Damage assessment of civil engineering structures by the observation of non-linear dynamic behaviour

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ABSTRACT: The detection of damages in civil engineering structures and bridges in particular is mainly done by visual examination. However, defects as for instance partial rupture of a prestressing cable or fatigue cracks in reinforcement can not be visually observed. It is well known that damage changes dynamic structural parameters like eigenfrequencies, eigenmodes and damping. However, the sensitivity to small damages is sometimes low. Therefore, as an alternative the occurrence and evaluation of non-linear dynamic behaviour is considered. The basic idea is that non-linear dynamic effects increase with growing cracks under forced excitation. The implementation of this idea in the regular inspection program of bridges presupposes exact knowledge of the eigenfrequencies of the undamaged structure that are also supposed to be force dependent. This paper presents the results of an experimental approach with three reinforced concrete beams of different damage states investigating the non-linear behaviour due to the excitation force.

1 INTRODUCTION

In many technical applications the dynamic behaviour of structures can be characterized by a linear model. When adapting a peal of bells to the natural frequency of a bell tower for example, the tower is regarded as a SDOF system (single degree of freedom) with constant natural frequency. This model is supposed to fulfill the requirements of a linear SDOF system for a sufficient distance between excitation frequency (peal of bells) and the natural frequency of the tower. As well, the strings of a piano are regarded as linear approximately, because the nonlinearities have only a very small part in the oscillation so that they are not audible.

Whenever nonlinear effects appear more strongly, a linear description of the system is no longer sufficient. These nonlinear effects are often not wanted, since the analysis of measurement data distorted by nonlinearities is quite difficult. However, in many cases nonlinearities can be a benefit for technical applications. An example is the friction damping through friction elements to turbine blades or in interstices of machine tools. Nonlinearities in stiffness and in damping, which are based on structural properties, can be used to identify the cause of their appearance e.g. cracks or damage due to corrosion. Often, a combination of different types of nonlinearities makes analyses difficult. In case of damage in reinforced concrete structures the following examples can be mentioned: friction in cracks, change in stiffness due to the alternately opening and closing of cracks under dynamic excitation or amplitude dependent material behaviour.

2 THE BASICS OF NON-LINEARITY

Assuming linear behaviour of a harmonically excited structure, the differential equation (1) describes the vibration adequately, where *M* is the mass matrix, *C* the damping matrix, *K* the stiffness matrix and *F* the excitation force.

$$M\ddot{y}(t) + C\dot{y}(t) + Ky(t) = F(t) \tag{1}$$

Disturbances in the structure like cracks lead to non-linear stiffness- and damping matrices. In this case the coefficients in the equation of motion are dependent on the vibration amplitude, velocity and thus on the excitation force.

Worden et al. (2001) and *Dimitriadis* et al. (2006) present an overview of the most studied types of nonlinearities. Much research is driven by the aerospace sector. The methodology is applicable to other domains of engineering. A distinction has to be made between nonlinearities depending on displacement and nonlinearities depending on velocity. Figure 1 shows common nonlinearities and their dependence.

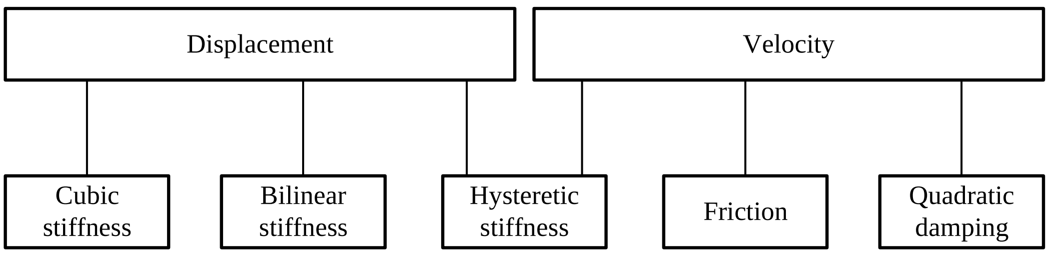

Figure 1. Common nonlinearities and their dependence on displacement and velocity

There is a large amount of different nonlinearities discussed in literature. The above-mentioned nonlinearities are representative for many different types occurring in vibratory systems. They allow a theoretical and approximate description of non-linear behaviour. Nonlinearities may occur, for which a classification according to the types in figure 1 is not possible. In this case a description can be based on a combination of nonlinearities or on a nonlinear material behaviour.

