Optimal mix of funded and unfunded pension systems: The case of Luxembourg

Jang SCHILTZ (University of Luxembourg)

joint work with
Jean-Daniel GUIGOU (University of Luxembourg),
& Bruno LOVAT (University of Lorraine)

2nd Luxembourg Workshop on Household Finance and Consumption

June 20, 2014
Outline

1 General context of the research project
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2. The salary trajectories in Luxembourg
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4. Outlook
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The Luxembourg pension system

Pay-as-you-go system + creation of a reserve (1.5 times the amount of the annual expenses).

Very high replacement rate (over 90%).

Due to several reasons (longevity risk and labor market explosion in the 1990s) the system is not sustainable.

Reform possibilities:

▶ Parameter adjustment in the Pay-as-you-go system
▶ and/or development of complementary systems (mix of funded and unfunded system)
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Our research project

We have analyzed a mix of funded and unfunded pension system:
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- a unique database
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- an innovative statistical methodology
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- a theoretical model based on a diversification principle
The data

Salaries of workers in the private sector in Luxembourg from 1940 to 2006.
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About 7 million salary lines corresponding to 718 054 workers.
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- gender (male, female)
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A statistical methodology based on homogeneous groups

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We try to divide the population into a number of homogenous sub-populations and to estimate, at the same time, a typical trajectory for each sub-population.
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Finite mixture model (Daniel S. Nagin (Carnegie Mellon University))
- mixture: population composed of a mixture of unobserved groups
- finite: sums across a finite number of groups
The finite mixture model

If the data are (censored) normally distributed

\[ L = \sigma N \prod_{i=1}^{r} \sum_{j=1}^{\sigma} \pi_j T \prod_{t=1}^{\phi} \left( y_{it} - \beta_j x_{it} \right) \]
The finite mixture model

If the data are (censored) normally distributed

\[
L = \frac{1}{\sigma} \prod_{i=1}^{N} \sum_{j=1}^{r} \pi_j \prod_{t=1}^{T} \phi \left( \frac{y_{it} - \beta_j x_{it}}{\sigma} \right). \tag{1}
\]
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(1)

This is too complicated to get closed-forms solutions.
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Software:
SAS-based Proc Traj procedure
by Bobby L. Jones (Carnegie Mellon University).

⇒ quasi-Newton procedure maximum research routine
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Proc Traj procedure

20 years of work for workers beginning their career between 1982 and 1987
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20 years of work for workers beginning their carrier between 1982 and 1987

Selection of the time period for macroeconomic reasons (Crisis in the steel industry and emergence of the financial market place of Luxembourg)
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Proc Traj Macro:
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Selection of the time period for macroeconomic reasons (Crisis in the steel industry and emergence of the financial market place of Luxembourg)

Proc Traj Macro:

```
DATA TEST;
    INPUT ID O1-O20 T1-T20;
    CARDS;
    data
RUN;
```
Proc Traj procedure

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Selection of the time period for macroeconomic reasons (Crisis in the steel industry and emergence of the financial market place of Luxembourg)

Proc Traj Macro:

DATA TEST;
    INPUT ID O1-O20 T1-T20;
    CARDS;

data
RUN;

PROC TRAJ DATA=TEST OUTPLOT=OP OUTSTAT=OS OUT=OF OUTTEST=OE ITDETAIL;
    ID ID; VAR O1-O20; INDEP T1-T20;
    MODEL CNORM; MAX 8000; NGROUPS 6; ORDER 4 4 4 4 4 4;
RUN;
Results for 9 groups
Results for 9 groups

![Graph showing outcomes for different groups]

Group Percentages:
- 13.4
- 16.9
- 20.8
- 7.9
- 14.9
- 4.8

Outcome vs. Time (T)
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Working hypotheses

- Hypothesis 1. Every salary trajectory has a constant growth rate $\lambda_j$. 

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Working hypotheses

- Hypothesis 1. Every salary trajectory has a constant growth rate $\lambda_j$.

