body motion method.

Impose boundary conditions in "weak" sense.

2. Boundary condition enforcement:

\[ \int_{S_u} R^T u \, dS = \int_{S_u} R^T \tilde{u} \, dS \quad \text{on } S_u, \]
\[ \int_{S_t} R^T t \, dS = \int_{S_t} R^T \tilde{t} \, dS \quad \text{on } S_t, \]

Impose boundary conditions in "weak" sense.
IGABEM Sensitivity Formulation

Recall the regularized BIE:

\[ \int_S T_{ij} (s, x) [u_j (x) - u_j (s)] \, dS(x) = \int_S U_{ij} (s, x) t_j (x) \, dS(x) \]

Direct differentiate the above equation:

\[ \int_S \left\{ \dot{T}_{ij} (s, x) [u_j (x) - u_j (s)] + T_{ij} (s, x) [\dot{u}_j (x) - \dot{u}_j (s)] \right\} \, dS(x) \]

\[ + \int_S T_{ij} (s, x) [u_j (x) - u_j (s)] [dS'(x)] \]

\[ = \int_S \left[ \dot{U}_{ij} (s, x) t_j (x) + U_{ij} (s, x) \dot{t}_j (x) \right] \, dS(x) \]

\[ + \int_S U_{ij} (s, x) t_j (x) [dS'(x)]. \]

\[ \dot{H}u + H\dot{u} = \dot{G}t + G\dot{t} \]

Critical points: analytical sensitivities of Green's functions \( \dot{T}_{ij}, \dot{U}_{ij} \).
**IGABEM Analysis (NURBS)**

Fig. 1: Problem definition

Fig. 2: Deformation

Fig. 3: Geometry description

Fig. 4: Convergence study

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**Notes:**
- Fig. 1: Problem definition
- Fig. 2: Deformation
- Fig. 3: Geometry description
- Fig. 4: Convergence study
IGABEM Analysis (T-splines)

Fig. 1: Propeller CAD model

Fig. 2: Displacement

Fig. 3: Stress

Fig. 4: Stress close-up
Sensitivity Analysis of Lamé problem

Problem definition

Displacement sensitivities on AB

Stress sensitivities on AB
Sensitivity Analysis of Kirsch problem

Problem definition

Design model

Displacement sensitivities on AB

Stress sensitivities on AB
Shape Optimization of a 2D Cantilever

Fig. 1: Problem definition

Fig. 2: Design model

Fig. 3: Optimal model

Fig. 4: Analysis model
Shape Optimization of a Fillet

Fig. 1: Problem definition

Fig. 2: Design model

Fig. 3: Analysis model

Fig. 4: Optimal models
Shape Optimization of a Connecting Rod

Fig. 1: Problem definition

Fig. 2: Analysis model

Fig. 3: Optimal shape

Fig. 4: Design model
Problem definition
Sensitivity Analysis of Cavity Problem

Displacement sensitivity comparison

Cavity problem

Displacement sensitivity comparison
Shape Optimization of a Hammer

Fig. 1: Problem definition

Fig. 2: CAD model

Fig. 3: Initial geometry

Fig. 4: Optimal geometry
Shape Optimization of a T-shape component

Fig. 1: CAD model

Fig. 2: Initial geometry

Fig. 3: Optimal geometry
Shape Optimization of a Chair

Fig. 1: CAD model

Fig. 2: Initial geometry

Fig. 3: Optimal geometry
Conclusions and Future Work

A shape optimization scheme using Isogeometric Boundary Element Methods (IGABEM) was presented, which possesses the following advantages:

1. Seamless integration with CAD.
2. No meshing during the steps throughout the iterative steps.
3. The CAD provides a natural parametrization for optimization.
4. NURBS for 2D and T-splines for 3D, so a water-tight geometry and local refinement is guaranteed.

Future work:
1. Acceleration algorithm, to save the memory and time.
2. Application in open domain problem optimization, such as acoustic and electromagnetics shape optimization.
Thank you!