3 HOW TO DETECT NON-LINEARITIES IN MEASUREMENTS

3.1 *Is the systems vibration non-linear?*

For the investigation of nonlinear dynamic behaviour it is initially essential to discover, whether the structure behaves nonlinear at all. For this purpose four elementary operations are presented.

For linear systems the principle of superposition is valid. This principal applies both to static systems and to dynamic systems. It reads as follows: The motions of a body proceeding at the same time do not affect each other mutually. The resulting quantities (displacement, velocity, acceleration) arise from a geometrical addition of the components.

$$y_1(t) \rightarrow x_1(t); y_2(t) \rightarrow x_2(t) \Rightarrow y_1(t) + y_2(t) \rightarrow x_1(t) + x_2(t) \tag{2}$$

Superposition is independent of the type of excitation. If this principle does not apply, nonlinear dynamical behaviour exists.

As well the distortion (harmonic distortion) of the response vibration, excited by a sinusoidal excitation force, is an explicit indicator for the existence of nonlinearity. In the case of linear vibration the ratio of the amplitudes of the input (*X(ω)*) and output (*Y(ω)*) signals is always constant. Hence the frequency response function (FRF) is independent of the excitation force level and can be expressed as:

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{\alpha Y(\omega)}{\alpha X(\omega)} \tag{3}$$

If the FRF does not comply with equation (3), it is possible to establish a connection between the input signal and the intensity of the nonlinearity when exciting the structure with excitation forces of different amplitude. The characteristic distortion of the FRF (FRF distortion) in comparison with the 'linear FRF' could give information about the type of nonlinearity.

An additional characteristic of linear systems is reciprocity. Reciprocity holds if an output Y_B at a point B due to an input X_A at a point A gives a ratio, that is numerical equal to that when the input and output points are reversed.

$$H_{AB}(\omega) = \frac{Y_B(\omega)}{X_A(\omega)} = \frac{Y_A(\omega)}{X_B(\omega)} = H_{BA}(\omega) \tag{4}$$

3.2 *How to detect non-linearity*

In order to construct more exact descriptions and characteristics of the nonlinear behaviour like the intensity and the type of nonlinearity, the application of several methods and the comparison of the respective results is advisable. There are a lot of practical methods presented in literature.

In general, two different methods to analyze nonlinear behaviour can be distinguished: the nonlinear time domain method (NL-TDM) and the nonlinear frequency domain method (NL-FDM). Within the NL-TDM a structure is excited sinusoidal near or at the natural frequency. When turning off the excitation, the systems vibration will die away. The frequency and damping is now observed during this process and a time dependent relationship of the modal parameters can be established. Using the NL-FDM the structure is excited with a swept sine or stepped sine signal over a certain frequency range (also with different excitation forces) encompassing the resonance frequency of interest and the vibration response is analyzed. An excitation dependent relationship will be observed.

Van Den Abeele & *De Visscher* (2000) discuss both methods in their research work. A reinforced concrete beam is damaged to failure in five steps. After each load step the modal parameters are measured. They conclude, that using either method, the increase in nonlinearity is the most sensitive indicator of cumulative microdamage. It is also shown, that it is possible to identify and detect damage in the midsection of the beam with the help of modal curvature. Therefore it is advantageous to have a dense measurement grid.

Tomlinson (1986) illustrates in his work the principal procedure of the identification of nonlinearities. He suggests among other things the use of the principles of superposition and reciprocity. To be able to detect different types of nonlinearities, he describes the characteristic distortions of the frequency response functions in the Nyquist plot compared to the linear case. For that the deviation of the frequency isochrones, being a circle in linear case, can be used.

In *Wordon* et al. (2001) the coherence function and especially the different display formats of the frequency response function are considered to be very useful to detect and identify nonlinearities. The already mentioned Nyquist plot and the inverse frequency response function (IFRF) are among the different graphic representations of the frequency response function. With the help of the IFRF it is possible to distinguish between stiffness- and damping nonlinearities. As another appropriate method the Hilbert transform is discussed in detail.

The Hilbert transform is also treated in the work of *Bruns* (2004). Different nonlinearities and their characteristic effects on the Hilbert transform are shown.

Gloth et al. (2002) analyze the so-called impedance plots as results of measurements applied to big aerospace structures. The impedance plots show the natural frequencies and the deflections of the structure in dependence on the excitation force.

Dimitriadis (2006) uses the occurrence of higher harmonics in the response spectrum, depending also on the excitation force. By means of a model he simulates different types of nonlinearities and analysis in each case the characteristic of the higher harmonics. User-friendly and applicatory "expert rules" are developed to describe the characteristic influences of diverse types of nonlinearities on the measured dynamic responses.