- Hypothesis 2. Let $d$ denotes the intergenerational demographical rate, i.e. at time $t$, if $N_0$ denotes the number of people beginning to work and $N_t$ the number of people working for $t$ years, then

$$N_t = \frac{N_0}{(1 + d)^t}.$$
Sustainability coefficient of the PAYG system

\[
\tau_1 = \frac{\text{sum of all salaries earned by active workers}}{\text{sum of all pensions paid to retirees at time } t}
\]

\[
\tau_1 = S_0 + \ldots + S_T (1 + d)^T + \ldots + k (1 + d)^T + \ldots + P_T (1 + d)^T + \ldots
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\[ \tau_1 = \frac{S_0 + \cdots + \frac{S_T}{(1+d)^T} k}{(1+d)^{T+1} P_{T+1} + \cdots + \frac{k}{(1+d)^{T+T^*}} P_{T+T^*}}. \]
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\]
Analysis of the current pension system
Analysis of the current pension system

Rate of necessary pension contributions to keep the system sustainable at the long run:

![Graph showing rate of pension contributions as a function of d]
Hypotheses 3. We suppose that every individual of group number $j$ invests every year of his activity a fixed amount $a_j$ which generates savings according to the market rate $i$. 
Sustainability coefficient of the funded system

\[ \tau_2 = \frac{\text{total sum earned by the individual during his period of activity}}{\text{sum of all the pensions that are paid to him thanks to the savings that he has accumulated}} \]
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\[ \tau_2 = \frac{\text{total sum earned by the individual during his period of activity}}{\text{sum of all the pensions that are paid to him thanks to the savings that he has accumulated}} \]

\[ \tau_2 = \frac{S_j}{a_j(i - \lambda_j)} \frac{(1 + i)^T - (1 + \lambda_j)^T}{(1 + i)^T - 1}. \]
Systemic risk

Modelisation based on portfolio type risk management principles
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Modelisation based on portfolio type risk management principles

<table>
<thead>
<tr>
<th></th>
<th>Market risk</th>
<th>Demographic risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Repartition</td>
<td>Negligeable</td>
<td>Extreme</td>
</tr>
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Global sustainability coefficient

\[ \tau = x\tau_1 + (1 - x)\tau_2 \]

is the number of euros necessary to pay 1 euro for the pension.

Here \( x \) euros come from the PAYG system and \( 1 - x \) euros from capitalization.
Global sustainability coefficient

\[ \tau = x\tau_1 + (1-x)\tau_2 \]

is the number of euros necessary to pay 1 euro for the pension.

Here \( x \) euros come from the PAYG system and \( 1 - x \) euros from capitalization.

We want to limit the risk of the hybrid system without reducing the pension and in the same time minimize the capitalization effort.
Gain of sustainability and optimal saving amount

\[ G(x) = \frac{\text{var}(\tau_1) - \text{var}[\tau(x)]}{\text{var}(\tau_1)} \]

measures the gain of sustainability of the mixed system with respect of the PAYG system.
Gain of sustainability and optimal saving amount

\[ G(x) = \frac{\text{var}(\tau_1) - \text{var}[\tau(x)]}{\text{var}(\tau_1)} \]

measures the gain of sustainability of the mixed system with respect of the PAYG system.

We suppose that the utility function \( U = U(a) \) of an active worker is decreasing in \( a \).
Gain of sustainability and optimal saving amount

Theorem. The value $x = x^*$ for which the utility function $U$ attains its maximum under the sustainability constraint

$$G(x) \leq G^*$$

is given by $x^* = 1 - G^*$. 
Gain of sustainability and optimal saving amount

Theorem. The value $x = x^*$ for which the utility function $U$ attains its maximum under the sustainability constraint $G(x) \leq G^*$ is given by $x^* = 1 - G^*$.

