Empirical Welfare Analysis: When Preferences Matter

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Empirical Welfare Analysis:
When Preferences Matter*

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Abstract

Fleurbaey and Maniquet have proposed the criteria of conditional equality and of egalitarian equivalence to assess the equity among individuals in an ordinal setting. Empirical applications are rare and only partially consistent with their framework. We propose a new empirical approach that relies on individual preferences, is consistent with the ordinal criteria and enables to compare them with the cardinal criteria. We estimate a utility function that incorporates individual heterogeneous preferences, obtain ordinal measures of well-being and apply conditional equality and egalitarian equivalence. We then propose two cardinal measures of well-being, that are comparable with the ordinal model, to compute Roemer’s and Van de Gaer’s criteria. Finally we compare the characteristics of the worst-off displayed by each criterion. We apply this model to a sample of US micro data and obtain that about 18% of the worst-off are not common to all criteria.

Keywords: Random Utility, Preference Heterogeneity, Welfare, Inequality of Opportunity, Labour Supply

JEL Codes: C35, D31, D63, H24, H31, J22

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1 Introduction

Inequality of opportunity is traditionally measured by selecting a particular outcome and decomposing it into two kinds of determinants: the factors beyond the individuals’ responsibility, often called circumstances, and the factors for which the individual can be held responsible, the responsibility factors. Equality of opportunity requires erasing unfair inequalities due to circumstances but maintaining fair inequalities due to responsibility factors. As a result, outcomes should be a function of responsibility factors only.

From this definition, two main criteria have been applied in empirical analysis: firstly, Roemer’s criterion requires that individuals who exert the same effort (being the responsibility factor) obtain the same outcome. This corresponds to the ex-post view of equality of opportunity. If this is not the case, an equality of opportunity policy would consist in maximizing the outcome of those who obtain the lowest outcome at each level of effort. A second criterion proposed by Van de gaer’s states there is equality of opportunity if the mean outcomes conditional to circumstances are equal. As people who share the same circumstances are called a type, the target of a policy aiming at reducing inequality of opportunity should give the priority to the type with the lowest average outcome. This latter criterion expresses an ex-ante view of equality of opportunity. In both cases, to apply these criteria, we need to be able to compare individuals’ outcome and to aggregate them such as to define the appropriate policy. To do so, Roemer and Van de gaer criteria rely on a cardinal approach.

Another stream of the literature is reluctant to use cardinal measures. By observing the outcome picked by an individual, we may derive information on individuals’ preferences but not necessarily on the intensity of these preferences. As a result, it would be more appropriate to work in an ordinal framework. Notwithstanding its relevance, using an ordinal setting might raise one problem in the context of equality of opportunity: the distinct level of effort may be interpreted as heterogeneous preferences, and it is impossible, as exposed in the Arrow’s impossibility theorem, to obtain a social ordering that respects fully heterogeneous preferences in an ordinal framework.

One contribution of Fleurbaey and Maniquet [12, 14, 15, 16] has been to define fairness criteria in an ordinal setting that allow interpersonal comparisons. Their solution consists in representing first heterogeneous preferences in an ordinal setting in which the way individuals rank situations would reveal their preferences. Then, they propose to rank individuals’ situations according to fairness criteria by making explicit the normative choices that will enable interpersonal comparisons.

Their work in the field of welfare economics is strongly related to the
concept of equality of opportunity [13] because the social rankings they propose are based on equity criteria very similar to the definition of equality of opportunity. They consider that individuals’ well being is the relevant outcome on which should be based social rankings and the observed choices made by the individual should be the method to identify their preferences. In addition, since individual’s well-being is a function of preferences and non-responsibility factors, policies should erase inequalities due to non-responsibility factors and be neutral\(^1\) with respect to preferences. In particular, fair policies should rely on two principles derived from the non-envy principle.

The first principle corresponds to the compensation principle. Compensation means we should compensate for factors for which the individual is not responsible. Therefore, people with the same preferences should achieve the same well-being. This is very similar to Roemer’s criterion that requires people with the same responsibility factors should end up with an equal outcome.

The second principle corresponds to the neutrality principle. Neutrality refers to the neutral treatment of individuals with respect to their preferences. This neutrality principle means that individuals should be treated equally with respect to their preferences\(^2\). As a result, redistribution mechanisms should be designed in such a way that individuals with equal circumstances will pay/receive the same taxes/transfers. This principle is closer to Van de Gaer’s criterion that recommends people to have equal opportunities whatever their non-responsibility factors. Because people should have the same opportunities before making their own choice (i.e. deciding on their responsibility factor), this principle encapsulates the idea of neutrality according to which the treatment of the individuals should be independent from their responsibility factors.

The compensation and neutrality principles cannot be both satisfied when individuals have heterogeneous preferences [6, 16, 21]. That is why, Fleurbaey and Maniquet measure unfairness through two criteria, each one giving the priority to one principle and fulfilling only partially the second one. The criterion of conditional equality fulfils the neutrality principle and compensates partially inequalities due to non-responsibility factors. The criterion of egalitarian equivalence gives priority to the compensation principle but does

\(^1\)Here neutral means that policies should not compensate for preferences. In this context, inequalities between individuals having distinct preferences do not give rise to redistribution.

\(^2\)This idea could be questioned if we consider that some goals or preferences should be avoided according to some prevalent values but this discussion falls outside the scope of this paper.
not satisfy neutrality with respect to all preferences.

Given the existence of these four criteria (Roemer’s ex-post approach, Van de gaer’s ex-ante view, conditional equality and egalitarian equivalence), the aim of the paper is twofold: (1) proposing a model in an ordinal set-up that approximates individuals’ preferences; (2) comparing the four criteria in a common framework by checking whether their identification of the worst-off is consistent.

Our first objective is to propose a model in an ordinal set-up that approximates individuals’ preferences through the observable choices made by the individuals. This enables us to apply Fleurbaey and Maniquet by respecting their theoretical approach. In fact, previous empirical applications of Fleurbaey and Maniquet’s criteria have been adapted in a cardinal framework: Almas [3] and Devooght [9] have used the definition of conditional equality and egalitarian equivalence to measure unfair inequalities for income, and because everyone is supposed to prefer more income to less, they did not use heterogeneous preferences. Then, Decoster and Haan [8] have proposed on German data the first application that follows the key points of the Fleurbaey and Maniquet’s approach, especially the identification of heterogeneous preferences and the use of an ordinal framework. Bargain et alii [4] extend their work and propose an international comparison for 11 European countries and the US based on egalitarian equivalence criteria. Notwithstanding their breakthrough in the field, the preferences they use are still collective (based on socio-demographic variables) and not genuinely individualistic. In this paper, we propose an extension of Decoster and Haan’s model such as to approximate the individualistic component of preferences.