4 REINFORCED CONCRETE UNDER DYNAMIC EXCITATION

Concrete is a heterogeneous construction material. It consists of visco-elastic cement mortar and embedded aggregates like crushed stone. In uncracked state, the strain of the concrete and the reinforcement of the beam are equal. In cracked state, relative displacements between the concrete and the reinforcing bars occurs. The level of cracks has an important influence on the damping of the structure. *Büttner* (1992) examined the characteristic damping of reinforced concrete and obtained some reference values. This characteristic is shown in figure 2.

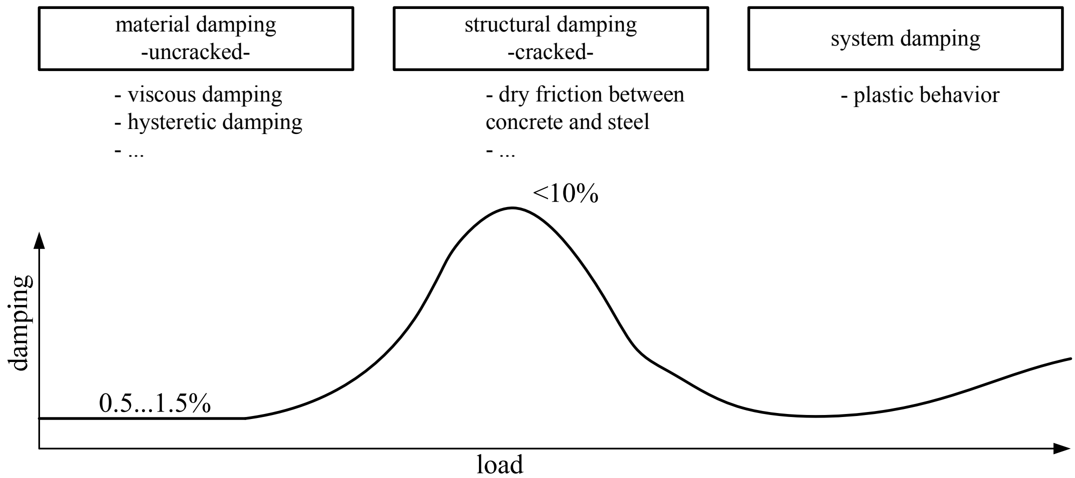

Figure 2. Damping of concrete in principle

In uncracked state, the damping behaviour is defined by the properties of the material. This kind of damping is called material damping. It is possible to describe the damping behaviour of concrete with the model of viscous damping. When applying a cyclic load, the occurrence of small nonlinear effects in the assumed linear material behaviour leads to an elliptic loop in the stress-strain diagram (hysteretic curve). Hysteretic behaviour results into damping effects. The stiffness is not influenced by hysteretic behaviour for small excitation forces. For larger values of the excitation force, the nonlinearity becomes more distinct and finally influences also the stiffness and consequently the natural frequencies of the system.

In cracked state, the effects in the cracks due to friction between the concrete and reinforcing bars have the biggest influence on the damping characteristic. This type of damping is called structural damping. Damping increases when the system passes into cracked state and takes the highest value in full cracked state. Additionally the damping value in cracked state is significantly dependent on the level of excitation force. While the damping value initially increases with small dynamic loads (the cracks are still closed - shear bond), it decreases with high values of dynamic loads (cracks are open – Coulomb friction). This behaviour is conform to the relationship between bond stress and the relative displacement shown in figure 3.

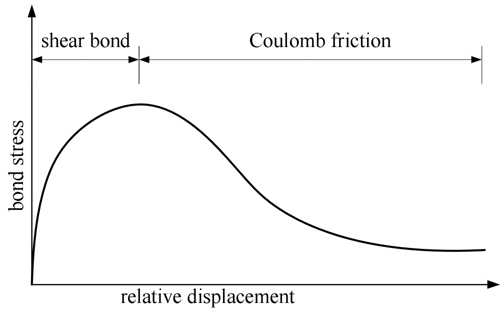

Figure 3. Bond stress vs. relative displacement of concrete and reinforcement

The behaviour of cracked reinforced concrete under forced excitation can easily be described by means of a stick-slip-model. *Magnevall* et al. (2006) describes the dynamic behaviour of a SDOF system considering a mass with a spring (constant stiffness) and a damping element (constant damping) connected in parallel. A further hysteretic element with nonlinear characteristic line is also connected in parallel. From a particular excitation force, the nonlinear influence breaks out. Up to a certain excitation force, shear bond is ruling. Afterwards shear bond is surmounted and the mechanism exchanges into friction. In case of sinusoidal excitation a permanent alteration between stick and slip condition occurs (stick-slip-condition). In case of further increasing excitation force the system is completely in slip-condition. Thus it is possible to determine approximately two different 'linear' natural frequencies for such a system. First, the natural frequency for low excitation force and stick-condition and secondly, the natural frequency for high excitation forces and slip-condition.