Moreover the individual needs a constant annual saving amount

$$a^* = \sqrt{\frac{G^* K}{\text{var}(\tau 1)(1 - G^*)}},$$

where $K = \frac{S_j}{a_j(i - \lambda_j)} \frac{i(1+i)^T - (1+\lambda_j)^T}{(1+i)^T - 1}$ depends on the salary trajectory.
Example

An individual worker wants to divide by 2 the variability of his PAYG sustainability constraint needs to save annually at least the following amount (depending on his salary evolution subgroup):

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<td>Annuity</td>
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<th>Curve 1</th>
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<th>Curve 4</th>
<th>Curve 5</th>
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<tr>
<td>$\lambda_1 = 3.07%$</td>
<td>$\lambda_2 = 0.96%$</td>
<td>$\lambda_3 = 1.45%$</td>
<td>$\lambda_4 = 2.82%$</td>
<td>$\lambda_5 = 0.19%$</td>
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<th>Curve 7</th>
<th>Curve 8</th>
<th>Curve 9</th>
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<td>$\lambda_6 = 2.58%$</td>
<td>$\lambda_7 = 1.28%$</td>
<td>$\lambda_8 = 0.48%$</td>
<td>$\lambda_9 = 1.09%$</td>
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A generalization of Nagin’s model

Let $x_1, \ldots, x_L$ and $z_{i1}, \ldots, z_{iT}$ be covariates potentially influencing $Y$.

We propose the following model:

$$
 y_{it} = \left( \beta_{j0} + \sum_{l=1}^{L} \alpha_{j0l} x_l + \gamma_{j0} z_{it} \right) + \left( \beta_{j1} + \sum_{l=1}^{L} \alpha_{j1l} x_l + \gamma_{j1} z_{it} \right) \text{Age}_{i,t} + \left( \beta_{j2} + \sum_{l=1}^{L} \alpha_{j2l} x_l + \gamma_{j2} z_{it} \right) \text{Age}_{2,i,t} + \left( \beta_{j3} + \sum_{l=1}^{L} \alpha_{j3l} x_l + \gamma_{j3} z_{it} \right) \text{Age}_{3,i,t} + \left( \beta_{j4} + \sum_{l=1}^{L} \alpha_{j4l} x_l + \gamma_{j4} z_{it} \right) \text{Age}_{4,i,t} + \epsilon_{it},
$$

where $\epsilon_{it} \sim N(0, \sigma)$, $\sigma$ being a constant standard deviation.
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Let $x_1 \ldots x_L$ and $z_{i_1}, \ldots, z_{i_T}$ be covariates potentially influencing $Y$. 

\[ y_{it} = \left( \beta_0 + \sum_{l=1}^{L} \alpha_0^l x_l + \gamma_0^l z_{it} \right) + \left( \beta_1 + \sum_{l=1}^{L} \alpha_1^l x_l + \gamma_1^l z_{it} \right) \times \text{Age}_{it} + \left( \beta_2 + \sum_{l=1}^{L} \alpha_2^l x_l + \gamma_2^l z_{it} \right) \times \text{Age}_{2it} + \left( \beta_3 + \sum_{l=1}^{L} \alpha_3^l x_l + \gamma_3^l z_{it} \right) \times \text{Age}_{3it} + \left( \beta_4 + \sum_{l=1}^{L} \alpha_4^l x_l + \gamma_4^l z_{it} \right) \times \text{Age}_{4it} + \epsilon_{it}, \]

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+ \left( \beta_4^j + \sum_{l=1}^{L} \alpha_{4l}^j x_l + \gamma_4^j z_{it} \right) \text{Age}_{it}^4 + \varepsilon_{it},
\]

where \(\varepsilon_{it} \sim \mathcal{N}(0, \sigma)\), \(\sigma\) being a constant standard deviation.
The new Database

**The data : second dataset** Salaries of all workers in Luxembourg which began to work in Luxembourg between 1980 and 1990 at an age less than 30 years.
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1,303,010 salary lines corresponding to 85,049 workers.
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- marital status
- year of birth of children
New Project

- Salary trajectories depending on socioeconomic and macroeconomic covariates.
New Project

- Salary trajectories depending on socioeconomic and macroeconomic covariates.
- More realistic hypotheses for the economic modeling (time dependent demographical and market rates).
Salary trajectories depending on socioeconomic and macroeconomic covariates.

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More precise use of the group trajectories.
Bibliography