Our second objective is to propose a way to compare the conditional equality and egalitarian equivalence criteria with Roemer’s and Van de gaer’s criteria. To this end, we have to solve two issues.

Firstly, we need a cardinal measure of well-being. To obtain it, we use two distinct strategies. On the one hand, we erase heterogeneity in preferences such as to obtain a cross-individual comparable measure of well-being. This corresponds to the assumption made to apply conditional equality. On the second hand, we take a money metric for utility that is used for implementing the egalitarian equivalence criterion. In both cases, we rely on explicit assumptions that make comparable the four criteria.

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3To our best knowledge, only a theoretical comparison of the four criteria has been performed [13].

4This necessary step for the comparison goes not without critics. Some consider Roemer’s and Van de gaer’s criteria cannot be applied to well-being, because of the practical difficulty of measuring well-being in a cardinally measurable and comparable way. The interpretations of the results remain subject to this limitation.
Secondly, we have to split between effort and non-responsibility factors instead of the split between preference and non-responsibility factors used in the ordinal model. We take the same non-responsibility factors as the one used for conditional equality and egalitarian equivalence. Concerning effort, since it is here not directly observable, we use Roemer’s Identification Axiom (RIA). By assuming that the outcome is a monotonous increasing function of effort, individuals who sit at the same percentile of the outcome’s distribution function of their type have exerted the same effort.

For illustrative purpose, we finally apply the model to a sample of US singles from the 2005 Cross-National Equivalent File (CNEF) dataset. We identify who are the worst-off and check if they change when we take, or not, into account individual preferences. We then compare the worst-off across the four criteria and check their socio-demographic characteristics. This comparison across criteria informs us about the impact of the normative choices that enable interpersonal comparisons on the measures of fairness.

The rest of this paper is organized as follows. Section 2 details the empirical approach to estimate utility and the way we extend it to incorporate individual preferences. Section 3 presents the way we measure conditional equality and egalitarian equivalence, while Section 4 compares with Roemer’s and Van de gaer’s criteria. The data and main results are presented in Section 5. Finally, Section 6 concludes.

2 A discrete choice model to apply the criteria

The objective is to identify heterogeneous preferences in an ordinal framework where the individual’s well-being depends on consumption and leisure. To this end, we start by presenting a utility model close to the one developed by Aaberge et al. [1, 2] and Decoster and Haan [8]. We then extend these approaches and detail how individual preferences can be genuinely approximated.

2.1 Estimation of groups’ preferences

We here rely on the models developed by Aaberge et al. [1, 2] and Decoster and Haan [8] where the labour time is discretized into a finite number of intervals. We assume that the individual’s well-being is representable through a utility function that depends on consumption $C$, leisure $L$, socio-demographic variables $X$ and a random error term $\epsilon_{ij}$ that varies independently among individuals. The subscript $j$ corresponds to the discretized labour time. The error term $\epsilon_{ij}$ is not observed but affects individuals’ choices. The utility
function can be written as follows:

\[ V(C, L, X, \epsilon) = U(C, L, X) + \epsilon \]  

(1)

and the utility of individual \( i \) when working \( L_{ij} \) hours is:

\[ V_{ij} = V(C_{ij}, L_{ij}, X_i, \epsilon_{ij}) \]  

(2)

The two sub-indices \( i \) and \( j \) may appear redundant. They aim to show that utility differs across individuals \( i \) and across alternatives over the working time \( j \). Indeed, the earnings \( C \) depend on the wage rate that varies across the individuals \( i \) and depend on the amount of working time. Also, the amount of leisure \( L \) depend on preferences for leisure, defined individually, and the working hours. Finally, \( \epsilon_{ij} \) is also assumed to vary across individuals and alternatives.

The individual \( i \) maximizes his utility by choosing his amount of working hours \( (h = 1 - L) \) from a set of the alternatives on working time \( j \in J^5 \). Empirically, as we use a discrete choice model, we restrict the individual’s choices such that the individual is free to select his weekly working time among 9 alternatives (less than 20 hours, around 20 hours, around 25 hours, around 30 hours, ... around 50 hours, and more than 50 hours).

The individual is subject to a budget set that depends on the wage rate received by the individual, the amount of labour time and the taxes he pays on his gross earnings. The wage rate is supposed to be constant whatever the amount of labour time picked by the individual. As the actual taxes are not linear, we use information on income taxes given by the OECD such as to estimate the actual individual’s budget set, that is to say, to calculate the net earnings an individual would receive for each of the 9 alternatives.

The budget set can be written as:

\[ C = w \times h - t(w, h) \]  

(3)

Where \( t(w, h) \) is the tax function that shifts gross earnings to disposable income (i.e. net wage).

Regarding the deterministic part of the utility function, we use the same specification as Aaberge et al. [1, 2] and Decoster and Haan [8]:

\[ U(C, L) = \beta_C \frac{C^{\alpha_C} - 1}{\alpha_C} + \beta_L(X) \frac{L^{\alpha_L} - 1}{\alpha_L} \]  

(4)

\( ^5 \)\( j \) and \( h \) both represent the working hours. Nevertheless we use both to maintain the intuitive meaning of \( h \) being the labour time, and \( j \) the discrete alternative.
The parameters $\beta_C$, $\alpha_C$, $\beta_L(X)$ and $\alpha_L$ determine preferences for consumption and leisure. $\beta_C$, $\alpha_C$ and $\alpha_L$ are common to all individuals. Heterogeneity among groups of individuals is introduced through $\beta_L$ that depends on socio-demographic variables:

$$\beta_L(X) = \beta_{L0} + \beta_{L1}X$$  \hspace{1cm} (5)

Where $X$ are gender, age, education and ethnic group.

The variables that explain the differences in the preference for leisure have been widely used in the literature to explain the determinants for labour supply. They are not really under the individual’s control but they are expected to explain differences in preferences for leisure. In fact, Dworkin [10, 11], Rawls [19], Fleurbaey and Maniquet [15] share the view that we should be neutral with respect to preferences as long as individuals identify themselves with these preferences. No matter what explains individuals’ differences in preferences. We follow the view of these authors, therefore $\beta_L$ represents individuals’ preferences, for which no compensation is required.