5 EXPERIMENT

5.1 *The beams*

Three reinforced concrete beams with different damage levels are examined concerning their non-linear dynamical behaviour. 6 m long beams are made of concrete C40/50 and six reinforcement bars of diameter 16 mm are equally distributed over the tension and the compression zone. Figure 4 shows the three beams.

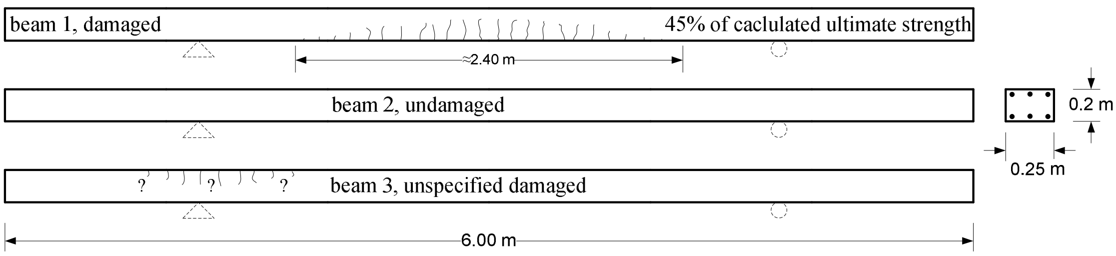

Figure 4. Three examined beams

Beam 1 has been loaded up to 45% of its calculated strength during a symmetrical three point bending test. Beam 2 can be considered as undamaged. The analysis of the dynamical behaviour of beam 3 (unspecified damaged) should detect a damage in the area of one support during storage.

5.2 *Measurement program*

Initially the natural frequencies are determined using hammer impact. For the investigation described in chapter 5.5 the beams are excited harmonically by means of a swept sine excitation starting with an excitation frequency smaller than the first natural frequency and ending with an excitation frequency higher than the third natural frequency of the beam. In order to transfer as much as possible of the oscillation energy into the nonlinear parts of the oscillation, the beams are excited with a swept sine with a small sweep-rate of 0.3 Hz/s. To avoid additional damage due to the test the excitation force for the undamaged beam 2 and unspecified damaged beam 3 has been restricted to 80 N in the first eigenfrequency. This results into maximum tensile stress of approximately 2,7 N/mm^2, which is smaller than the nominal tensile stress of concrete C40/50.

5.3 *Measurement setup*

Figure 5 illustrates the beam in a free-free set-up. The shaker excites the beam via a force sensor. The dynamic responses are measured with acceleration sensors at three points on the beam.

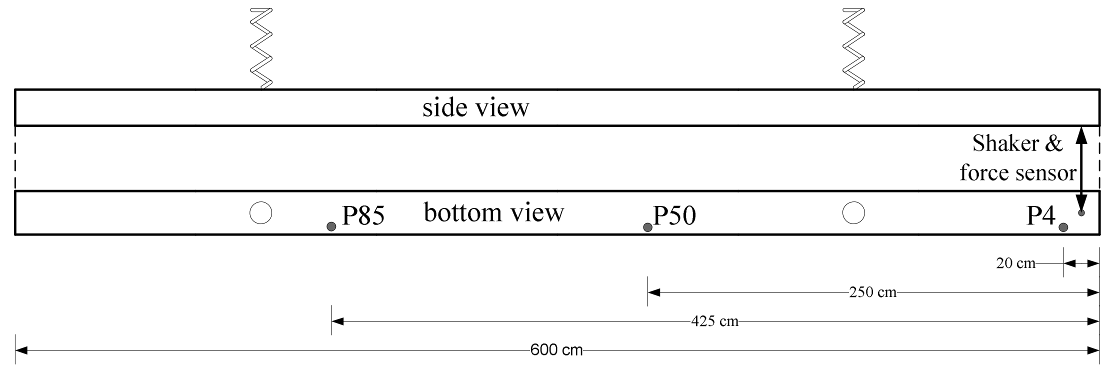

Figure 5. Measurement set-up, measurement points P4, P50, P85, Shaker & force sensor

5.4 *Hammer-impact measurement*

Initially the natural frequencies of the three beams are measured using hammer impact. The energy associated with an individual frequency is small and nonlinear effects do not appear clearly. So only the quasi-linear dynamic properties of the system can be observed, when using the hammer impact method. Table 1 figures the first eight natural frequencies of the three beams.

