Parallel simulations of soft-tissue using an adaptive quadtree/octree implicit boundary finite element method

Jack S. Hale\textsuperscript{1}, Pierre Kerfriden\textsuperscript{2}, J. J. Ródenas\textsuperscript{3}, Stéphane P. A. Bordas\textsuperscript{1}

1. University of Luxembourg, Luxembourg
2. Cardiff University, Wales, UK
3. Universidad Politécnica de Valencia, Spain
How can we move from an image...
…or a series of images…

Source: COLONIX, OSIRIX
to a full mechanical analysis?
...with the following constraints

• The developed methods should be usable in a day to day clinical environment.

• What does that mean?
  
  • Ease of use - hospital technician, surgeon.
  
  • Sit comfortably within an existing image segmentation and analysis pipeline.
  
  • Timeframe for results in seconds to one hour, not weeks.
    
    • CPU time - increasingly cheap.
    
    • User time - increasingly expensive.
  
  • Guaranteed results.
    
    • Adaptivity, error estimation (ongoing work with Pierre Kerfriden).
Pipeline to analysis

- Acquire images
- Segment images
- Mesh Surfaces
- Mesh Volume
- Perform analysis

Traditional

IGA

Geometry data
Loss of data
Time consuming

No loss of data
Each voxel $j$ is a 32-bit floating point measurement.
Soft or ‘fuzzy’ segmentation

\[ 0 < m_k(j) < 1 \quad \forall j, k \]
\[ \sum_{k=1}^{K} m_k(j) = 1 \quad \forall j \]

Hard segmentation

\[ \Omega = \bigcup_{k=1}^{K} S_k \quad S_k \cap S_j = \emptyset \quad \forall k \neq j \]
Hard Segmentation at 0.2f, OpenVDB and ITK

float / class unknown
38 x 50 x 60 / voxel size 3.943 (ScaleMap)
22,490 active voxels
Pipelines to analysis

Adaptive Implicit Boundary Pipeline

Acquire images

Segment images

Perform analysis

Geometry data

Low data loss

Implicit
Influences

• **Uncoupling discretisation and approximation.**
  - Bordas et al. - *Geometry-Independent Field Theory.*
  - Legrain et al. - *Double Nested Quadtree.*

• **PDEs on octrees.**
  - Museth et al. - *OpenVDB Library.*
  - Bangerth et al. - *deal.ii Library.*

• **Numerical methods for softly segmented medical images.**
  - J. J. Ródenas et al. - *Adaptive Quadtree.*
  - Miller et al. - *Adaptive Meshfree.*

• **Implicit boundary/immersed boundary/XFEM etc.**
  - Belytschko et al. - *XFEM.*
  - Hansbo and Burman - *Ghost penalty methods.*
  - Peskin et al. - *Immersed boundary methods.*
    • And many, many others…
The method
1-irregular mesh/2:1 balance

Octree or Quadtree data structure
Uncoupling discretisation and geometry
Nested Octree

Discretisation

\[ O_d \]

Geometry

\[ O_g \]

\[ M \]
Strategy 1: Soft or ‘fuzzy’ segmentation

Fast, automatic. But no so good for contact mechanics and setting boundary conditions…
Strategy 2: Hard Segmentation

\[ V_{h_d}(\mathcal{O}_d) \oplus E[V_{h_g}(\mathcal{O}_g)] \]

\[ V_{h_d}^{p_d}(\mathcal{O}_d) \]

\[ p_d > p_g \]

\[ h_d > h_g \]

\[ V_{h_g}^{p_g}(\mathcal{O}_g) \]

\[ p_g = 1 \]
For each enriched cell in the discretisation...
generate local Delaunay triangulation…
Case 1: boundary
Case 2: Material Interface

\[ u_h(x) = \sum_{i=1}^{N} N_i u_i + \sum_{i=1}^{N} N_i \sum_{j=1}^{M} \psi_j(x) a^j_i \]
Case 3: Dirichlet Boundary

Nitsche’s method, Lagrange multipliers…
Distributed memory parallelisation
Two work units.

$D - 1$

$\Omega_1$

$n_b$

$n_i$

$n_b \gg n_i$
The problem.

- Workload is no longer approximated well by the sparsity pattern of the problem because workload is quadrature bound.
- Some processors are assigned many boundary cells.
- Poor scaling as longest running workers dominate overall runtime.

**Conclusion**: Superior balancing algorithms are required for optimal scaling.
Conditioning number. $K$
Conclusion: Methods to keep condition number bounded are a necessity for *practical* computations using iterative solvers.
Summary and Outlook

• We are developing a cartesian grid implicit boundary/enriched finite element method specifically designed for rapid and automatic image based analysis.

• We plan to release the code as an open-source framework. Based upon deal.ii/PETSC/CGAL backend.

• Still outstanding issues with parallelisation and conditioning.

• Complexity is shifted from segmentation and mesh generation (clinician) to the numerical method (the developer); we believe the proposed method has potential to be used as a reference method for automatic image based analysis.
Acknowledgements

Jack S. Hale would like to acknowledge the support of the Marie Curie COFUND scheme through the FNR, Luxembourg and the structural position under the chair of Prof. Stephane P. A. Bordas at the University of Luxembourg.

Stephane P. A. Bordas would like to acknowledge partial support for his time from European Research Council Starting Independent Research Grant (ERC Stg grant agreement No. 279578) entitled ‘Towards real time multiscale simulation of cutting in non-linear materials with applications to surgical simulation and computer guided surgery’.