To obtain the parameters of the deterministic part of the utility function, the estimation relies on a rationality assumption. It states that, if the $i$th individual makes the choice $j$, $V_{ij}$ is the maximum among the $j$ alternatives. In other words, the probability that the $i$th individual makes the choice $j$ is:

$$\text{Prob}(h_i = j) = \text{Prob}(V_{ij} > V_{ik}) \forall k \neq j$$  \hspace{1cm} (6)

We replace $V_{ij}$ and $V_{ik}$ by its expression, rearrange and obtain:

$$\text{Prob}(h_i = j) = \text{Prob}(\epsilon_{ik} - \epsilon_{ij} < -(U_{ik} - U_{ij})) \forall k \neq j$$  \hspace{1cm} (7)

The resulting multinomial model is treatable if we assume that $\epsilon_{ij}$ is $i.i.d$ random value with type I extreme value distribution, then differences in epsilon follow a standard logistic distribution [7]. In this case, we estimate the parameters of the utility function by maximum likelihood as a conditional logit model where:

$$\text{Prob}(l_i = j) = \frac{\exp U_i(C_{ij}, L_{ij})}{\sum_{k=1}^{g} \exp U_i(C_{ik}, L_{ik})}$$  \hspace{1cm} (8)

We obtain as many utility functions as numbers of groups having the same socio-demographic characteristics. Formally, we have:

$$V_{ij}(C, L) = \beta_C \frac{C_{ij}^{\alpha_C} - 1}{\alpha_C} + (\beta_{L0} + (\beta_{L1}X_i)) \frac{L_{ij}^{\alpha_L} - 1}{\alpha_L} + \epsilon_{ij}$$  \hspace{1cm} (9)
Individuals who have the same leisure and the same consumption may obtain a distinct well-being if they have a distinct $\beta_L$. In other words, the form of the utility function varies across individuals who have a different age or gender or education or ethnic group.

In the end, with this estimation proposed by Aaberge et al. [1, 2] and Decoster and Haan [8], we estimated groups’ preferences for leisure. However, Fleurbaey and Maniquet recommend to identify individuals’ preferences, as the criteria they propose allow us to establish individuals’ rankings. Identifying groups’ preferences is not really satisfactory to establish individuals’ rankings. This is why, we propose an extension of this model.

2.2 Approximation of individual’s preferences

$\beta_L$ is determined by socio-demographic variables and does not include information specific to the individuals. On the contrary, $\epsilon_{ij}$ is specific to the individual $i$ and to the alternative $j$ and has an impact on the well-being $V_{ij}$. The error term, $\epsilon_{ij}$, captures the individualistic component of preferences not explained by $\beta_L$. The point is here to show how to derive some information related to this unobserved heterogeneity.

After estimating the parameters of $U(C, L)$, we can compute the utility an individual would obtain for each alternative $j \in J$. It may occur that the individual maximises his utility for the amount of labour time he actually chose. In this case, to the extent the model is well specified, the individual has the same preferences as the group to which he belongs.

But, we can also have a situation where the individual maximises his utility for an amount of labour time he did not choose. Formally, we can observe $U(C_{ij}, L_{ij}) < U(C_{ik}, L_{ik})$ where $j$ is the amount of labour time the individual has actually chosen and $k$ is one or some other possible alternatives. However, following the assumption of our model according to which the individual is rational, we should have $V(C_{ij}, L_{ij}) > V(C_{ik}, L_{ik})$ for all the other possible alternatives $k$.

Assuming the model is well specified and using the expression of $V_{ij}$, we deduce that $\epsilon_{ij}$ may explain why, at the same time, the individual does not maximise his utility $U_{ij}$ for his actual labour time $j$, whereas he actually maximizes his utility $V_{ij}$ when picking a labour time $j$. In such a case, $\epsilon_{ij}$ captures the individualistic component of preferences not explained by $\beta_L$. Still, $\epsilon_{ij}$ is not observable after the estimation. In this sense, we cannot capture the individual’s preferences. Nevertheless, building on the work of Bonin and Schneider (2006) [5] who propose an analytical approach to extract information from the $\epsilon_{ij}$ in the conditional logit model, we can use the assumptions of the model to get a proxy for differences in epsilon as follows:
if the individual has picked the alternative $j$, this means that the utility he obtains with this option is superior to any other alternative. Formally, it requires that:

$$
\epsilon_{ik} - \epsilon_{ij} < -(U_{ik} - U_{ij}) \forall k \neq j
$$  \hspace{1cm} (10)

Omitting indexes, we can say that because individuals are rational, we have to satisfy:

$$
\Delta \epsilon < -\Delta U
$$  \hspace{1cm} (11)

Figure 2 shows how the maximum value of $\Delta \epsilon$ is computed for each individual whose chosen number of working hours ($j$ hours) differs from the numbers of working hours maximizing his estimated utility ($k$ hours).

(2) The hypothesis of the estimation, according to which differences in epsilon follow a standard logistic distribution, requires that the density function of $\Delta \epsilon$ be:

$$
f(\Delta \epsilon) = \frac{\exp \Delta \epsilon}{(1 + \exp \Delta \epsilon)^2}
$$  \hspace{1cm} (12)

Using (1) and (2) permits us to rescale the density function of $\Delta \epsilon$ so that:

$$
f(\Delta \epsilon | \Delta \epsilon < \Delta U) = \frac{f(\Delta \epsilon)}{\text{Prob}(\Delta \epsilon < \Delta U)} = \frac{f(\Delta \epsilon)}{P} \hspace{1cm} \forall \epsilon < -\Delta U
$$  \hspace{1cm} (13)

$$
= 0 \hspace{1cm} \forall \epsilon \geq -\Delta U
$$  \hspace{1cm} (14)

where $P$ is:

$$
P = 1 - \frac{\exp \Delta U}{1 + \exp \Delta U}
$$  \hspace{1cm} (15)

This rescaling matters for our purpose, since it allows us to improve the identification of the form of the indifference set (or utility curve) of every individual by using part of the individual unobserved preferences.

With our estimation we can compute the utility received by an individual for the bundle $(C_i, L_i)$ he actually chose. With this information, we can compute the level of consumption an individual would need to obtain the same utility if he choose another labour time. As a consequence we can compute the 9 points of the indifference set (since the individual has 9 alternatives of labour time).

The points of an indifference set are such that $V_{ik} = V_{ij} \forall k \neq j$. By replacing $V_{ik}$ and $V_{ij}$ by their expressions and omitting the index $i$, each point of the indifference set is as follows:

$$
(\tilde{C}_k, L_k) = \left( [C_j^{\alpha C} + \alpha C (\Delta \epsilon + \frac{\beta L}{\alpha L} (L_j^{\alpha L} - L_k^{\alpha L}))]^{\frac{1}{\beta C}}, L_k \right)
$$  \hspace{1cm} (16)
Where $\hat{C}_k$ is the level of consumption conditional on labour time $1 - L_k$ required to have a utility equal to $V_{ij}$.

We observe that the formula requires a value for $\Delta \epsilon$. This is why the rescaling matters. Given the unconditional density function of $\Delta \epsilon$, we generate one million drawings of a random variable that follows a standard logistic distribution so as to obtain one million possible values for $\Delta \epsilon$. Then, for each alternative and each individual, we measure $\Delta U$. We finally plug these pieces into the formula given in Equation 13 to compute conditional expected values of $\Delta \epsilon$.