Table 1. Eigenfrequencies [Hz] using hammer-impact

Mode	damaged beam 1	undamaged beam 2	unsp. dam. beam 3
B1	16.6	22.8	21.7
B2	52.2	63.1	57.2
B3	106	123	113
T1	165	194	187
B4	172	201	191
B5	250	295	283
T2	350	391	375
B6	361	407	390

5.5 *Forced excitation – frequency response function (FRF)*

In case of linear systems the FRF is independent of the excitation force and takes constant values when exciting the structure with different force levels. However, as already mentioned in chapter 3.1, the stiffness and damping values are dependent on the excitation force for nonlinear systems. Therefore, observing nonlinear systems necessitates a comparison between the parameters of the system under different excitation forces. Figures 6,7 and 8 show the frequency response functions for the first three modes of the beams.

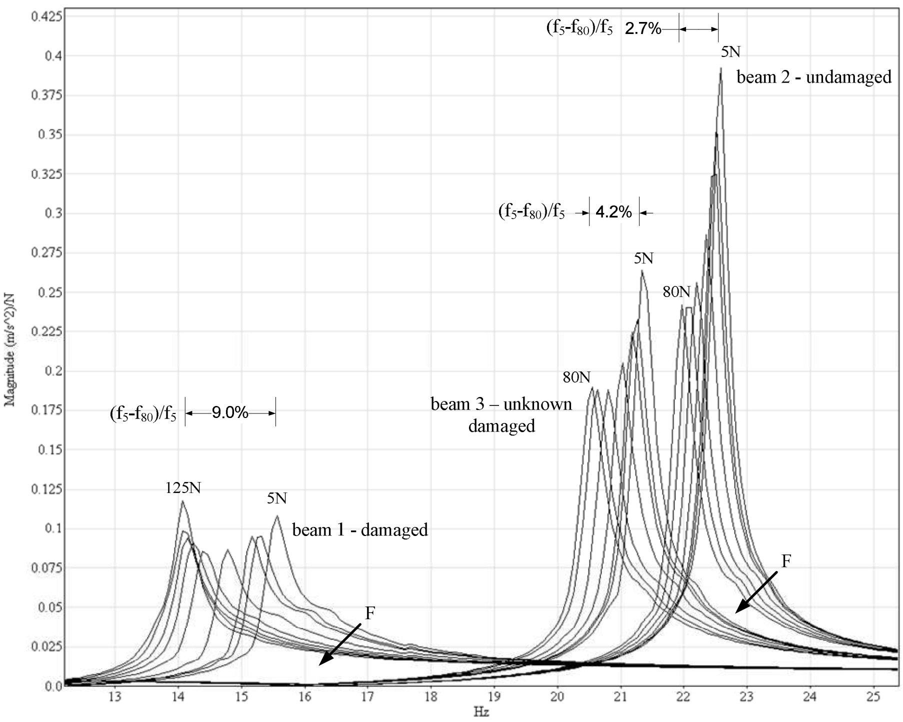

Figure 6. FRF, mode 1 different force level, measurement point 4

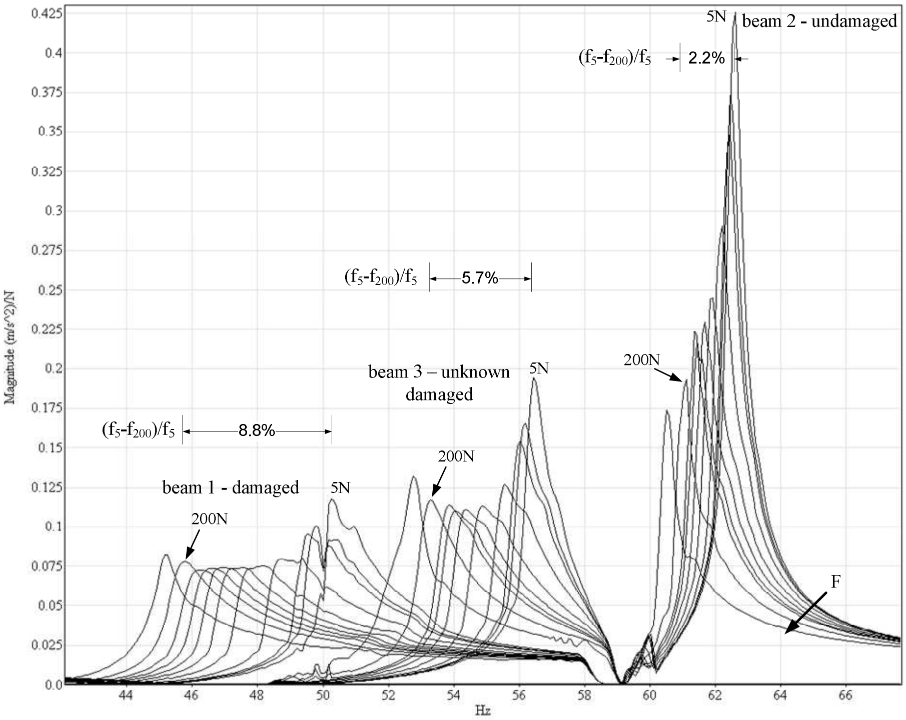

Figure 7. FRF, mode 2 different force level, measurement point 4

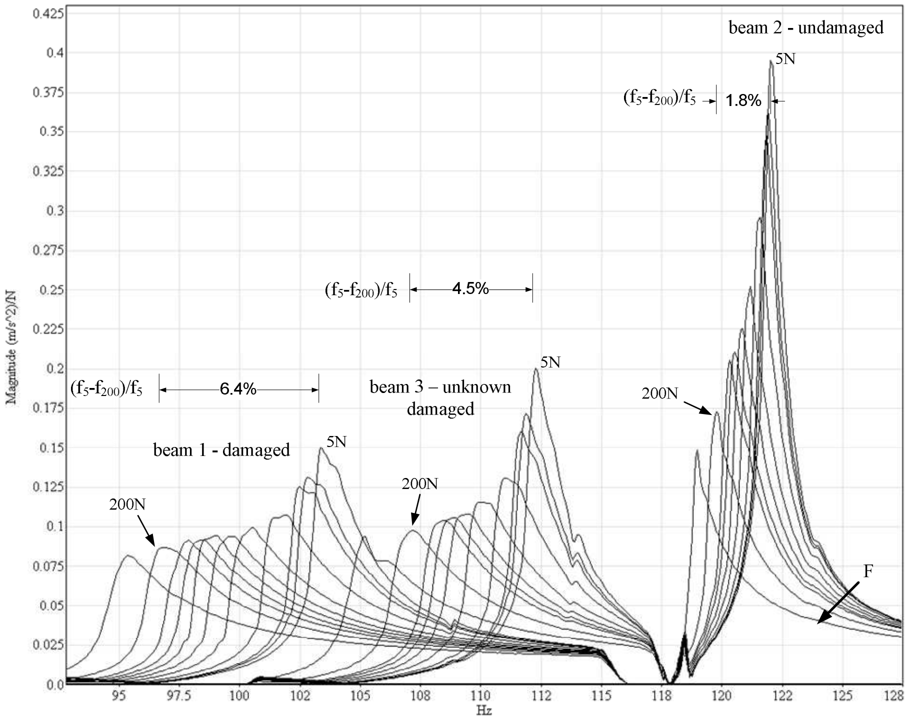

Figure 8. FRF, mode 3 different force level, measurement point 4

For the undamaged beam 2 both the natural frequencies and the amplitudes of the FRF decrease with increasing excitation force. This property can be explained by means of the strongly dependent hysteretic material behaviour of concrete, probably due to inevitable microcracks.

Comparing the behaviour of the three beams in the first mode, it has to be noted, that beam 3 and beam 2 indicate qualitative analogue behaviour. Differences in the amplitudes of the FRFs are due to varying material behaviour. The properties of the damaged beam 1 show a clear difference. It is obvious that the frequency decreases up to a certain excitation force more than the frequencies of the undamaged beam 2. At higher excitation forces there is a remarkable increase of amplitudes. According to the excitation force the cracks are opened and thus the stiffness is reduced. This results in an intense decrease of the eigenfrequencies with increasing excitation force. Due to the bond between concrete and reinforcement (a hysteretic behaviour) the damping ratio increases and according to this the amplitude of the FRF decreases. In case of low excitation forces there is only shear bond (static friction, stick-condition); in case of higher force values bond changes between static and dynamic friction (permanent change between stick and slip condition) and thus increasing damping. In this state the resonance frequency is still decreased. Exciting the beam with higher excitation forces, the bond is just ensured by dynamic friction (slip-condition). In this state the stiffness, so the eigenfrequency is no longer influenced and keeps a constant value. As the friction force still slightly decreases with increasing excitation force, the damping ratio is also decreasing. So the FRF amplitude increases again. The described behaviour can be regarded as an indicator for a typical damage in RC structures.