To summarize, what we have done is to use the assumptions about the distribution of $\Delta \epsilon$ and the rationality hypothesis to define the conditional distribution $\Delta \epsilon$. This allows us to better approximate the indifference set of every individual as the form of the indifference set is specific to the individual when $E(\Delta \epsilon)$ is different from zero. Still, two individuals who (1) share the same socio-demographic characteristics and (2) maximize $U$ for their chosen labour time have the same indifference set.

3 Identification of the worst-off according to conditional equality and egalitarian equivalence

We start this section by some methodological preliminaries and then present the way each criterion is empirically implemented and the worst-off identified, starting by conditional equality and following by egalitarian equivalence.

3.1 Methodological preliminaries

3.1.1 A simplified budget set

The economy is characterized by a set of $N$ agents $i$ that maximize their utility $u_i(C, L)$ over consumption (i.e. disposable income) and leisure $(C, L) \in X = \{R^+ \times (0, 1)\}$. Since we allow individuals to have heterogeneous preferences for leisure, a same bundle $(C, L)$ will, in general, not lead to an equal well-being.

Every individual maximizes his utility subject to a budget set $B \subseteq X$. The budget represents the level of consumption accessible to the individual. It varies for each individual because it depends on (1) the individual’s wage rate $w$ that is assumed to be constant whatever the individual’s labour time, (2) the amount of labour time $h = 1 - L$ and (3) the tax rate $t$ applied to the gross labour earnings.
This budget set is not linear, since \( t \) is progressive and varies along with \( w \times h \). This is a problem, since we aim to preserve as far as possible the neutrality principle (see Section 1). The non-linearity of the budget set is incompatible with that principle. Taxing more those who like to work is not neutral.

This is why, before implementing the conditional equality and egalitarian equivalence criteria, a preliminary transformation of the actual budget set into a simplified (linear) budget set is necessary. The simplified budget set is composed of the lump sum transfer that would make the individual just as well-off as he is in his current situation (given his observed wage rate and his freedom to choose the bundle \((C, L)\) according to his preferences). Figure 3 illustrates how a progressive tax scheme is approximated by a lump-sum tax design or, in other words, how a non-linear budget set (the net wage curve) is replaced by a linear budget set (the simplified net wage line).

We obtain nested budget sets by replacing the actual budget set by a lump-sum transfer. Given every individual maximizes his utility subject to a budget set \( B \subseteq X \), the utility function derived from any subset \( B \) is

\[
u_i(B) = \{ \max u_i(C, L) | (C, L) \in B \} \tag{17}\]

The simplified budget set \( B^* \subseteq X \), is determined by the gross income and a lump-sum transfer \( \hat{t} \) such that

\[
B^*(w, \hat{t}) = \{ (C, L) \in X | C \leq w \times l + \hat{t} \} \tag{18}
\]

and

\[
u_i(B) = u_i(B^*(w, \hat{t})) \tag{19}\]

### 3.1.2 Responsibility versus non-responsibility factors

Before defining the fairness criteria, we also need to precise the split between responsibility and non-responsibility factors.

In Fleurbaey and Maniquet setting, the non-responsibility factor is the wage rate. The wage rate is likely to represent skills that are mostly the product of genetics, family background, luck but also effort. Despite this choice appears to be quite controversial, we aim to implement the criteria following the spirit of their authors. We therefore take the same hypothesis. The responsibility factors are individuals’ preferences and correspond to the preferences for leisure and consumption. Once identified the wage rate, the individual’s preferences and the implicit budget set, we now explain in further detail the equity criteria.
3.2 Conditional equality

Conditional equality permits to fulfil completely the neutrality principle and achieve partially the compensation for circumstances. Formally defined by Fleurbaey [12], conditional equality can be computed as follows:

“Define a reference value for responsibility characteristics and give priority (according to the leximin criterion) to individuals, who, with their current resources and circumstances and this reference value of responsibility characteristics, would be the worst-off.”

In a nutshell, the idea enclosed in this principle is the following: in a fair economy, if individuals had the same preferences, they would end up with the same well-being. Thus, the individuals who would obtain the lowest well-being, given a reference value for preferences, are those whose well-being is the most negatively affected by the non-responsibility factors. They are the worst-off. A conditional equality rule would consist in neutralizing the effect of circumstance for the people having the same reference value for preferences.

In this framework, implementing conditional equality requires defining a reference value for preferences. This is done by fixing a reference utility function. Individuals with the same preferences should obtain the same well-being.

Formally, individuals are ranked according to:

\[ \tilde{u}(B(w, \hat{t})) \]  

where \( \tilde{u} \) (note the absence of the subscript \( i \)) is the reference utility function that results from fixing a reference value for preferences\(^6\) and where \( \hat{t} \) is the lump-sum transfer corresponding to the simplified budget set such that:

\[ u_i(C, L) = u_i(B(w, \hat{t})) \]  

Equivalently, in programmatic terms, we estimate for each of the 9 working time intervals the pairs \((\hat{C}_k, \hat{L}_k)\) defining the utility derived from the choice made by the individual, as defined in Formula 16. We then estimate, given the gross wage rate, the tax required to reach the indifference curve for each working time. The optimizing consumption leisure bundle is the one where the tax is the largest. The worst-off are those on the lowest utility curve.

\(^6\)Instead of fixing a reference value for the form of the utility function, an alternative would be to fix a reference value for labour time. It is equivalent with stating that all individuals have preferences such that they decide to work the same amount of working hours.
Figure 4 illustrates the conditional equality criterion for two reference value for the preferences, one where $\beta_L$ is set to 10 (low preference for leisure), and one where $\beta_L$ is set to 70 (larger preference for leisure).

\[\text{INSERT FIGURE 4 HERE}\]

Figure 5 shows a case where the worst-off is the one with the higher (simplified) net wage rate (John) contrary to the case depicted in Figure 4 where the worst-off is the one with the lower (simplified) net wage rate (Mary).

\[\text{INSERT FIGURE 5 HERE}\]

Since the results change depending on the reference value for $\beta_L$ and on the inclusion or not of the $\Delta \epsilon$, we estimate five variants. First we set $\beta_L$ to its average (close to 10), and include the unobserved heterogeneity through $\Delta \epsilon$. Second, we keep $\beta_L$ at its average but set the $\Delta \epsilon$ to zero. These first two variants allow to assess the impact of considering the individual preferences. In variants three to five, we keep $\Delta \epsilon$ and increase the preference for leisure by incrementing $\beta_L$ to 20, 70 and 100 respectively. For each variant, the worst-off are the 10% of individuals with the lowest utility.

### 3.3 Egalitarian Equivalence

The second equity criterion, egalitarian equivalence, fulfils completely the compensation principle and partially the neutrality principle. As defined by Fleurbaey [12], it requires the following:

“Define a reference type of circumstances and give priority (leximin) to individuals whose current level of well-being would be obtained with the least resources if their circumstances were of the reference type”

This method mimics a situation where inequalities would be due to responsibility characteristics only. As a consequence, the redistribution rule that emerges from this principle fulfils the compensation principle but may treat unequally individuals with the same circumstance.

Regarding the application, Fleurbaey proposes to use different reference values for the wage rate as the reference value affects people ranking. The ranking depends on the apportion between preferences and income. The higher the reference wage rate, the larger the priority given to people having lower preferences for leisure, that is to say the hard-working people.
If the reference wage rate is equal to zero, we obtain the zero egalitarian equivalence. To this end, we rank \( \hat{t}_i \) in ascending order as defined by:

\[
u_i(C, L) = u_i(B(0, \hat{t}))
\]

(22)

Figure 6 illustrates the zero-wage egalitarian equivalence for two heterogeneous individuals (John has a relative preference for income/work and Mary for leisure). We see that the lump-sum tax required to compensate the loss of disposable income is higher for John. Mary is the worst-off.

**INSERT FIGURE 6 HERE**

If the reference wage rate is equal to the minimum wage rate (set at 5 USD per hour), we will obtain the minimum-wage egalitarian equivalence. It consists in ranking in ascending order the \( \hat{t} \) as defined by

\[
u_i(C, L) = u_i(B_{\min_{j \in N} w_j, \hat{t}})
\]

(23)

Figure 7 illustrates the minimum-wage egalitarian equivalence for John and Mary. We see that the lump-sum tax required to compensate the loss of disposable income is higher for John. Mary is the worst-off in this case again.

**INSERT FIGURE 7 HERE**

Lastly, the wage egalitarian equivalence or equivalent wage is defined in Fleurbaey [12] as:

“For each individual, compute the counterfactual wage rate (with no transfer) that would make the individual as happy as in his current situation, and give priority (leximin) who are the worst-off in these terms.”

Formally, it is equivalent to ranking the individuals in ascending order according to \( \hat{w} \) that satisfies:

\[
u_i(C, L) = u_i(B(\hat{w}, 0))
\]

(24)

Figure 8 illustrates the wage egalitarian equivalence for John and Mary again. The minimum wage required to allow the individuals to maintain their utility in a no-tax context is larger for Mary than for John. John is the worst-off in this case.
In programmatic terms, we estimate for each of the 9 working time intervals the pairs \((\tilde{C}_k, L_k)\) defining the utility derived from the choice made by the individual, as defined in Formula 16. We then fix a reference value for the wage rate (zero or 5 USD for the minimum wage approach) and then compute the maximum lump-sum transfer that gives to the individual his actual level of well-being. The worst-off are the 10% individuals with the lowest lump-sum taxes. Regarding the wage egalitarian equivalence, we compute for each of the 9 working time intervals the ratio \(\tilde{C}_k\). The smallest value is the smallest net wage required to reach the actual level of utility. The worst-off are the 10% individuals with the lowest required wage rates.

4 Comparison with Roemer’s and Van de gaer’s criteria

4.1 Methodological preliminaries

We present in this subsection the methodological choices to make Roemer’s and Van de gaer’s criteria comparable with conditional equality and egalitarian equivalence (see Ramos and Van de gaer [18] for a recent survey of the empirical literature on these measures). We start by discussing the cardinalization of well-being, the identification of responsibility and non-responsibility factors and finally discuss Roemer’s and Van de gaer’s criteria.

4.1.1 Cardinalization of well-being

For the application of conditional equality and egalitarian equivalence, the form of the utility function itself does not matter, we only need to know the way people rank the bundles \((C, L)\) according to their preferences in order to make orderings. Instead, Roemer’s and Van de gaer’s criteria use a cardinal measure of well being to make interpersonal comparisons. Taking the observed earnings is not an appropriate solution as conditional equality and egalitarian equivalence account for another dimension of well-being. A possible solution is to take a measure of well-being. This is consistent with the hypothesis made above about individuals’ behaviour.

The question is thus to select an appropriate cardinal measure of well-being. We thus follow two strategies, one inspired by the conditional equality approach, the other by the egalitarian equivalence.
We detailed in the precedent Section that, to implement the conditional equality criterion, we need to choose a reference utility function for everybody in order to remove heterogeneity in preferences. It means that the value of $\tilde{u}(B(w_i, t_i))$ that is obtained for conditional equality is (1) comparable across individuals because the same utility function is used for everybody, and is (2) an approximation of the current individual’s well-being. The point is that we also need a unique utility function to implement Roemer’s and Van de gaer’s criteria. In consequence, this conditional equality measure can be used to implement Roemer’s and Van de gaer’s criteria and will be easily comparable with the results given by the conditional equality criterion.

Regarding the egalitarian equivalence, we saw that it consists in estimating the amount of resources that should be given to an individual such that he reaches his current level of well-being if all individuals had the same circumstances. In consequence, this amount of resources reflects the current well-being of the individuals and respects individuals’ preferences. This is a second money metric for individual’s well-being. For example, if we take the zero equivalence criteria, we obtain the resources needed by an individual if he were not working to be as well off as he is in his current situation. One limit is that this money metric corresponds to a virtual situation that may not occur ever. Still, this measure enables us to implement Roemer’s and Van de gaer’s criteria and is directly comparable with the results given by the egalitarian equivalence criterion.

4.1.2 Criteria compatibility

The second problem we face is the fact that Roemer’s and Van de gaer’s criteria make a distinction between effort and circumstance, and Fleurbaey and Maniquet between preferences and non-responsibility factors.

To make the comparison more reliable among the four criteria, it is natural to define the circumstance in the same way as the non-responsibility factors because both terms reflect the factors that are beyond individual’s responsibility. Thus, the circumstance is unique and corresponds to the wage rate$^7$.

In the framework of Fleurbaey and Maniquet, the concept most similar to effort is the difference in preferences which leads to distinct utility functions across individuals. In this setting, it is impossible however to find one single continuous value for effort. A way out is to use the RIA to determine one continuous index of responsibility: firstly, the population is partitioned into

$^7$More complex specifications could be of main interest but are left to other studies since here the purpose is to offer a consistent comparison among several criteria and not a detailed implementation of one specific criterion.
types according to the wage rate. Then, we draw the outcome’s distribution function of each type, the outcome being the well-being as defined in the Subsection on the cardinalization problem. Finally, we assume that the individuals belonging to the same percentile of their outcome’s distribution function have exerted the same effort. In this way, we obtain an index of individuals’ effort. This method is coherent with Roemer’s definition of effort and allows us to compute Roemer’s and Van de gaer’s criteria consistently within their original framework.