The described effect can also been observed when investigating the FRF of the second mode of the unspecified damaged beam 3. This might be an indicator for a damage concerning mode 2.

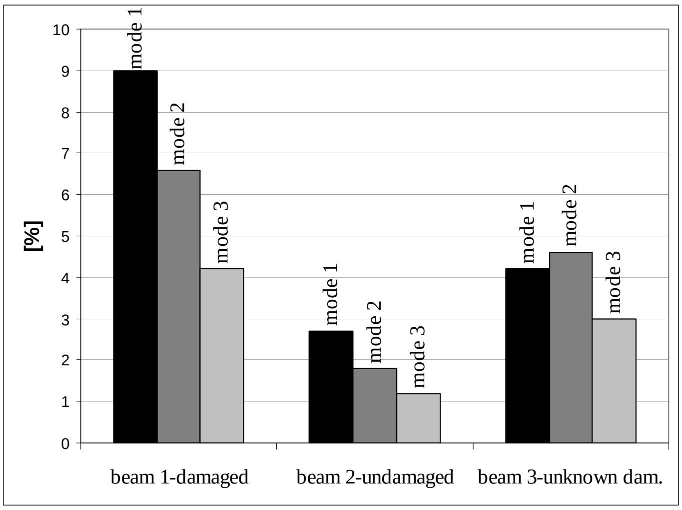

Figure 9. Decrease of the first three eigenfrequencies from 5N to 80N

Figure 9 contains the percentage decrease of the resonance frequencies from an excitation force from 5 N to 80 N. The most significant decrease can be denoted for the first mode of the damaged beam 1. The natural frequencies of the undamaged beam 2 show comparatively small changes. Likewise the first mode is influenced the most. As already mentioned this is due to the strong distortion dependent hysteretic material behaviour. As the deflection of mode 1 is always greater than the deflections of mode 2 and 3, the influence on the first mode is more significant. This is different when inspecting the unspecified damaged beam 3. Here the decrease of the second natural frequency (f_2) is dominant. The decrease of f_1 is similar to f_1 of the undamaged beam 2. f_3 of beam 3 tends to the behaviour of f_3 of the damaged beam 1.

Dimitriadis (2006) illustrates in his expert rules the influence of different types of nonlinearities on the natural frequencies and the FRFs in general. The rules read as follows:

- (a) If the frequencies of the modes vary consistently with excitation level then the nonlinearity is likely to lie in the stiffness.
- (b) If the frequencies of the modes are completely unaffected by the excitation level and only their amplitudes change then the nonlinearity is polynomial damping (e.g. quadratic damping).
- (c) If the impact of the nonlinearity decreases continuously with excitation amplitude then the nonlinearity is friction.

Rule ‘a’ can be applied to the frequency dependent behaviour of f_1 of beam 1 up to an excitation force of 80 N. In this range there is a strong dependence due to a stiffness nonlinearity. For excitation forces greater than 80 N this dependence does no longer exist and the natural frequency keeps a constant value with increasing excitation force (rule ‘b’). Furthermore the influence of the nonlinearity on the dynamic behaviour decreases (rule ‘c’), because the FRF amplitudes begin to increase again, despite increasing excitation force. Thus it is a matter of a damping-related nonlinearity.

5.6 *Result*

Fitting the above-mentioned results into the big picture, the state of the three beams can easily be assessed. The investigation of the dependence of the natural frequencies on the excitation force has been very useful, because it showed a strong nonlinear behaviour especially for the damaged beam 1 and for mode 2 of the unspecified damaged beam 3. There is nonlinear behaviour of beam 2 (undamaged) as well, but this is due to hysteretic material behaviour. As already mentioned in the introduction, beam 3 is supposed to be unspecified damaged. With the help of the different methods explained above, it can be assumed, that beam 3 is damaged at the zone of big curvatures of modes 2 and 3. The result of a visual inspection of the beam is presented in figure 10.