4.2 Implementing Roemer’s criterion

There is equality of opportunity according to Roemer when people exerting the same effort obtain the same outcome. Measuring the fairness of any distribution consists in comparing individuals’ well-being at each level of effort across types.

With such a criterion, the worst-off may be defined as the individuals with the lowest well-being at each level of effort. Improving equality of opportunity requires maximizing the mean outcome of the individuals who have the lowest well-being at each level of effort.

To implement this criterion, we firstly divide the population into types (based on wage quantiles, following Fleurbaey and Maniquet approach). We then take our cardinalization of well-being and apply the RIA to obtain an indirect measure of effort: people who sit at the same decile of the utility distribution function of their type have exerted the same effort. Lastly, we take the individuals with the lowest well-being for each value of effort and obtain who the worst-off are according to Roemer’s criterion. Figure 9 represents the worst-off according to Roemer’s criterion.

4.3 Implementing Van de gaer’s criterion

Equality of opportunity defined by Van de gaer’s is characterized by an economy where the average well-being of each type is equal. With such a definition, the worst-off cannot be identified individually. Instead they can be defined as the type with the lowest average well-being. Here we do not need to implement RIA. To identify the worst-off, we use the cardinalization of the well-being detailed above and then measure the average well-being conditional on the type. The worst-off are the individuals who belong to the type with the lowest average well-being. Figure 10 represents the worst-off according to Van de gaer criterion.
5 Data and Results

5.1 Data

The empirical analysis is based on US data from the Cross National Equivalent File (CNEF) for 2005 that provides information on incomes in 2004. It includes detailed information on socio-demographic variables. To keep things simple and avoid discussions on the problems related to considering household’s interests, we focus on singles without children. In addition, we restrict our sample to people who work at least twenty hours a week\(^8\). We also limit our sample to individuals aged between 20 and 65 years old who are not self-employed, retired or fully engaged in education. We finally exclude individuals whose capital incomes have a value superior to 10% of the labour income\(^9\). This gives us a sample of 914 individuals. Table 1 provides some descriptive statistics on the sample, where we see a quite large variability of the annual net wage across socio-demographic characteristics.

Even though we restrict our sample to individuals who work at least twenty hours, we estimate our model by letting to the individuals the freedom to choose among all possible alternatives of working hours (including less than 20 hours). The wage rate is assumed to be constant whatever the number of working hours, this is in line with Decoster and Haan model [8]. Distribution of working hours is given in Figure 11.

To build the budget set, we use the 2004 OECD report for the US to derive the net income for each possible working hours. Precisely, we first calculate the gross wage rate by dividing the individual gross labour earnings by the annual working hours. Then, we make discrete the labour time such as to vary between 0 and 55. For each discrete alternative, we compute the

\(^8\)As we have poor information on transfers given to people who do not work, we restrict our analysis to individuals least affected by transfers.

\(^9\)Capital income would affect the level of the budget set and probably the decision of working or not. Therefore, not taking into account capital income should affect the slope of the utility function for low level of working hours. This is why we excluded individuals with a capital income superior to 10% of labour income.
corresponding gross total earnings and we use the report on tax income to simulate the corresponding net earnings. Leisure is mapped in the space zero-to-one and net income is expressed in units of 10000 dollars per year.

We do not include capital income in the estimation. The gross amount of capital income the individual receives is available but there is no disaggregation by type of capital income and no data on net capital income. As taxes depend on the sources of this income, net capital income cannot be properly computed.

5.2 Estimation Results

Table 2 presents the parameter estimates of the utility function for 6 variants. The benchmark case is estimated on the full sample, with heterogeneous preferences captured by the full set of socio-demographic variables (age, gender dummy, ethnic dummies and education dummies). We first find that the marginal utility of consumption and leisure ($\beta > 0$) is positive as expected, and decreasing ($\alpha < 1$), as expected. We then note that the age and gender variables are the sole significant regressors capturing the heterogeneity of preferences for leisure. Being a woman and being out of an intermediate age (so being young or old) means a preference for leisure. High education has also a positive impact on the preference for leisure but is slightly insignificant. Other variables are clearly insignificant. These results do not differ from those reported in the Decoster and Haan study for Germany [8] (excepted that they found that a low education level has a significant positive impact on preference for leisure).

INSERT TABLE 2 HERE

To the benchmark model, we add 5 variants to check the robustness of the estimates. The first variant (column 2) reports the estimates for a sample where individuals with hourly wages inferior to 5 USD are excluded. The second variant (column 3) excludes the same individuals and the ones earning more than 100 USD per hour. The variants reported in column 4, 5 and 6 rely on the benchmark full sample but exclude ethnic variables, education variables and both, respectively. No significant changes occur in the estimates. The same regressors are significant. Signs remain unchanged. On basis of the significativity of the regressors, we keep the model presented in column 6 as our preferred model. The following steps rely on the estimates of this variant.

Before diving into the worst-off analysis, we check the performance of the utility model by comparing the working hours maximizing the utility of
the individuals with actual choice made by the individuals. We find that 57% of the maximizing number of hours are equal or adjacent to the actual number of hours of the individuals. This relatively low percentage, to some extent, means that some unobserved preferences had an impact on the choice of the individuals. Our approach to include a part of the information on the unobserved preferences is in this context strongly recommended.

5.3 Who are the worst-off?

As a preliminary, we measure how taking individual preferences into account affects the estimates. As illustrated in Table 3, the percent of worst-off commonly identified by models considering and, respectively, not considering, individual preferences oscillates between 82.4% and 95.6% for conditional equality (depending on the reference value for $\beta_L$) and from 91.3% to 100% for egalitarian equivalence. For each of the four criteria, results will be reported for both the cases with and without consideration of the unobserved heterogeneity.


table 3

Firstly, regarding conditional equality, we fix four different reference values for $\beta_L$. We first use the average of $\beta_L$ (around 10), then increase the preference for leisure by setting $\beta_L$ equal to 20, 70 and 100. Intuitively, by increasing the value of $\beta_L$, higher wage-rate individuals should be the worst-off. This is concordant with our results, as reported in Table 4. As we increase the reference value for $\beta_L$ the results change significantly. If we assume strong preferences for leisure, the worst-off turn to be white individuals with higher education. Instead, when we took a reference value of 10, the worst-off tend to be people black individuals with a rather low educational level. It is also worth noting that not taking into account the individual preferences ("without" column) can lead to different conclusions, as in the low education case.


table 4

Regarding the egalitarian equivalence criteria, results, reported in Table 5 change slightly depending on the reference value for the wage rate. When the reference value is equal to zero, the worst-off tend to be people with high distaste for working, so that they need low transfer to be as happy as they would be if they did not work. Here, in comparison with the composition of the sample, the worst-off profile is a white woman with a high education level. Again, not considering the individual preferences change the results for the eduction parameter (cf "without" column).
Regarding the wage egalitarian equivalence criterion and the egalitarian equivalence when the reference value for the wage rate is equal to 5 dollars per hour, the results are quite similar in terms of gender and ethnicity. The proportion of men increases slightly as well as the proportion of black people.

Again, considering the unobserved heterogeneity leads to different results as reported by the differences between "with" and "without" columns. It is however worth noting that no difference occurs in the case of zero egalitarian equivalence.

Regarding Roemer’s criterion, we proposed and tested different cardinalization of welfare, based either on utility derived from the conditional equality approach or on the amount of the compensation tax derived from the egalitarian equivalence approach. Results are reported in column one under the label Roemer of Table 6. We do not detail results per cardinalization because the results are identical. The reason is related to the way circumstances are defined. Since our aim is to make consistent comparisons with the results of conditional equality and egalitarian equivalence, we follow Fleurbaey and Maniquet and define the wage as the circumstance. Necessarily, a higher wage extends the budget set and the potential combinations of consumption-leisure available to the individuals. Hence, there is no surprise that the worst-off are those with the lowest wage rates. Notwithstanding these results, the cardinalizations remain relevant for cases where the circumstances would include other variables.

5.4 Differences and Similarities among the criteria

The final question we raise here is the following: are the worst-off common to all criteria? In other words, can we raise a unique conclusion about the victims of unfairness regardless of the criteria of inequality of opportunity and the cardinalization we use?

\[\text{Equation}\]

\[\text{Equation}\]

\[\text{Equation}\]

Since the information derived from the $\epsilon$ is only provided in regions around or above 20 hours per week, no additional information is provided for the unobserved heterogeneity when the working hours are close to zero. The equality of "with" and "without" columns is therefore no surprise.

Though this is out of the scope of our analysis, we explored the sensibility of the cardinalization to inclusions of other circumstances such as gender and ethnicity and found identifications changes in the worst-off identifications.
To this end, we take one particular version of each criterion. We compare the min egalitarian equivalence with the conditional equality with the average $\beta_L$, with Van de gaer’s and Roemer’s criteria with a cardinalized outcome following the conditional equality approach based on the average $\beta_L$. Comparisons based on other variants of the criteria do not change drastically. We report in Figure 12 the socio-demographic characteristics of each criterion and in Figure 13 the percentage of individuals who belong to multiple criteria simultaneously.

We first note that 82% of the worst-off are worst-off according to the 4 criteria. As a consequence, 18% of the individuals are worst-off through 1, 2 or 3 criteria only. We secondly note that 6% of the worst-off are worst-off according to the min egalitarian equivalence criterion only. This is half a surprise since the 3 other criterion are derived from a same conditional equality approach (through cardinalization for Roemer’s and Van de gaer’s criteria). A similar analysis with zero wage egalitarian equivalence increases the share of criterion specific worst-off (not reported here). We also note that Roemer and Van de gaer criteria identify identically the worst-off. Again, this comes from the choice made by Fleurbaey and Maniquet to define wages as circumstances. Since wages directly determine the budget set, a larger wage necessarily makes individuals better-off. Notwithstanding its limitations, this exercise is instructive in order to evidence the impact of the normative choice we make when enabling interpersonal comparisons. The fact that about 18% of the individuals are not the worst-off across all the criteria shows that the hypothesis that allows us to make interpersonal comparisons has an impact on the identification of the proper target of any redistribution policy aiming at reducing unfairness.

6 Conclusion

This paper proposes an empirical application of distinct fairness criteria. Our contribution is twofold: Firstly, we define a model that allows us to better

\footnote{Due to the linearisation of the budget sets, there could be a case of worst-off differences across Roemer’s and Van de gaer’s criteria, as illustrated in Figure 5, but this is not observed in our sample. In general, Roemer’s and Van de gaer’s worst-off should no longer be the same if we choose other variables to define the circumstances (socio-demographic ones for example).}
approximate individuals’ preferences such as to apply the criteria of conditional equality and egalitarian equivalence. Secondly, we propose to compare these criteria with the criterion proposed by Roemer and the one proposed by Van de gaer. In this way, we propose different approaches for translating the ordinal model into a cardinal model. Each method derives from the normative choices raised by the criteria of conditional equality and egalitarian equivalence in order to make the comparisons as reliable as possible.

We apply our model to a sample of singles from the CNEF dataset and identify who are the worst-off according to each criterion. We find that our model makes possible refining groups’ preferences to better approximate individuals’ preferences. Then, we find that 18% of the individuals are not commonly identified as worst-off across the criteria.

We conclude that (1) our model for identifying individuals’ preferences allows to follow an empirical approach closer to the spirit of the Fleurbaey and Maniquet measure of equality of opportunity, and (2) the discrepancies in the identification of the worst-off across criteria and their variants shows how important is to make explicit the normative assumptions on which rely each criterion.

In terms of research agenda, we believe that studying the comparability of criteria is a promising future area of exploration. Two combined challenges are to determine how the definition of circumstances affects the comparability and to measure the impact of the cardinalization approaches, that we proposed in this paper, to make comparable the Roemer’s and Van de gaer’s criteria.

References


## Tables and Figures

Table 1: Descriptive statistics on heterogeneity variables

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Share</th>
<th>working hours per week</th>
<th>mean annual net wage ($)</th>
</tr>
</thead>
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<td>Less than high school</td>
<td>8.4%</td>
<td>42.9</td>
<td>21,205</td>
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<td>Education: high school</td>
<td>34.7%</td>
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<td>26,187</td>
</tr>
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<td>Education: more than high school</td>
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Table 2: Parameters of the utility function

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<th>(4)</th>
<th>(5)</th>
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Standard errors in brackets.
* significant at 10%; ** significant at 5%; *** significant at 1%.