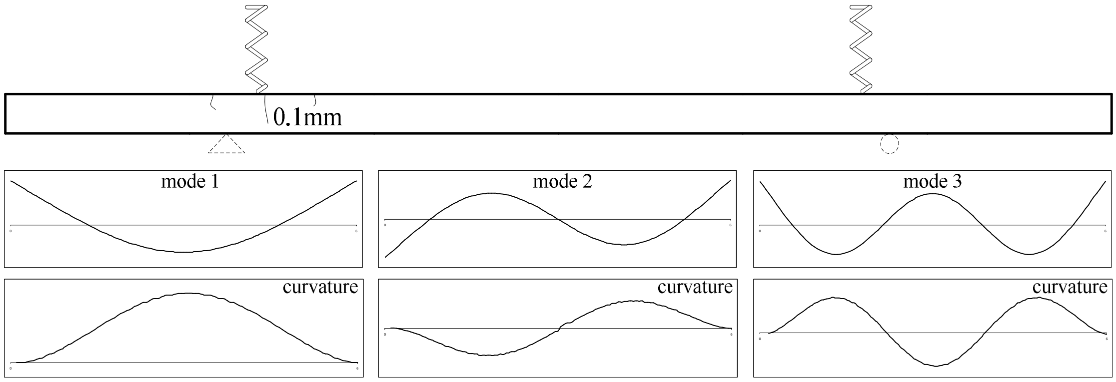

Figure 10. Predicted damage, beam 3; modeshapes and modal curvatures

There is a clear crack near to the supports. It is possibly due to unscheduled loads during storage. Cracks with a width of 0.1 mm in bridges are already problematic concerning corrosion of reinforcement or prestressing cable. There are a few more cracks visible, cause of insufficient reinforcement for shear.

6 CONCLUSION AND PROSPECT

Using non-linear vibration analysis for detecting damage in civil engineering structures is still in the early stages. The presented experimental investigation showed promising results. There is an obvious relationship between excitation force and eigenfrequencies. For the undamaged beam the decrease of the eigenfrequencies with increasing excitation force is small but not negligible. This and the change of damping is due to non-linear material behaviour. The damaged beam showed a very noticeable relationship between modal parameters and excitation force. The described stick-slip effect can be used as an indicator for damaged reinforced concrete. By means of the most affected modes, first appraisal of damage location is possible.

As the force dependent non-linear effects seem to be a sensitive indicator for damage it has to be determined whether these effects can be used as well in practice. It will be further investigated if this approach can be implemented in the regular inspection program of bridges to document their actual force dependent behaviour and thus, their actual structural state.

REFERENCES

Akesson, H., Sällberg, B.: Identification and Analysis of Nonlinear Systems, Department of Telecommunication and Signal Processing, December, 2003

Bruns, J.-U.: Detektion und Identifikation von Nichtlinearitäten in mechanischen Schwingungssytemen. Dissertation, Fachbereich Maschinenbau, Universität Hannover, 2004

Büttner, A. Beitrag zur Beschreibung des Dämpfungsverhaltens von Stahlbetonbalken, Fakultät Bauingenieurwesen, Universität Weimar, 1992

Dimitriadis, G.; Vio, G.A.: Nonlinearity Characterization for Nonlinear Dynamic System Identification Using an Expert Approach, *Proceedings of ISMA 2006*, Leuven, Belgium, 2006

Gloth, G.; Sinapius, M.: Swept-Sine Excitation During Modal Identification of Large Aerospace Structures, Deutsches Zentrum für Luft- und Raumfahrt, Forschungsbericht 2002-18, 2002

Magnevall, M.; Josefsson, A.; Ahlin, K.: On Nonlinear Parameter Estimation, *Proceedings of ISMA 2006*, Leuven, Belgium, 2006

Neild, S.A.: Using Non-Linear Vibration Techniques to Detect Damage in Concrete Bridges, Department of Engineering Science, University of Oxford, July, 2001

Tomlinson, G.R.: Detection, Identification and Quantification of Nonlinearity in Modal Analysis - A Review, *Proceedings of the 4th International Modal Analysis Conference (IMAC)*, Los Angeles, USA, 1986

Van Den Abeele, K.; De Visscher, J.: Damage assessment in reinforced concrete using spectral and temporal nonlinear vibration techniques, *Cement and Concrete Research 30*, p. 1453-1464, 2000

Wordon, K.; Tomlinson, G.R.: *Nonlinearity in Structural Dynamics*. IOP Publishing Ltd., London, 2001