Specification (1) is the benchmark case. Workers earning less than 5 USD per hour are excluded from specification (2). Workers earning less than 5 USD per hour or more than 100 USD per hour are excluded from specification (3). Specifications (4), (5) and (6) are based on the benchmark case but vary on the regressors of $\beta_l$, the preference for leisure. $\bar{\beta}_l$ is the sample average of $\beta_l$. Less than high school and High school are dummy variables with More than high school as reference. Black and Other are dummy variables with White as reference value.
Table 3: Percent of worst-off commonly identified by models considering, and respectively not considering, the unobserved heterogeneity

<table>
<thead>
<tr>
<th>Conditional Equality</th>
<th>( \beta_L )</th>
<th>95.6%</th>
<th>( \beta_L = 20 )</th>
<th>91.2%</th>
<th>( \beta_L = 70 )</th>
<th>82.4%</th>
<th>( \beta_L = 100 )</th>
<th>82.4%</th>
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<tr>
<td>Egalitarian Equivalence</td>
<td>Zero wage</td>
<td>100.0%</td>
<td>Min wage</td>
<td>91.2%</td>
<td>Wage</td>
<td>93.4%</td>
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Table 4: Conditional Equality: percent of worst-off per social category - comparison of estimates considering, and not considering, the unobserved heterogeneity

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<th>CE ( \beta_L )</th>
<th>CE ( \beta_L = 20 )</th>
<th>CE ( \beta_L = 70 )</th>
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<td>With</td>
<td>Without</td>
<td>With</td>
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<td>With</td>
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<td>Women</td>
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<td>6.4%</td>
<td>7.0%</td>
<td>6.4%</td>
</tr>
<tr>
<td>Other</td>
<td>9.6%</td>
<td>9.6%</td>
<td>9.6%</td>
<td>9.6%</td>
</tr>
<tr>
<td>Less than high school</td>
<td>20.8%</td>
<td>20.8%</td>
<td>20.8%</td>
<td>19.5%</td>
</tr>
<tr>
<td>High school</td>
<td>14.5%</td>
<td>14.5%</td>
<td>14.5%</td>
<td>15.5%</td>
</tr>
<tr>
<td>More than high school</td>
<td>5.6%</td>
<td>5.6%</td>
<td>5.6%</td>
<td>5.2%</td>
</tr>
</tbody>
</table>

The "with" and "without" headers refer to the approaches where the unobserved heterogeneity is considered, respectively not considered. Information on unobserved heterogeneity is derived from the difference of \( \epsilon \) referred to in Equation 10.
Table 5: Egalitarian Equivalence: percent of worst-off per social category - comparison of estimates considering, and not considering, the unobserved heterogeneity

<table>
<thead>
<tr>
<th>Category</th>
<th>zero EE With</th>
<th>zero EE Without</th>
<th>min EE With</th>
<th>min EE Without</th>
<th>wage EE With</th>
<th>wage EE Without</th>
</tr>
</thead>
<tbody>
<tr>
<td>Women</td>
<td>11.6%</td>
<td>11.6%</td>
<td>9.0%</td>
<td>8.3%</td>
<td>9.2%</td>
<td>9.9%</td>
</tr>
<tr>
<td>Men</td>
<td>8.6%</td>
<td>8.6%</td>
<td>10.8%</td>
<td>11.4%</td>
<td>10.6%</td>
<td>10.0%</td>
</tr>
<tr>
<td>Black</td>
<td>13.3%</td>
<td>13.3%</td>
<td>14.5%</td>
<td>14.7%</td>
<td>14.7%</td>
<td>14.2%</td>
</tr>
<tr>
<td>White</td>
<td>7.8%</td>
<td>7.8%</td>
<td>7.0%</td>
<td>6.8%</td>
<td>6.8%</td>
<td>7.2%</td>
</tr>
<tr>
<td>Other</td>
<td>9.6%</td>
<td>9.6%</td>
<td>9.6%</td>
<td>9.6%</td>
<td>9.6%</td>
<td>9.6%</td>
</tr>
<tr>
<td>Less than high school</td>
<td>19.5%</td>
<td>19.5%</td>
<td>20.8%</td>
<td>18.2%</td>
<td>19.5%</td>
<td>19.5%</td>
</tr>
<tr>
<td>High school</td>
<td>14.2%</td>
<td>14.2%</td>
<td>13.9%</td>
<td>15.1%</td>
<td>14.5%</td>
<td>13.6%</td>
</tr>
<tr>
<td>More than high school</td>
<td>6.0%</td>
<td>6.0%</td>
<td>6.0%</td>
<td>5.6%</td>
<td>5.8%</td>
<td>6.4%</td>
</tr>
</tbody>
</table>

The "with" and "without" headers refer to the approaches where the unobserved heterogeneity is considered, respectively not considered. Information on unobserved heterogeneity is derived from the difference of $\epsilon$ referred to in Equation 10.

Table 6: Ex-post Roemer and ex-ante Van de gaer criteria: characteristics of the worst-off

<table>
<thead>
<tr>
<th>Category</th>
<th>Roemer</th>
<th>Van de gaer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Women</td>
<td>9.5%</td>
<td>9.5%</td>
</tr>
<tr>
<td>Men</td>
<td>10.6%</td>
<td>10.6%</td>
</tr>
<tr>
<td>Black</td>
<td>14.7%</td>
<td>14.7%</td>
</tr>
<tr>
<td>White</td>
<td>7.0%</td>
<td>7.0%</td>
</tr>
<tr>
<td>Other</td>
<td>9.6%</td>
<td>9.6%</td>
</tr>
<tr>
<td>Less than high school</td>
<td>20.8%</td>
<td>20.8%</td>
</tr>
<tr>
<td>High school</td>
<td>14.5%</td>
<td>14.5%</td>
</tr>
<tr>
<td>More than high school</td>
<td>5.8%</td>
<td>5.8%</td>
</tr>
</tbody>
</table>

Percent of worst-off per category. Results constant through the different cardinalizations.
Figure 1: Heterogeneous preferences

Figure 2: Inferring unobserved preferences
Figure 3: A simplified (linear) budget set

Figure 4: Conditional Equality (a)
Figure 5: Conditional Equality (b)

**Conditional Equality** — a case where the individual with the lowest utility (the worst-off) is the one with the highest wage rate (John).

Figure 6: Zero-wage egalitarian equivalence

**Zero-wage egalitarian equivalence** — the minimal lump-sum tax required to help John keep his utility is larger than Mary’s one. She is the worst-off.
Figure 7: Minimum-wage egalitarian equivalence

Minimum-wage egalitarian equivalence – the minimal lump-sum tax required to help John keep his utility is larger than Mary’s one. She is the worst-off.

Figure 8: Wage egalitarian equivalence

Wage egalitarian equivalence – the minimum wage required to help Mary keep her utility in a no-tax context is larger than John’s one. He is the worst-off.
Figure 9: Roemer ex-post approach

Roemer criterion – Conditional on effort (approximated by quantiles according to the RIA principle), the worst-off are those with the lowest utility across circumstances (types)

Figure 10: Van de gaer ex-ante approach

Van de gaer criterion – Conditional on circumstances (wages), the group of the worst-off is the one with the lowest average (homogenized via CE) utility
Figure 13: The worst-off according to each criterion